IB Math SL Exam Study Sheet

Written by Melissa Yuce and Morgan Bowling. Edited by Mr. Gowdy

Not all formulas and calculator moves written on this sheet are given to you on the formula sheet.
KNOW YOUR FORMULA SHEET WELL!

TOPIC 1 & 2 = Algebra, Functions

Sequences

- All formulas for this chapter are given on the formula sheet
- Arithmetic- addition
- Geometric- multiplication
- *D* is the slope
  For example, if they ask for *d* and all they give you is U_{21}= -37 and U_4= -3, solve using the slope formula: $\frac{-37-(-3)}{21-4}$ = -2 = *d*

Quadratic Equations

$y = ax^2 + bx + c$ $\qquad$ $y = a(x-h)^2 + k$, vertex @ (h, k)

Axis of Symmetry: $x = \frac{b}{2a}$ $\qquad$ $x = h$

To find y-intercept: plug in 0 for x

To find x-intercept: plug in 0 for y

Exponential Growth and Decay

$A = P(1+r)^t$ Basic Growth, Decay or Yearly compound interest:

A = Current or final amount $\qquad$ P = Principal or starting amount

r = interest rate $\qquad$ t = time (generally in years – units must match interest rate)

$A = P\left(1+\frac{r}{n}\right)^{nt}$ Compound interest multiple times per year (= # of times)

$A = Pe^{rt} = Ce^{rt}$ Continuous compounding, or continuous growth, decay

Inverse Functions

$f^{-1}(x)$ = the inverse of f(x)

$y = e^x \overset{inverse\ functions}{\Longleftrightarrow} y = \ln(x)$ $\qquad$ $y = x^2 \overset{inverse\ functions}{\Longleftrightarrow} y = \sqrt{x}$

Logs

- All laws of logs are given on the formula sheet
- If there is a log problem and you are having trouble with it, most likely you need to use one of the log laws given on the sheet to solve it
- e and ln are inverses (IB loves to ask questions to see if you know that)

$a = b^x \overset{equilvalent\ functions}{\Longleftrightarrow} y = \log_b a$ $\qquad$ $a = e^x \overset{equivalent\ functions}{\Longleftrightarrow} x = \ln(a) = \log_e a$

$\log A + \log B = \log(AB)$

$\log A - \log B = \log \frac{A}{B}$

$\log a^n = n \log a$

Binomial Expansion

$(a+b)^n = \binom{n}{r}(a)^r(b)^{n-r} = \_a^n b^0 + \_a^{n-1}b^1 + \_a^{n-2}b^2 + \ldots + \_a^1 b^{n-1} + \_a^0 b^n$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1

Number of terms in the expansion is one more than the exponent given
Ex) $(3+x)^4$ has five terms

Ex) $(2x-3y)^{12}$ find the 4th term → the second exponent will always equal one less that the term number
Set up: $\binom{12}{9}(2x)^9(-3y)^3$ **or** $\binom{12}{3}(2x)^9(-3y)^3$

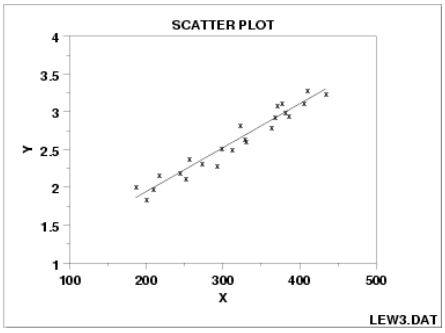
To find the $\binom{n}{r}$ part on the calculator, type *n* first, then go to Math→ PRB→ nCr and then type in *r*

Remember:
1. *n*= Exponent given on top of the entire equation
2. The two exponents in the set up have to add up to *n*
3. *r* can be either of the two exponents in the set up

Linear Regressions

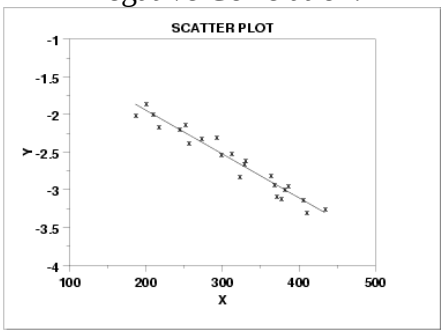
- Diagnostics and stat plot need to be on while graphing
  Turn diagnostics on: 2^{nd}→0→ diagnostics on
  Turn stat plot on: y= → click the stat plot icon (in the upper left hand corner)
- Extrapolation- when the values given stretch outside a given range of data
- Intrapolation- when the values given are inside a range of data
- *r* is the correlation coefficient
  The highest *r* can be is 1 (also means strong positive correlation)
  The lowest *r* can be is -1 (also means strong negative correlation)
  The worst correlation is 0
- To get a linear regression line: Stat→ Calc→ LinReg
- If asked for the mean, don't freak out! Just type in the values in L_1(and L_2 if necessary)
  Then go to Stat→ calc→ 1 var stats (just like we did in the unit with mean and standard deviation)
- Some graphs to recognize:

Positive Correlation:



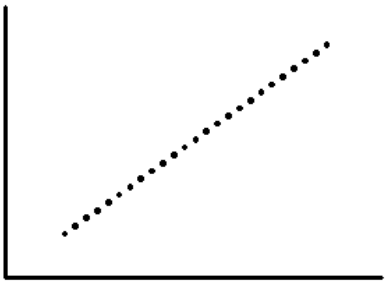
http://www.itl.nist.gov/div898/handbook/eda/section3/eda33q2.htm

Negative Correlation:

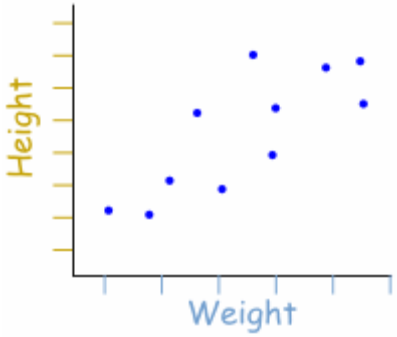


http://www.itl.nist.gov/div898/handbook/eda/section3/eda33q3.htm

Strong Correlation:



Weak Correlation



http://blog.majesticseo.com/research/majesticseo-beginners-guide-to-correlation-part-1/

Topic 3 – Circular Function and Trigonometry

Sine Equations to Model Table of Values

- General sine equation: $y = A\sin B(x - C) + D$
- A= Amplitude which is: $\frac{Max-min}{2}$
- B= $\frac{2\pi}{Period}$
- C= phase shift or shift left/right
- D= Vertical shift and center line: $\frac{Max+Min}{2}$

Trig Intro

- All trig IDs are on the formula sheet
- When in doubt, graph it (if you have a calculator)
- Remember to check your calculator's mode. Do you want radians or degrees?
- Check to make sure that your answer is within the restrictions they gave you, some might not give restrictions
- Signs for each quadrant:

Sine is positive	Always positive
Tangent is positive	Cosine is positive

Topic 3: Circular functions and trigonometry

- Trigonometry

SOHCAHTOA

$\sin\theta = \frac{opp}{hyp}$ $\quad csc\theta = \frac{hyp}{opp}$

$\cos\theta = \frac{adj}{hyp}$ $\quad sec\theta = \frac{hyp}{adj}$

$\tan\theta = \frac{opp}{adj}$ $\quad cot\theta = \frac{adj}{opp}$

$\sin 2\theta = 2\sin\theta\cos\theta$

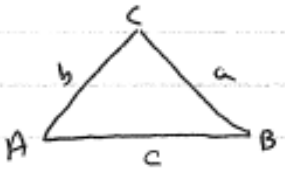
$\cos 2\theta = \cos^2\theta - \sin^2\theta$

$= 2\cos^2\theta - 1$

$= 1 - 2\sin^2\theta$

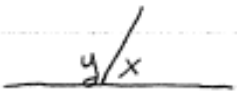
- Sine & Cosine Rules

* calc degree mode



Sine rule

$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$



$\sin x = \sin(180 - x)$

Cosine rule

To find a side:

$a^2 = b^2 + c^2 - 2bc\cos A$

to find an angle:

$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$

Area of triangle

$A = \frac{1}{2}bh$

$= \frac{1}{2}ab\sin C$

- Triangles and sectors

* calc ~~degree~~ radian mode

arc length $l = \theta r$

area of sector $A = \frac{1}{2}\theta r^2$

- Graphs of Sine and Cosine

$y = A\sin B(x - c) + D$

$|A|$ = amplitude → the distance above/below the center line the graph reaches

→ if negative the graph flips

B = period → Period $= \frac{2\pi}{|B|} = \frac{360}{|B|}$

C - phase shift → left (+) / right (−)

D - vertical shift → up (+) / down (−)

sin graph starts at 0

Cos graph starts at 1

Topic 4 - Vectors

Vectors describe a speed or amount of force moving in a certain direction.

- Represented by arrows
  - $\overrightarrow{AB} = -\overrightarrow{BA}$
  - Vector = [end point – starting point]
- Moving in the direction of the arrows on the vector= positive
  Moving opposite to the arrow= negative
- Vectors are named by their endpoints or one lower case letter.
- Magnitude- length of the vector
- Use the slope vector when finding the magnitude
- Position vector- vector to get from the origin to a point
- Vectors are perpendicular if their dot product= 0
- Dot Product: $(x_1 * x_2) + (y_1 * y_2) + (z_1 * z_2)$ *remember to use the direction vectors if using a line*
- If the dot product is negative, then the vectors make an obtuse angle
- Unit Vector- any vector with a magnitude of exactly 1
- Parallel vectors- have the same slope, that means if you can multiply one vector's direction vector by a scalar (k) and get the other vector's direction vector, then they are parallel
- You might get a problem like this: what is the vector with a magnitude of 20 and parallel to $\binom{3}{-4}$?
  1. Find magnitude of original direction vector: $\sqrt{3^2 + (-4)^2} = 5$
  2. $5x = 20$
  3. Scalar = 4
  4. New velocity vector: $4\binom{3}{-4} = \binom{12}{-16}$
- Ways to write equations:
  Parametric: $x = 5 + 2t \quad y = 8 - 5t$
  Unit Vector form: $\vec{r} = \vec{i} + \vec{j} + \vec{k}$
  Cartesian: $y - y_1 = m(x - x_1)$
- XOY plane→ when z=0, what is x and y?
  XOZ plane→ when y=0, what is x and z?
  YOZ plane→ when x=0, what is y and z?
- To find if a point is on the line, set the parametric equation equal to the x and y value, if they have the same t value, then the point is on the line
- They might give you a problem like this: Write the vector equation for a vector that starts at (4,-1), has a speed of 30mph, and is parallel to $\binom{6}{-8}$
  1. Find the magnitude of the given direction vector → $\sqrt{6^2 + (-8)^2}$= 10
  2. Find scalar → $10k = 30 \quad k = 3$
  3. Find velocity vector using k→ $3\binom{6}{-8} = \binom{18}{-24}$
  4. Write new equation→ $\vec{r} = \binom{4}{-1} + t\binom{18}{-24}$

- vector - describe a speed of force in a certain direction
  - represented by arrows
  - $\overrightarrow{AB} = -\overrightarrow{BA}$
  - vector: [end pt - starting pt]
- magnitude - length of vector
  - magnitude = $\sqrt{x^2+y^2}$
- parallel vectors
  - ↳ is there a value of k that can be multiplied to get other vector
- unit vector
  - ↳ any vector with a magnitude (length) of exactly 1
  - ex. $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$
  - to turn any vector into a unit vector
    - find magnitude then divide it through
- scalar product → dot product
  - $= x \cdot x + y \cdot y$
- to find $\angle$ formed by two vectors $\vec{a}$ and $\vec{b}$
  - $\cos\theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| \cdot |\vec{b}|} = \frac{\text{dot product}}{\text{magnitude}}$
- when vectors are $\perp$ their dot product $= 0$
- slope = $\frac{x}{y}$ $\frac{y}{x}$
  - vector = $\frac{x}{y}$
- 3D vectors
  - (x,y,z) $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ front/back $\vec{i}$; right/left $\vec{j}$; up/down $\vec{k}$
  - $\cos = \frac{\text{dot product}}{\text{magnitude}} = \frac{x_1 x_2 + y_1 y_2 + z_1 z_2}{\sqrt{x^2+y^2+z^2} \cdot \sqrt{x^2+y^2+z^2}}$
  - $\perp$ → dot product $= 0$
  - $\parallel$ → is there a common value you can multiply the first by to get second
- Vector equation of a Line
  - ↳ vector from the origin to a point on the line
  - $\vec{r} = \vec{a} + \epsilon \vec{b}$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \vec{r} = \begin{bmatrix} 3 \\ 4 \end{bmatrix} + t\begin{bmatrix} -2 \\ 1 \end{bmatrix} = \begin{bmatrix} 3-2t \\ 4+t \end{bmatrix} \qquad \begin{aligned} x &= 3-2t \\ y &= 4+t \end{aligned}$$

initial position vector — direction vector, velocity vector, slope of line — parametric equations

- if point is not on the line → ts aren't equal
- magnitude of velocity vector = speed of object
- due North $x = 0$
- due east $y = 0$
- mag = distance from origin

↳ when 2 objects are moving:

① to find when they intersect set $x = x$, $y = y$, and solve

② If they don't hit, to find out how close they are use distance formula

$$d = \sqrt{(x-x)^2 + (y-y)^2}$$

③ when one object moves, and one is stationary do dot product = 0 of position vector and direction vector

- on calc → graph it → find minimum x value = value of y value = distance

*either you don't see = 0

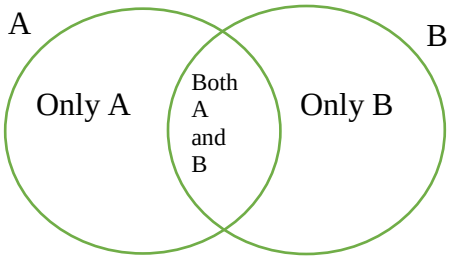
When crosses xOy plane $z = 0$

xOz plane $y = 0$

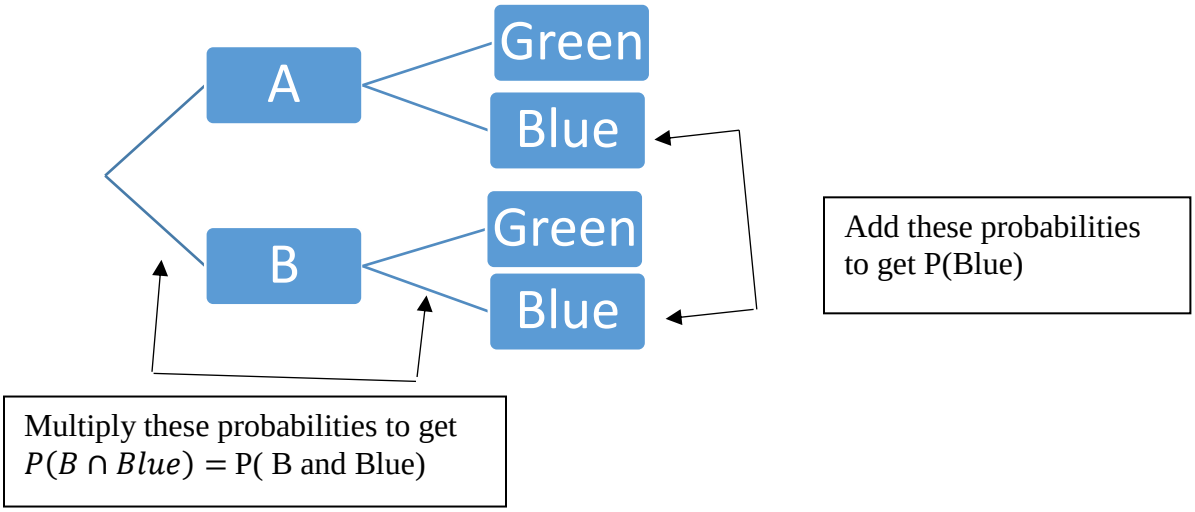
yOz plane $x = 0$

Topic 5 – Statistics and probability

Venn and Tree Diagrams



- All formulas for probability are on formula sheet
- Combined events and conditional probability can be used anywhere
- For tree probabilities:



Discrete Probability Distributions

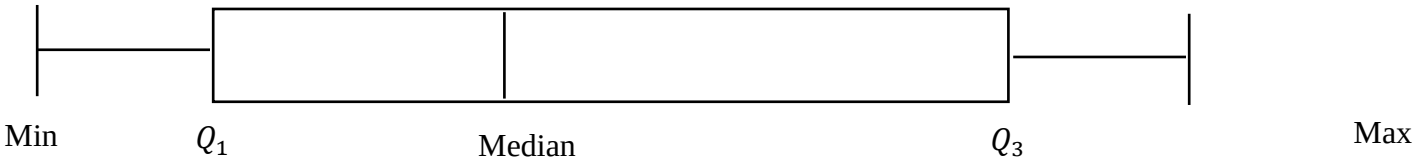
- All probabilities add up to 1
- If asked to find the mean, do each X term times its probability and add all the answers up. Then divide by the number of terms to get the mean.
- E(X)= mean= average result (Formula given on sheet)

Descriptive Stats

- Mean- average
- Median- middle number: $\frac{n+1}{2}$ → Gives you which term # the median is
- Mode- most frequent number
- Remember this is the formula for any probability: $\frac{Part\ you\ want}{Everything}$
- Independent events- don't influence each other
- Conditional probability- probability when one event occurred when another event is known to have occurred

Central Tendency

- Range- Max-min
- Q_1= lower quartile→ which is median of bottom half or lower 25% of the entire thing
- Q_3= upper quartile→ which is the median of the top half or upper 25% of the entire thing
- Q_2= median
- IQR- Inter quartile range→ $Q_3 - Q_1$
- Parts of a box and whiskers plot:



- To determine if something is an outlier: Q_1- 1.5(IQR) or Q_3+ 1.5(IQR)
- Variance- $mean^2$
- Percentile- the % of data less than a certain number

Cumulative Frequency

- If it is easy to point out, the middle of the cumulative frequency graph is the median
- Check to make sure to take your answers from the correct axis, check the axis labels
- Same rules for IQR and Range as above

Finding Standard Deviation by Hand

- Find the mean ($\overline{X}$) first

X	*Frequency*	$X - \overline{X}$	$(X - \overline{X})^2$	*Frequency x* $(X - \overline{X})^2$
				Add all
				Values
				In this
				column

After adding up the last column: $\sqrt{\frac{Sum\ of\ last\ column}{\#\ of\ terms}}$ to get standard deviation

- On the calculator, plug in data into L_1 and L_2, then press Stat→Calc→ 1 var status
  ****Remember if you have two columns, after pressing 1 var stats, type in L_1, L_2

Binomial Distribution

- $\binom{n}{r}(p)^r(q)^{n-r}$
- P= probability of successes n= number of trials
  q= probability of failures r= number of successes
- To find standard deviation: $\sqrt{npq}$
- For most binomial distribution questions you will use either of these two calculator moves:
  1. Given a number of trials, they can ask a specific number of outcomes from that given amount of trials (ex. If the probability of red coins in a jar is .08, out of 10 trials, what is the probability that you will pick 7 red coins?)
     Use: binompdf(10, .08, 7)

  2. Given a number of trials, they can ask for multiple different outcomes. Watch out for their wording on these problems. (ex. If the probability of red coins in a jar is .08, out of 10 trials, what is the probability that you pick at most 5 coins?)
     Use: binomcdf(10, .08, 5)

****Watch out for this kind of wording: if the probability of red coins in a jar is .08, out of 10 trials, what is the probability that you pick at least 5 coins?

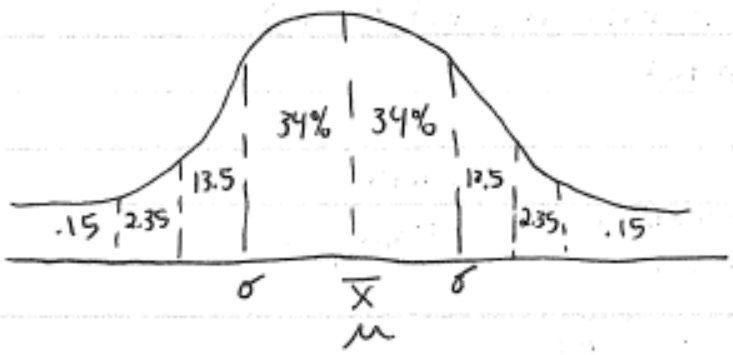
Here, you would use 1-binomcdf(10,.08,4)

- All of these calculator moves can be found by hitting 2nd→Vars

Normal Distribution

- They have to tell you if the problem is a normal distribution one
- With this type of problem, use:
  normalcdf(lower boundary, upper boundary, mean, standard deviation)
- If you're given a probability and asked to find the mean, use:
  Invnorm(probability)
- Z-score- the number of standard deviations away from the mean
- One way to find the z-score:
  Invnorm(probability, mean, standard deviation)
- If you have an unknown mean and standard deviation:
  1. Use invnorm(probability)= $\frac{x-\mu}{\sigma}$
  2. Get the X one one side
  3. Set up a matrix using the two equations, make sure they are in the same order
     Here is what the matrix should look like:
     $\begin{bmatrix} Coeff.of\ \mu & Coeff.of\ \sigma \\ Coeff.of\ \mu & Coeff.of\ \sigma \end{bmatrix}^{-1}$ x $\begin{bmatrix} X\ value \\ X\ value \end{bmatrix}$
- Remember, if they give you a probability above a certain value, when you do the invnorm move, you need to do the probability below that value

- Quartiles and Box plots
  - min - lowest value
  - Q_1 - Lower quartile - median of bottom half
  - Med or Q_2 - middle number
  - Q_3 - Upper quartile - median of top half
  - max - highest value
  - range = Max - min
  - IQR - interquartile range $IQR = Q_3 - Q_1$
- Central Tendency
  - mean = average = $\frac{\text{sum of all \#}}{\text{\# of items}} = \frac{\Sigma x}{n}$
    - $\bar{x}$ or μ
  - median = middle #
    - $= \frac{n+1}{2}$
    - If middle # is between 2 #s, the median is an a
  - mode = the number that occurs the most
    - up to 2 modes
  - data is skewed when mean & median do not match
    - mean < median - negatively skewed
    - mean > median - positively skewed
- Cumulative Frequency Graph
  - Percentiles → the percent of data a score is higher than
    - 0% – lowest
    - 99% - highest
  - Standard deviation = $\sqrt{\text{variance}}$
    - $= s$ or σ
- Normal Curve



- Probability
  - the chance an event occurs = $\frac{\#\text{ outcomes}}{\text{possible outcomes}} = P(E)$

$P(E')$: the complement of E

$P(E) + P(E') = 1$

- Compound event = more than one event occuring
  - independent event = one event does not affect the other
  - dependent = one event affects the other
  - independent = $P(A \text{ and } B) = P(A) \cdot P(B)$
- Combined events

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

- If independent then $P(A \cap B) = P(A) \cdot P(B)$
- Mutually exclusive

No overlap $\rightarrow P(A \cap B) = 0$

$P(A \cup B) = P(A) + P(B)$

- Conditional Probability

$P(A/B) = \frac{P(A \cap B)}{P(B)}$

- Discrete Random Variables
  - standard deviation
    1. Find mean = expected value
    2. Subtract mean from each x value
    3. square each difference
    4. multiply by probability
    5. add together (variance)
    6. square root (std)
  - expectated value

$exp = np$

$E(x) = (\#\text{ of trials})(\text{probability an event occurs})$

- all probabilities add up to 1

Binomial Distribution

$P(X=r) = \binom{n}{r} p^r q^{n-r}$

$\binom{n}{r}$: Pascals coefficient; p: prob success; q: prob failure

n = trials; r = # success

$mean = np \qquad std = \sqrt{npq}$

- Normal Distribution

Z score = the number of standard deviations away from mean

= if positive → above the mean

= if negative → below the mean

normalcdf (lower #, upper #)

$z = \frac{x - \mu}{\sigma}$: $\frac{\text{value} - \text{mean}}{\text{std}}$

normcdf(lower, upper, mean, std) → give %

invnorm (percentage below z) → give #

invnorm (prob lower than, mean, std)

Topic 6 - Calculus

Limits

- The answer to any limit problem is a y value
- $x \to^{-} 0$ = read from left to right
- $x \to^{+} 0$ = read from right to left
- Top heavy= plus or minus infinity
- Bottom heavy= 0
- Equal powers= divide the coefficients
- When given equations in limit problems, sometimes it if helpful to factor what you can before plugging in the limit

Continuity

- A function is continuous at x=a if….
    1. $\lim_{x \to a}$ exists
    2. F(a) exists
    3. $\lim_{x \to a} f(x) = f(a)$
- Use lift the pencil test, if you have to lift the pencil, then the function is not continuous
- With piece-wise functions, if the top equation equals the bottom one when x equals the breaking point, then it is continuous
- Watch for domain restrictions in these problems

Derivatives

- Derivative= gradient=slope=rate of change
- Remember you can't derive a square root and you can't have negative exponents in your answer
- Second derivative= the rate that your change is changing= acceleration
- Differentiability- if you are able to take the derivative
- A function is not differentiable at x=a if…
    1. f(x) is not continuous
    2. f(x) has a vertical tangent line
    3. the graph has a sharp point at x=a
- All derivative formulas are given on the formula sheet
- To find the equation of a tangent line, you need a point and a slope, remember the slope of the normal line is just the opposite reciprocal of the slope of the tangent

Implicit Differentiation – will NOT be on the exam

- Use this when you take the derivative of equations with multiple variables
    1. Take the derivative of each variable just like we normally do
    2. Taking the derivative of any other variable besides x requires that you multiply that derivative by $\frac{dy}{dx}$
    3. Simplify and get final answer in $\frac{dy}{dx}$ form
- Ex) $x^2 + y^2 = 9$

  $2x + (2y)(\frac{dy}{dx}) = 0$

  $\frac{dy}{dx} = \frac{-2x}{2y}$

Graph Theory

Original	Derivative	2nd Derivative
Increasing	Positive	
Decreasing	Negative	
Relative Max	=0 AND switches from positive to negative	Negative
Relative Min	=0 AND goes from negative to positive	Positive
Concave Up	Increasing	Positive
Concave Down	Decreasing	Negative
POI	Has a relative max or min	=0 AND signs change

- Finding the absolute max or min:

  Y= $\frac{-1}{3}x^3 - 3x^2 + 8x - 4$ with a domain of [0,6]

  1. Take the derivative → $x^2 - 6x + 8$
  2. Set the derivative equal to 0 and factor for values of x→ (x-4)(x-2)
  3. Plug those factors AND the domain into the original equation→
     f(0)= -4
     f(2)= $\frac{8}{3}$
     f(4)= $\frac{4}{3}$
     f(6)= 8
  4. The highest and lowest values are your absolute max and mins, respectively

- Finding the chart information by hand:

  Y= $\frac{-1}{3}x^3 - 3x^2 + 8x - 4$

  1. Find derivative→ $x^2 - 6x + 8$
  2. Find critical values by setting equation =0→ (x-4)(x-2)
  3. Create sign line

++ - - - - - - - - - - ++
------|------------------|-------
2 4

F(x) increases on $(-\infty, 2)\ U\ (4, \infty)$ and decreases on (2,4)

f(x) has a relative max at x=2 and has a relative min at x=4

- Do the same steps but with the second derivative to find concavity and POI
- Rule of thumb for graphing:
  1. The only way *e* will be negative is if it has a negative INFRONT of it, the sign of its exponent will not affect the sign of *e*
  2. If there is a horizontal tangent, make sure to make that very clear on the graph

Antiderivatives/Integration

- Use this when you are given a derivative and you need to get back to the original equation
- Always put "+ C" at the end of the integration
- The only formulas for this unit that are not on the formula sheet are these: $\int sec^2xdx = tanx + C$
  $\int cscxcotx(dx) = -cscx + C$
  $\int csc^2xdx = -cotx + C \qquad \int secxtanx(dx) = secx + C$
- If they want you to find the value of C, they will give you a point. Plug the given x and y values into the equation after integrating and solve for C
  Ex) Derivative= 5x+ 2 point(1,3)
  $\int(5x + 2)(dx) = \frac{5}{2}x^2 + 2x + C$
  $3= \frac{5(1)^2}{2} + 2(1) + C$
  C= -3/2
- When you substitute U in, here are some possible places to sub in U:
  1. Item being raised to a power
  2. Angle of a trig function
  3. Exponent of e, or another constant
  4. Entire bottom of a fraction
- Properties of integrals with limits:
  $\int_b^c f(x)dx = \int_a^c f(x)dx \qquad \int_a^a f(x)dx = 0 \qquad \int_a^b f(x)dx = -\int_b^a f(x)dx \qquad \int_a^b f(x)dx +$

Area Using Integrals

- $\int_{startpoint}^{endpoint} top\ curve - bottom\ curve$
- Sometimes they will not tell you the end or start point, so you have to look at where the graphs intersect to get those values
- If the area has two different bottom curves, split the graph according to those bottom curves and add the separate areas once you've found them

Solids of Rotation

- The volume of a solid is formed by revolving a region about the x-axis
- Disk- after revolving the equation, there is no space between the function and x-axis
  V= $\int_a^b \pi r^2 dx$
- Washer- After revolving the equation, there is space between the function and x-axis
  V= $\pi \int_a^b (R^2 - r^2)dx$
- R= top curve, r= bottom curve

Motion

- Watch out for restrictions in these types of problems
- s(t)→ normal, displacement function
  v(t)→ velocity (1st derivative)
  a(t)→ acceleration (2nd derivative)

Position	Velocity	Acceleration
Moving right/up/increasing	Positive	
Moving left/down/decreasing	Negative	
At rest/changes directions	0	
Relative max	=0 AND switches from positive to negative	Negative
Relative min	=0 AND switches from negative to positive	Positive
	Increases	Positive
	Decreases	Negative
Speeding up	Positive	Positive
Speeding up	Negative	Negative
Slowing down	Positive	Negative
Slowing down	Negative	Positive

- Speed is always positive, but velocity can be either positive or negative

Distance and Displacement

- Distance has to be positive, displacement can be positive or negative
- Displacement= end-start or s(end)-s(start)
- To do distance by hand, you need a start point, stopping point, and when it changed directions
- On the calculator:
  Distance= $\int_{start}^{when\ it\ changes} v(t)dt \quad + \quad \int_{when\ it\ changes}^{end} v(t)dt$

OR (this way is a lot easier):

$$\int_{start}^{end} |v(t)|dt$$

Displacement= $\int_{start}^{end} v(t)dt$

- Limits

$\lim_{x \to a} f(x)$

$\lim_{x \to a^+} (f(x))$ → right hand limit

$\lim_{x \to a^-} f(x)$ → left hand limit

if left doesn't equal right overall limit DNE

- Continuous if
  1. $\lim f(x)$ exists
  2. $f(a)$ exists
  3. $\lim_{x \to a} f(x) = f(a)$

  2 types

  removable - can move 1 pt to make continuous

  non removable - can't move 1 pt make continuous

  given 2 equations

  plug in - do get same value

  match → limit exists (= function value)

- Finding Asymptotes
  - to find horizontal asymptote - find the limit as $x \to \pm\infty$
    - the answer is the equation of asymptote
  - to find vertical asymptote
    1. find pts of discontinuity (values of x that make denominator equal zero and thus function undefined)
    2. double check those #s by plugging back into numerator
       - a. if you get a non-zero # then there is a vertical asymptote at that x value
       - b. if you get 0 for numerator - no vertical asymptote at that x value

  Horizontal

  - top heavy limit = ∞ no HA
  - bottom heavy limit = 0 asymptote $y = 0$
  - equal powers limit = fraction created by this · asymptote $y =$ fraction

- Instantaneous Rate of Change = slope of tangent = derivative

$$\lim_{a \to b} \frac{f(b)-f(a)}{b-a} \quad ; \quad \frac{y-y}{x-x}$$

- Power rule for derivatives
  - $y = x^a$

    $\frac{dy}{dx} = y' = f'(x) = a x^{(a-1)}$
  - polynomials → do each term separately
- constant rule → derivative of constant = 0
- A function is not differentiable at an x-value if any of the following occur
  - $f(x)$ is not continuous
  - sharp pt in the graph
  - vertical tangent

↳ plug x value into $f(x)$ get answer if = continuous

plus x value into $f'(x)$ get answer if = differentiable

- Finding the Equation of a Tangent Line

↳ need point and slope

- point

  to find point must use original function

  $y - y_1 = m(x - x_1)$
- slope

  to find slope of tangent line must use derivative of original.
  - plug x value for pt into derivative

    resulting value = slope of tangent
  - once you know the slope of the tangent find the negative reciprocal to get slope of normal

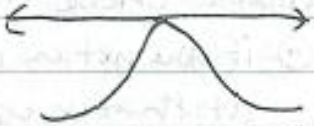
When does $f(x)$ have horizontal tangent?

① find $f'(x)$

② set $f'(x) = 0$

③ solve for x

④ plug x into original to find y



- Product Rule

$y = u \cdot v \quad \frac{dy}{dx} = u'v + uv'$

- Quotient Rule

$$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$$

- Chain Rule

$y = (5x^2 + 3x + 5)^9$

$y' = 9(5x^2 + 3x + 5)^8 \cdot (10x + 3)$

- Polynomials

$f(g(x))' = f'(g(x)) \cdot g'(x)$

- Derivatives of trig functions

$f'(\sin x) = \cos x$ $\quad f'(-\sin x) = -\cos x$

$f'(\cos x) = -\sin x$ $\quad f'(-\cos x) = \sin x$

$f'(\tan x) = \sec^2 x$

$f'(\sec x) = \sec x \tan x$

$f'(\csc x) = -\csc^2 x$

$f'(\cot x) = -\csc x \cot x$

- Derivatives of e^u and $\ln|u|$

$y = e^u \quad y' = e^u \cdot u'$

$y = \ln|u| \quad y' = \frac{1}{u} \cdot u'$

- to find critical values of ~~derivative~~ function set equal to 0

- Antiderivative / Integration

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C$$

$$\int e^x dx = e^x + C$$

$$\int \frac{1}{x} dx = \ln|x| + C$$

- Integration w/ chain rule

U substitution

- what is being raised to a power
- angle of trig function
- exponent of e, or other constant
- the bottom of fraction

- Definite Integrals

$$\int_a^b f(x) dx = F(b) - F(a)$$

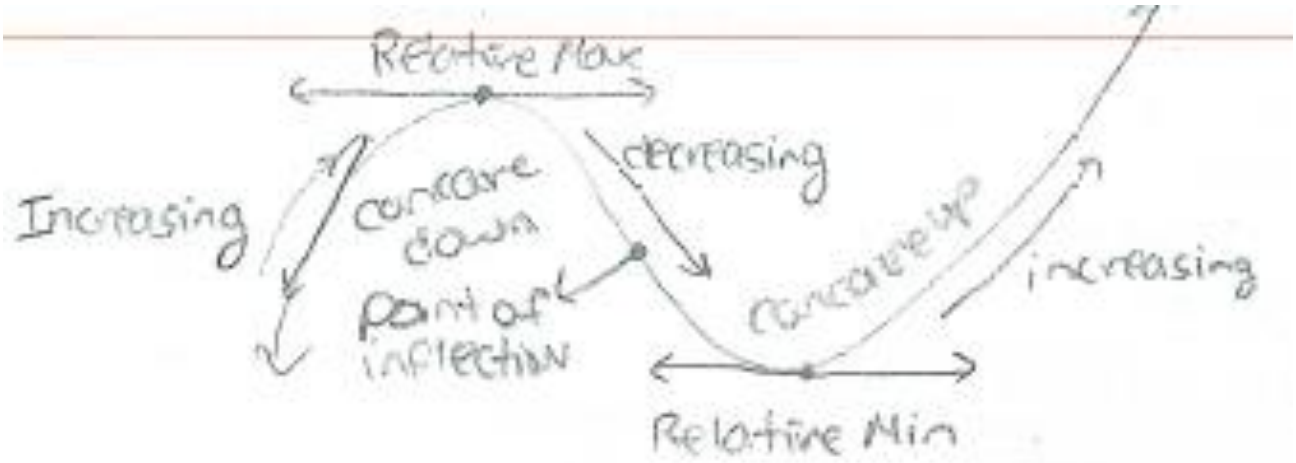
$$\int_a^b f(x) dx = -\int_b^a f(x) dx \qquad \int_a^b f(x) dx + \int_b^c f(x) dx = \int_a^c f(x) dx$$

- Area = top curve - bottom curve

$$A = \int_a^b (f(x) - g(x))\,dx$$

$$\left[\right]_a^b \quad \text{integration}$$

- Volume = $\int_a^b \pi y^2\,dx$



 = concave up

 = concave down

One last thing:

Good luck and don't stress, you can do this!!!!