1. The Complex Numbers

The general form of a complex number is a + bi; where $i = \sqrt{-1}$, a is the real part and b is the imaginary part of the complex number.

Geometrical Representation of Complex Number

We can represent or locate complex number a + bi on the coordinate plane as an ordered pair of real numbers i.e. (a, b). X-axis is called or labeled as the real axis and the Y-axis is called or labeled as the imaginary axis.

The plane on which we represent complex numbers is called complex plane.

How to Enter in a Complex Mode of the Calculator:

To enter in the complex mode of the calculator, press



As long as you are in complex mode of the calculator, you will see CMPLX at the top of the screen.

(Pressing MODE 1 (COMP) will take you back to the normal computation mode.)

Entering a Complex Number

To enter a complex number, for example 1+2i, use the following sequence of keys:



Storing a Complex Number in a Memory Variable of the Calculator

During your learning you will find exercises where you require calculating higher powers, conjugate, modulus etc. of a complex number. To use a complex number repetitively you can store the complex number into a memory variable of the calculator.

Guided Activity 1:

To store a complex number 1 + 2i into a memory variable A, press the following sequence of buttons:



To view or to use the complex number stored in the memory variable A, press



Practice Activity:

Store a complex number $-\frac{1}{2} - \frac{2}{3}i$ into the memory variable B. You will have to use the stored number in upcoming activities.

Guided Activity 2:



Steps to follow

Press the following sequence of buttons



Guided Activity 3:

Calculate conjugate of a complex number 3 + 5i.

Steps to follow

Press the following sequence of buttons



Guided Activity 4:

Calculate conjugate of complex number 1 + 2i; already stored in memory variable A.

Steps to follow

Press the following sequence of buttons



Guided Activity 5:



in memory variable B after practice activity 1)

<u>Steps to follow</u>

Press the following sequence of buttons



OR

If the complex number is not already stored, then press



Guided Activity 6:

Prove the following properties of complex numbers:

i.
$$|-z| = |z| = |\overline{z}| = |-\overline{z}|$$

ii. $z \,\overline{z} = |z|^2$

by using z = 3 - 2i.

<u>Steps to follow</u>

Press the following sequence of buttons

As we have to use the same complex number for various calculations, it is convenient to store this complex number in a memory variable, say C.



Math 🔺

Guided Activity 3:

Calculate conjugate of a complex number 3 + 5i.

Steps to follow

Press the following sequence of buttons



Guided Activity 4:

Calculate conjugate of complex number 1 + 2i; already stored in memory variable A.

Steps to follow

Press the following sequence of buttons



Guided Activity 5:



in memory variable B after practice activity 1)

<u>Steps to follow</u>

Press the following sequence of buttons



OR

If the complex number is not already stored, then press



Guided Activity 6:

Prove the following properties of complex numbers:

i.
$$|-z| = |z| = |\overline{z}| = |-\overline{z}|$$

ii. $z \,\overline{z} = |z|^2$

by using z = 3 - 2i.

<u>Steps to follow</u>

Press the following sequence of buttons

As we have to use the same complex number for various calculations, it is convenient to store this complex number in a memory variable, say C.



Math 🔺



Guide Book for CASIO fx-991ES PLUS | 1. The Complex Mathematics Association of Pakistan ook for CASIO fx-991ES PLUS | 1. The Complex Numbers Mathematics Association of Pakistan



Guide Book for CASIO fx-991ES PLUS | 1. The Complex Mathematics Association of Pakistan ook for CASIO fx-991ES PLUS | 1. The Complex Numbers Mathematics Association of Pakistan



Hence,
$$z \overline{z} = |z|^2$$
 (Proved)

Guided Activity 7:

Prove the following properties of complex numbers

i.
$$\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\overline{z_1}}{\overline{z_2}}; z_2 \neq 0$$

ii. $|z_1 z_2| = |z_1| |z_2|$
by using $z_1 = 2 - i$ and $z_2 = 1 + i$

Steps to follow

As we have to use the given two complex numbers for different calculations, it is better to store these complex numbers in memory variables, say A and B.



i.e.
$$\overline{\left(\frac{z_1}{z_2}\right)} = \frac{1}{2} + \frac{3}{2}i$$

And $\overline{\frac{z_1}{z_2}} = ?$

i.e. $\frac{\overline{z_1}}{\overline{z_2}} = \frac{1}{2} + \frac{3}{2}i$

iii. Finding $|z_1 z_2|$



Math 🔺





Hence,
$$z \overline{z} = |z|^2$$
 (Proved)

Guided Activity 7:

Prove the following properties of complex numbers

i.
$$\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\overline{z_1}}{\overline{z_2}}; z_2 \neq 0$$

ii. $|z_1 z_2| = |z_1| |z_2|$
by using $z_1 = 2 - i$ and $z_2 = 1 + i$

Steps to follow

As we have to use the given two complex numbers for different calculations, it is better to store these complex numbers in memory variables, say A and B.



i.e.
$$\overline{\left(\frac{z_1}{z_2}\right)} = \frac{1}{2} + \frac{3}{2}i$$

And $\overline{\frac{z_1}{z_2}} = ?$

i.e. $\frac{\overline{z_1}}{\overline{z_2}} = \frac{1}{2} + \frac{3}{2}i$

iii. Finding $|z_1 z_2|$



Math 🔺



```
Now, finding |z_1||z_2|
```



Guided Activity 8:

Calculate arg(2-3i).

Steps to follow

9



Note: The calculator is in *Degree Mode*, so the answer of $\arg(2-3i) = -56.30993247^{\circ}$. If answer is required in radian, then press the keys as follows:



Guided Activity 9:

Convert (-2, 2) or -2+2i into polar co-ordinate system (r, θ) .

Steps to follow:

AC (→ 2 + 2 ENG (i) SHFT 2 (CMPLX	1:ar9 2:Conj9 3:⊅r20 4:⊅a+bi
3 (3)	-2+2i▶r∠θ ^{α Ματι} Α 2√2∠ <u>3</u> π

(Note: The calculator is in Radian Mode)

The answer
$$2\sqrt{2} \angle \frac{3}{4}\pi$$
 means $r = |-2+2i| = 2\sqrt{2}$ and $\theta = Arg(-2+2i) = \frac{3}{4}\pi$, so $-2+2i$ can be written in polar co-ordinate system as $\left(2\sqrt{2}, \frac{3}{4}\pi\right)$

Guided Activity 10:

Convert $(3, 30^\circ)$ into Cartesian co-ordinate system (x, y).

Steps to follow:

To bring calculator to degree mode – if it is already not in the degree mode – press



So (3,30°) in Cartesian coordinate system is (2.598076211,1.5)

```
Now, finding |z_1||z_2|
```



Guided Activity 8:

Calculate arg(2-3i).

Steps to follow

9



Note: The calculator is in *Degree Mode*, so the answer of $\arg(2-3i) = -56.30993247^{\circ}$. If answer is required in radian, then press the keys as follows:



Guided Activity 9:

Convert (-2, 2) or -2+2i into polar co-ordinate system (r, θ) .

Steps to follow:

AC (→ 2 + 2 ENG (i) SHFT 2 (CMPLX	1:ar9 2:Conj9 3:⊅r20 4:⊅a+bi
3 (3)	-2+2i▶r∠θ ^{α Ματι} Α 2√2∠ <u>3</u> π

(Note: The calculator is in Radian Mode)

The answer
$$2\sqrt{2} \angle \frac{3}{4}\pi$$
 means $r = |-2+2i| = 2\sqrt{2}$ and $\theta = Arg(-2+2i) = \frac{3}{4}\pi$, so $-2+2i$ can be written in polar co-ordinate system as $\left(2\sqrt{2}, \frac{3}{4}\pi\right)$

Guided Activity 10:

Convert $(3, 30^\circ)$ into Cartesian co-ordinate system (x, y).

Steps to follow:

To bring calculator to degree mode – if it is already not in the degree mode – press



So (3,30°) in Cartesian coordinate system is (2.598076211,1.5)

Activity Sheet (The Complex Number)



6. 6. Convert (2,3) of Cartesian coordinate system into Polar coordinate system (θ should be in degree measure).





11 Guide Book for CASIO fx-991ES PLUS | Activity Sheet (The Complex Number) Mathematics Association of Pakistan

2. Quadratic Equations

The standard form of a quadratic equation is $ax^2 + bx + c = 0$; $a \neq 0$.

A quadratic equation has two roots which may be equal or unequal, real or complex in nature. The real roots can be either rational or irrational.

The quadratic formula to solve the quadratic equation $ax^2 + bx + c = 0$ is $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. The two values of *x* are called roots of the quadratic equation.

Through scientific calculator fx-991ES PLUS we can find the real roots as well as complex roots, if exist, of standard quadratic equations.

The calculator can also be used to solve the equations reducible to quadratic equation; but in such cases the calculator can solve the equations that have at least one real root.

Entering in the Equation Mode (EQN) of the Calculator

To enter in the equation mode of the calculator, press



To solve a quadratic equation, press



Enter the values of *a*, *b* and *c* to get the roots of a quadratic equation.

(Note: Pressing MODE 1 (COMP) will take you back to the normal computation mode.)

Activity Sheet (The Complex Number)



6. 6. Convert (2,3) of Cartesian coordinate system into Polar coordinate system (θ should be in degree measure).





11 Guide Book for CASIO fx-991ES PLUS | Activity Sheet (The Complex Number) Mathematics Association of Pakistan

2. Quadratic Equations

The standard form of a quadratic equation is $ax^2 + bx + c = 0$; $a \neq 0$.

A quadratic equation has two roots which may be equal or unequal, real or complex in nature. The real roots can be either rational or irrational.

The quadratic formula to solve the quadratic equation $ax^2 + bx + c = 0$ is $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. The two values of *x* are called roots of the quadratic equation.

Through scientific calculator fx-991ES PLUS we can find the real roots as well as complex roots, if exist, of standard quadratic equations.

The calculator can also be used to solve the equations reducible to quadratic equation; but in such cases the calculator can solve the equations that have at least one real root.

Entering in the Equation Mode (EQN) of the Calculator

To enter in the equation mode of the calculator, press



To solve a quadratic equation, press



Enter the values of *a*, *b* and *c* to get the roots of a quadratic equation.

(Note: Pressing MODE 1 (COMP) will take you back to the normal computation mode.)

Guided Activity 1:

Solve $x^2 - 4x + \frac{1}{2} = 0$.

Steps to follow:

Press



Now, enter the coefficients as



to view the roots of the quadratic equation, press

$$\begin{array}{c} X_{1} = \\ X_{1} = \\ X_{2} = \\ \frac{4 - \sqrt{14}}{2} \\ X_{2} = \\ \frac{4 - \sqrt{14}}{2} \\ X_{3} = \\ X_{4} = \\ X_$$

So, the solution set of the quadratic equation $x^2 - 4x + \frac{1}{2} = 0$ is $\left\{ \frac{1}{2} \right\}$.

How to View the Decimal Equivalent of a root?

Press [940] to view the decimal equivalent of a root. (Note: To view the first root again, press ().)

X2=	۵	Math 🔺	
0.	12917	13066	

To edit or change the coefficients of the quadratic equation press 🖃 after getting the second root of the equation or press AC any time during these operations.

Guided Activity 2:

Solve $5y^2 + 4 = 0$.

Steps to Follow

Press



Math

2√5

5 Math 🔺

Enter the coefficients, as



to view the roots of the quadratic equation



Guided Activity 3:

Solve $x^2 - 4x = 5$.

Steps to Follow:

Press



1:anX+bnY=Cn

3:

2:anX+bnY+cnZ=dn

aX3+bX2+cX+d=0

aX2+bX+c=0

As, the format of quadratic equation, shown in the screen , we will have to enter the constant term as -5.

Guided Activity 1:

Solve $x^2 - 4x + \frac{1}{2} = 0$.

Steps to follow:

Press



Now, enter the coefficients as



to view the roots of the quadratic equation, press

$$\begin{array}{c} X_{1} = \\ X_{1} = \\ X_{2} = \\ \frac{4 - \sqrt{14}}{2} \\ X_{2} = \\ \frac{4 - \sqrt{14}}{2} \\ X_{3} = \\ X_{4} = \\ X_$$

So, the solution set of the quadratic equation $x^2 - 4x + \frac{1}{2} = 0$ is $\left\{ \frac{1}{2} \right\}$.

How to View the Decimal Equivalent of a root?

Press [940] to view the decimal equivalent of a root. (Note: To view the first root again, press ().)

X2=	۵	Math 🔺	
0.	12917	13066	

To edit or change the coefficients of the quadratic equation press 🖃 after getting the second root of the equation or press AC any time during these operations.

Guided Activity 2:

Solve $5y^2 + 4 = 0$.

Steps to Follow

Press



Math

2√5

5 Math 🔺

Enter the coefficients, as



to view the roots of the quadratic equation



Guided Activity 3:

Solve $x^2 - 4x = 5$.

Steps to Follow:

Press



1:anX+bnY=Cn

3:

2:anX+bnY+cnZ=dn

aX3+bX2+cX+d=0

aX2+bX+c=0

As, the format of quadratic equation, shown in the screen , we will have to enter the constant term as -5.



to view the roots of the quadratic equation, press



So, the solution set of $x^2 - 4x = 5$ is $\{5, -1\}$.

Solutions of Equations Which are Reducible to a Quadratic Equation

To solve the equations reducible to a quadratic equation, convert your calculator in COMP mode by pressing III (COMP).

The calculator can solve the equations which are reducible to quadratic equation with following limitations:

- 1. It can solve the reducible equations, if the roots are real.
- 2. It can provide you only one root of the equation; however, you can also find other roots, if they are real, by applying basic concepts of Algebra.

Guided Activity 4:

Solve
$$4x + \frac{3}{x} = 8$$
.

Steps to Follow ::



(The value of X appearing on the screen is not the solution; it is just reflecting the currently stored value of X. Your calculator's screen may have different value at this point depending upon the value of X stored in your calculator's memory. It is recommended to assign the value of X as 0 to start the calculation. To assign X=0, press

To see the solution of the equation, press



So, one root of the equation $4x + \frac{3}{x} = 8$ is 1.5. The L-R= 0

indicates the accuracy of the answer. The answer would be more accurate if the value of L - R is nearest to or equal to 0. Please also note that the memory value of X will be automatically updated by the answer i.e. 0.5.

So, if one root of the equation is 0.5, then one factor of $f(x) = 4x + \frac{3}{x} - 8$ is x - 0.5. So, by using the concept of Algebra, we can write that $\frac{(4x + \frac{3}{x} - 8)}{x - 0.5} = 0$.

We can find the second root by solving this equation through calculator. We can reenter the complete equation or edit the previously entered equation as



So, the roots of the equation are $\{0.5, 1.5\}$



to view the roots of the quadratic equation, press



So, the solution set of $x^2 - 4x = 5$ is $\{5, -1\}$.

Solutions of Equations Which are Reducible to a Quadratic Equation

To solve the equations reducible to a quadratic equation, convert your calculator in COMP mode by pressing III (COMP).

The calculator can solve the equations which are reducible to quadratic equation with following limitations:

- 1. It can solve the reducible equations, if the roots are real.
- 2. It can provide you only one root of the equation; however, you can also find other roots, if they are real, by applying basic concepts of Algebra.

Guided Activity 4:

Solve
$$4x + \frac{3}{x} = 8$$
.

Steps to Follow ::



(The value of X appearing on the screen is not the solution; it is just reflecting the currently stored value of X. Your calculator's screen may have different value at this point depending upon the value of X stored in your calculator's memory. It is recommended to assign the value of X as 0 to start the calculation. To assign X=0, press

To see the solution of the equation, press



So, one root of the equation $4x + \frac{3}{x} = 8$ is 1.5. The L-R= 0

indicates the accuracy of the answer. The answer would be more accurate if the value of L - R is nearest to or equal to 0. Please also note that the memory value of X will be automatically updated by the answer i.e. 0.5.

So, if one root of the equation is 0.5, then one factor of $f(x) = 4x + \frac{3}{x} - 8$ is x - 0.5. So, by using the concept of Algebra, we can write that $\frac{(4x + \frac{3}{x} - 8)}{x - 0.5} = 0$.

We can find the second root by solving this equation through calculator. We can reenter the complete equation or edit the previously entered equation as



So, the roots of the equation are $\{0.5, 1.5\}$

Guided Activity 5:

Solve
$$2\sqrt{x+1} + \frac{2}{\sqrt{x+1}} - 5 = 0$$

Steps to follow:



$$\frac{2\sqrt{x+1} + \frac{2}{\sqrt{x+1}} - 5}{x-3} = 0.$$

Renter the above equation or edit the already entered equation as



i.e. the second root of the equation is -0.75 and hence the solution set of the equation is $\{-0.75,3\}$

Activity Sheet (Quadratic Equations)





2. Solve $\frac{x-1}{x+1} + \frac{x+1}{x-1} = \frac{13}{6}$.

(Hint: While solving an equation for X=0, if calculator displays "Can't Solve", then attempt to start the solution for some other value of X. e.g. X=1 or X=2 etc.)



4. Find one root of
$$\sqrt{2x^2 - 5x} + \frac{1}{\sqrt{2x^2 - 5x}} = 2$$
.
Ans:
$$\begin{bmatrix}
\frac{\sqrt{2x^2 - 5x}}{\sqrt{2x^2 - 5x}} \\
X = -0.186140568 \\
L - R = 0
\end{bmatrix}_{\&} \begin{bmatrix}
\frac{\sqrt{2x^2 - 5x}}{\sqrt{2x^2 - 5x}} \\
X = 2.686140851 \\
L - R = 0
\end{bmatrix}_{\&}$$

(Hint: You will get the first answer when we start with X=0. When you

will be trying to solve $\frac{\sqrt{2x^2 - 5x} + \frac{1}{\sqrt{2x^2 - 5x}} - 2}{x + 0.186140568} = 0$ for X=0, the

calculator will show the same answer x = -0.186140568 again. For any such situation, gradually increase the value of X i.e. 0,1, 2, 3.....to start the solution process for calculator. The second answer shown here will be obtained when X=3.

Guided Activity 5:

Solve
$$2\sqrt{x+1} + \frac{2}{\sqrt{x+1}} - 5 = 0$$

Steps to follow:



$$\frac{2\sqrt{x+1} + \frac{2}{\sqrt{x+1}} - 5}{x-3} = 0.$$

Renter the above equation or edit the already entered equation as



i.e. the second root of the equation is -0.75 and hence the solution set of the equation is $\{-0.75,3\}$

Activity Sheet (Quadratic Equations)





2. Solve $\frac{x-1}{x+1} + \frac{x+1}{x-1} = \frac{13}{6}$.

(Hint: While solving an equation for X=0, if calculator displays "Can't Solve", then attempt to start the solution for some other value of X. e.g. X=1 or X=2 etc.)



4. Find one root of
$$\sqrt{2x^2 - 5x} + \frac{1}{\sqrt{2x^2 - 5x}} = 2$$
.
Ans:
$$\begin{bmatrix}
\frac{\sqrt{2x^2 - 5x}}{\sqrt{2x^2 - 5x}} \\
X = -0.186140568 \\
L - R = 0
\end{bmatrix}_{\&} \begin{bmatrix}
\frac{\sqrt{2x^2 - 5x}}{\sqrt{2x^2 - 5x}} \\
X = 2.686140851 \\
L - R = 0
\end{bmatrix}_{\&}$$

(Hint: You will get the first answer when we start with X=0. When you

will be trying to solve $\frac{\sqrt{2x^2 - 5x} + \frac{1}{\sqrt{2x^2 - 5x}} - 2}{x + 0.186140568} = 0$ for X=0, the

calculator will show the same answer x = -0.186140568 again. For any such situation, gradually increase the value of X i.e. 0,1, 2, 3.....to start the solution process for calculator. The second answer shown here will be obtained when X=3.

3. Matrices

The general form of a matrix having *m* rows and *n* columns is a_{11} a_{12} a_{1n} *a*₂₁ a_{22} a_{2n} : the order of the matrix is $m \times n$ a_{m1} a_{m2} a_{mn}

Converting the Calculator to Matrix Mode

enter in the Matrix mode То press MODE 6 (MATRIX) and observe the screen shown here.

	Matrix? 1:MatA 3:MatC	2:MatB	
--	-----------------------------	--------	--

You can enter either Matrix A, B or C.

To enter a matrix A (MatA), press 1 and you will MatA(mxn) mxn? 1:3×3 observe this screen. 3:3×1

By selecting a number from 1 to 6, you can select the order of matrix A as per requirement

To view further possible orders for matrix A, press . ⊙



2:3x2

4:2×3

0]

n.

(To go back to previous screen press \bigcirc).

Let $A = \begin{bmatrix} 3 & -2 \end{bmatrix}$. So our selection in the above screen should be [2] and after pressing [2], this screen would appear to enter the first element of matrix A.

Press **3 =** to enter the first element of matrix A. The cursor will move to the next position to enter second element of matrix A.





Now matrix matA will remain stored until you change the elements or order of matA.

Attention:

Re-entering in the Matrix Mode or switching to any other mode will result in lost the data stored in the different matrices i.e. matA, matB, matC or matAns.

MAT written at the top of the screen indicates that currently you are in *Matrix* Mode.

Guided activity 1:

Find transpose of matrix $A = \begin{bmatrix} 3 & -2 \end{bmatrix}$ (the matrix is already stored as matA after performing above operation).

Steps to follow:

If the matrix A is not already stored in matA, then enter the matrix as described in the previous activity.



Use navigation keys $(\mathbf{A}, \mathbf{E}, \mathbf{A})$ and $(\mathbf{\nabla})$ to clearly view various elements of the answer matrix.

3. Matrices

The general form of a matrix having *m* rows and *n* columns is a_{11} a_{12} a_{1n} *a*₂₁ a_{22} a_{2n} : the order of the matrix is $m \times n$ a_{m1} a_{m2} a_{mn}

Converting the Calculator to Matrix Mode

enter in the Matrix mode То press MODE 6 (MATRIX) and observe the screen shown here.

	Matrix? 1:MatA 3:MatC	2:MatB	
--	-----------------------------	--------	--

You can enter either Matrix A, B or C.

To enter a matrix A (MatA), press 1 and you will MatA(mxn) mxn? 1:3×3 observe this screen. 3:3×1

By selecting a number from 1 to 6, you can select the order of matrix A as per requirement

To view further possible orders for matrix A, press . ⊙



2:3x2

4:2×3

0]

n.

(To go back to previous screen press \bigcirc).

Let $A = \begin{bmatrix} 3 & -2 \end{bmatrix}$. So our selection in the above screen should be [2] and after pressing [2], this screen would appear to enter the first element of matrix A.

Press **3 =** to enter the first element of matrix A. The cursor will move to the next position to enter second element of matrix A.





Now matrix matA will remain stored until you change the elements or order of matA.

Attention:

Re-entering in the Matrix Mode or switching to any other mode will result in lost the data stored in the different matrices i.e. matA, matB, matC or matAns.

MAT written at the top of the screen indicates that currently you are in *Matrix* Mode.

Guided activity 1:

Find transpose of matrix $A = \begin{bmatrix} 3 & -2 \end{bmatrix}$ (the matrix is already stored as matA after performing above operation).

Steps to follow:

If the matrix A is not already stored in matA, then enter the matrix as described in the previous activity.



Use navigation keys $(\mathbf{A}, \mathbf{E}, \mathbf{A})$ and $(\mathbf{\nabla})$ to clearly view various elements of the answer matrix.

Guided Activity 2:

For two matrices
$$A = \begin{bmatrix} 3 & -2 \end{bmatrix}$$
 and $B = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$, find AB and AB if possible.

Steps to follow:

To enter the second matrix B, use the following sequence:



Now, matrix B is stored in matB.

Finding AB:





So, $AB = \begin{bmatrix} -1 & -2 \end{bmatrix}$

Finding BA:



The *Dimension ERROR* indicates that B and A are not confirmable for multiplication.

Guided Activity 3:

	Γa	1	1]	$\left[\frac{1}{2}\right]$	1	0		[1	2	0	
Enter three matrices	$A = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$	$A = \begin{bmatrix} 2 & -1 \\ 2 & 0 \end{bmatrix}$	$\begin{bmatrix} 1 \\ 2 \end{bmatrix}, B = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$	2	0	and $C = -1$	2	0	to		
	Ľ	0	3	0	1	3		3	0	1	

evaluate the following.

- i. 2B-3C and then also find $A \times (2B-3C)$.
- ii. multiplicative inverse of matrix B. i.e. B^{-1} .
- iii. determinant of matrix B i.e. |B|.
- iv. adjoint of matrix C.
- v. B^3
- vi. prove that $(AB)^t = B^t A^t$

Guided Activity 2:

For two matrices
$$A = \begin{bmatrix} 3 & -2 \end{bmatrix}$$
 and $B = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$, find AB and AB if possible.

Steps to follow:

To enter the second matrix B, use the following sequence:



Now, matrix B is stored in matB.

Finding AB:





So, $AB = \begin{bmatrix} -1 & -2 \end{bmatrix}$

Finding BA:



The *Dimension ERROR* indicates that B and A are not confirmable for multiplication.

Guided Activity 3:

	Γa	1	1]	$\left[\frac{1}{2}\right]$	1	0		[1	2	0	
Enter three matrices	$A = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$	$A = \begin{bmatrix} 2 & -1 \\ 2 & 0 \end{bmatrix}$	$\begin{bmatrix} 1 \\ 2 \end{bmatrix}, B = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$	2	0	and $C = -1$	2	0	to		
	Ľ	0	3	0	1	3		3	0	1	

evaluate the following.

- i. 2B-3C and then also find $A \times (2B-3C)$.
- ii. multiplicative inverse of matrix B. i.e. B^{-1} .
- iii. determinant of matrix B i.e. |B|.
- iv. adjoint of matrix C.
- v. B^3
- vi. prove that $(AB)^t = B^t A^t$

Steps to follow:

(If your calculator is not already in MATRIX mode, then enter in the MATRIX mode first)

To store A as mat A, press









1:Dim 2:Data 3:MatA 4:MatB AC SHIFT 4 5:MatC 6:MatAns 7:det 8:Trn Matriz? 2:MatB 1:MatA 1 3:MatC MatC(mxn) mxn? 1:3×3 2:3x2 3 3:3×1 4:2×3 5:2×2 6:2×1 Mat С 1 1 = 2 = 0 = -1 = 2 = 0 = 3 = 0 = 1 =C. 2 Π Ē -1 ÷. п

Now three matrices A, B and C are stored in matA, matB and matC respectively.

2B-3Ci.

Steps to Follow:

And



Steps to follow:

(If your calculator is not already in MATRIX mode, then enter in the MATRIX mode first)

To store A as mat A, press









1:Dim 2:Data 3:MatA 4:MatB AC SHIFT 4 5:MatC 6:MatAns 7:det 8:Trn Matriz? 2:MatB 1:MatA 1 3:MatC MatC(mxn) mxn? 1:3×3 2:3x2 3 3:3×1 4:2×3 5:2×2 6:2×1 Mat С 1 1 = 2 = 0 = -1 = 2 = 0 = 3 = 0 = 1 =C. 2 Π Ē -1 ÷. п

Now three matrices A, B and C are stored in matA, matB and matC respectively.

2B-3Ci.

Steps to Follow:

And





The answer of last performed calculation automatically stores in the matrix variable matAns. So to find $A \times (2B - 3C)$ perform the following



Note that, the matAns will automatically save the new answer i.e. $A \times (2B - 3C)$.

ii. Multiplicative inverse of matrix B. i.e. B^{-1} .

Steps to Follow:







To view an element in simple fraction, use navigational keys $(\mathbf{D}, \mathbf{O}, \mathbf{O})$ and (\mathbf{O}, \mathbf{O})

iii. determinant of matrix B.

Steps to Follow



Note: This will not change the matrix stored in matAns as the answer of above calculation is not a matrix. However, the answer of this calculation would store in Ans.



The answer of last performed calculation automatically stores in the matrix variable matAns. So to find $A \times (2B - 3C)$ perform the following



Note that, the matAns will automatically save the new answer i.e. $A \times (2B - 3C)$.

ii. Multiplicative inverse of matrix B. i.e. B^{-1} .

Steps to Follow:







To view an element in simple fraction, use navigational keys $(\mathbf{D}, \mathbf{O}, \mathbf{O})$ and (\mathbf{O}, \mathbf{O})

iii. determinant of matrix B.

Steps to Follow



Note: This will not change the matrix stored in matAns as the answer of above calculation is not a matrix. However, the answer of this calculation would store in Ans.

iv. adjoint of matrix C.

Steps to Follow

The calculator does not have a feature to calculate adjoint of a matrix. However, we can find it by using concept of inverse of a matrix i.e. $C^{-1} = \frac{1}{|C|} \cdot adjC \Rightarrow adjC = |C| \cdot C^{-1}$

So, to find adjoint of C, press following sequence of keys:



To view the complete entry, press



so the adjoint of C is
$$\begin{bmatrix} 2 & -2 & 0 \\ 1 & 1 & 0 \\ -6 & 6 & 4 \end{bmatrix}$$

v. B^3

Steps to Follow



vi. prove that $(AB)^t = B^t A^t$

Steps to Follow:

Finding $(AB)^t$



iv. adjoint of matrix C.

Steps to Follow

The calculator does not have a feature to calculate adjoint of a matrix. However, we can find it by using concept of inverse of a matrix i.e. $C^{-1} = \frac{1}{|C|} \cdot adjC \Rightarrow adjC = |C| \cdot C^{-1}$

So, to find adjoint of C, press following sequence of keys:



To view the complete entry, press



so the adjoint of C is
$$\begin{bmatrix} 2 & -2 & 0 \\ 1 & 1 & 0 \\ -6 & 6 & 4 \end{bmatrix}$$

v. B^3

Steps to Follow



vi. prove that $(AB)^t = B^t A^t$

Steps to Follow:

Finding $(AB)^t$





so
$$(AB)^t = \begin{bmatrix} 2 & 1 \\ 1 & 5 \\ 3 & 9 \end{bmatrix}$$
.

Now finding $B^t A^t$







Hence proved that $(AB)^t = B^t A^t$

Guided Activity 4:

Solve the following system of linear equations by using AX = B method and also verify your answer by using EQN mode of calculator.

$$x + y = 3$$
$$2x - 3y + z = -1$$
$$3y + z = 9$$

Steps to Follow:

The matrix equation for above system of linear equation is

[1	1	0	$\int x^{-}$		[3]
2	-3	1	y	=	-1
0	3	1	z		9

i.e.

To find the multiplicative inverse of matrix A, apply following operations:





so
$$(AB)^t = \begin{bmatrix} 2 & 1 \\ 1 & 5 \\ 3 & 9 \end{bmatrix}$$
.

Now finding $B^t A^t$







Hence proved that $(AB)^t = B^t A^t$

Guided Activity 4:

Solve the following system of linear equations by using AX = B method and also verify your answer by using EQN mode of calculator.

$$x + y = 3$$
$$2x - 3y + z = -1$$
$$3y + z = 9$$

Steps to Follow:

The matrix equation for above system of linear equation is

[1	1	0	$\int x^{-}$		[3]
2	-3	1	y	=	-1
0	3	1	z		9

i.e.

To find the multiplicative inverse of matrix A, apply following operations:





And now, enter the data as follows:



To find the inverse of matrix A,



To store matrix B,





the A^{-1} is stored in MatAns. To multiply A^{-1} and B , we proceed as follows:



Hence, $A^{-1}B = X = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ and therefore, x = 1, y = 2, z = 3.

Verification using EQN mode

The EQN mode facilitates to solve a system of three linear equations directly.

To convert your calculator in EQN mode, press



And now, enter the data as follows:



To find the inverse of matrix A,



To store matrix B,





the A^{-1} is stored in MatAns. To multiply A^{-1} and B , we proceed as follows:



Hence, $A^{-1}B = X = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ and therefore, x = 1, y = 2, z = 3.

Verification using EQN mode

The EQN mode facilitates to solve a system of three linear equations directly.

To convert your calculator in EQN mode, press





- 5. Adjoint of C (Hint: $adj C = |C| \cdot C^{-1}$)

4. Multiplicative inverse of C

Activity Sheet (Matrices)

2. Find transpose of AB

3. |A|

망 : 달 Ans:

Ans

Ans



-11

 $\mathbf{2}$

1. 416 0. 25









So, the answers have been verified.

1:anX+bnY=Cn 2:anX+bnY+cnZ=dn AC MODE 5 (EQN) 3: aX2+bX+c=0 (3+bX2+cX+d=0)



Now, enter the values of a, b, c and d





- 5. Adjoint of C (Hint: $adj C = |C| \cdot C^{-1}$)

4. Multiplicative inverse of C

Activity Sheet (Matrices)

2. Find transpose of AB

3. |A|

망 : 달 Ans:

Ans

Ans



-11

 $\mathbf{2}$

1. 416 0. 25









So, the answers have been verified.

1:anX+bnY=Cn 2:anX+bnY+cnZ=dn AC MODE 5 (EQN) 3: aX2+bX+c=0 (3+bX2+cX+d=0)



Now, enter the values of a, b, c and d

4. Statistics

The calculator can be used to calculate Permutation, Combination and Probabilities. The calculations can be performed in the COMP or STAT mode of the calculator.

To bring calculator in COMP mode, press MODE 1 (COMP).

Guided Activity 1:

Find the value of 5!.

Steps to follow:



Guided Activity 2:

Find the value of $\frac{5!}{2!\times 3!}$.

Steps to Follow:



Guided Activity 3:

Find the value of ${}^{5}C_{2}$.

Steps to Follow:



Guided Activity 4:

Find the value of ${}^{5}P_{2}$

Steps to Follow:



Guided Activity 5:

In a Mathematics quiz, scores of 20 students are given as follows:

9 3 5 3 8 7 9 4 3 6 10 5 7 8 9 10 5 3 8 6

For the given data, calculate

- i. Mean
- ii. Range of the data
- iii. $\sum X^2$

Steps to follow:

To perform these calculations your calculator should be in *Statistics Mode*. We can enter in the Statistics Mode by pressing



To apply simple statistical operations based on data related to one variable, press



(Upon pressing 1, if your calculator shows screen as



Then turn the frequency off by pressing



Guide Book for CASIO fx-991ES PLUS | 4. Statistics 36 Mathematics Association of Pakistan

4. Statistics

The calculator can be used to calculate Permutation, Combination and Probabilities. The calculations can be performed in the COMP or STAT mode of the calculator.

To bring calculator in COMP mode, press MODE 1 (COMP).

Guided Activity 1:

Find the value of 5!.

Steps to follow:



Guided Activity 2:

Find the value of $\frac{5!}{2!\times 3!}$.

Steps to Follow:



Guided Activity 3:

Find the value of ${}^{5}C_{2}$.

Steps to Follow:



Guided Activity 4:

Find the value of ${}^{5}P_{2}$

Steps to Follow:



Guided Activity 5:

In a Mathematics quiz, scores of 20 students are given as follows:

9 3 5 3 8 7 9 4 3 6 10 5 7 8 9 10 5 3 8 6

For the given data, calculate

- i. Mean
- ii. Range of the data
- iii. $\sum X^2$

Steps to follow:

To perform these calculations your calculator should be in *Statistics Mode*. We can enter in the Statistics Mode by pressing



To apply simple statistical operations based on data related to one variable, press



(Upon pressing 1, if your calculator shows screen as



Then turn the frequency off by pressing



Guide Book for CASIO fx-991ES PLUS | 4. Statistics 36 Mathematics Association of Pakistan Enter the data as follows



i. <u>To calculate Mean of the data</u>

Press the following sequence of the keys and observe the screen.



ii. <u>To calculate Range of the data</u>

Press the following sequence of the keys and watch the screens.





So, Range of the data is 7.

iii. To Calculate
$$\sum X^2$$

Press the following sequence of the keys and watch the screens.



Guided Activity 6:

The following table gives the number of goals scored in last 40 matches in the National Hockey Championship.

No. of Goals	Number of Matches
	(Frequency)
0	7
1	8
2	5
3	4
4	12
5	3
6	1

Calculate the mean, standard deviation and variance of the data.

Enter the data as follows



i. <u>To calculate Mean of the data</u>

Press the following sequence of the keys and observe the screen.



ii. <u>To calculate Range of the data</u>

Press the following sequence of the keys and watch the screens.





So, Range of the data is 7.

iii. To Calculate
$$\sum X^2$$

Press the following sequence of the keys and watch the screens.



Guided Activity 6:

The following table gives the number of goals scored in last 40 matches in the National Hockey Championship.

No. of Goals	Number of Matches
	(Frequency)
0	7
1	8
2	5
3	4
4	12
5	3
6	1

Calculate the mean, standard deviation and variance of the data.

Steps to Follow:

Since this question requires frequencies, so to enter the frequencies we need to press the following sequence of calculator keys:

1:MthIO 2:LineIO 4:Rad 3:Deg AC SHIFT MODE 5:Gra 6:Fix 7:Sci 8:Norm 2:d/c 1:ab/c 3:CMPLX 4:STAT $\overline{}$ 5:Disp 6: CONT Frequency? 1:0N 2:0FF **4** 1 n.

Now it's time to enter the data



To enter the frequency, we make use of navigation/scroll keys



Press the following sequence of the keys and watch the screens.







Since we are successfully entered the data, now we can find the mean as follows



And we can find standard deviation as follows



Since, variance = $(standard deviation)^2$



Steps to Follow:

Since this question requires frequencies, so to enter the frequencies we need to press the following sequence of calculator keys:

1:MthIO 2:LineIO 4:Rad 3:Deg AC SHIFT MODE 5:Gra 6:Fix 7:Sci 8:Norm 2:d/c 1:ab/c 3:CMPLX 4:STAT $\overline{}$ 5:Disp 6: CONT Frequency? 1:0N 2:0FF **4** 1 n.

Now it's time to enter the data



To enter the frequency, we make use of navigation/scroll keys



Press the following sequence of the keys and watch the screens.







Since we are successfully entered the data, now we can find the mean as follows



And we can find standard deviation as follows



Since, variance = $(standard deviation)^2$



Activity Sheet (Statistics)

The following table give information about number of employees absent during the first 15 working days in a company i

No. of employees absent	Frequency	
15	3	
16	2	
18	6	
19	1	
20	3	

calculate

1. Mean

3. $\sum X$

4. $\sum X^2$



5. Trigonometry

Most of the trigonometric calculations can be performed in simple computational (COMP) mode. Carry out following actions to convert your calculator in Computational Mode if it is already not in this mode.



'Math' at the top right corner of the screen indicates that you are in simple computation mode.

Guided Activity 1:

Find the sexagesimal equivalent of the angle measuring 2.31° .

Steps to Follow:



i.e.

 $2.31^\circ = 2^\circ 18'36''$

Guided Activity 2:

Convert sexagesimal angle $0^{\circ}25'10''$ into decimal fraction.

Steps to Follow:



Activity Sheet (Statistics)

The following table give information about number of employees absent during the first 15 working days in a company i

No. of employees absent	Frequency	
15	3	
16	2	
18	6	
19	1	
20	3	

calculate

1. Mean

3. $\sum X$

4. $\sum X^2$



5. Trigonometry

Most of the trigonometric calculations can be performed in simple computational (COMP) mode. Carry out following actions to convert your calculator in Computational Mode if it is already not in this mode.

'Math' at the top right corner of the screen indicates that you are in simple computation mode.

Guided Activity 1:

Find the sexagesimal equivalent of the angle measuring 2.31° .

Steps to Follow:

i.e.

 $2.31^\circ = 2^\circ 18'36''$

Guided Activity 2:

Convert sexagesimal angle $0^{\circ}25'10''$ into decimal fraction.

Steps to Follow:

So $0^{\circ}25'10''$ is equal to 0.4194444444° .

Guided Activity 3:

Convert $\frac{19\pi}{32}$ radian into sexagesimal system.

Steps to Follow:

First we ensure to convert the calculator in degree mode, we proceed as follows:

First we will convert
$$\frac{19\pi}{32}$$
 radian into the degree measures by Pressing

Guide Book for CASIO fx-991ES PLUS | 5. Trigonometry

Mathematics Association of Pakistan

 $\operatorname{So}\frac{19\pi}{32}$ radian is equivalent to $106^{\circ}52'30$.

Guided Activity 4:

Simplify $10^{\circ}10'' + 2'25''$.

Steps to Follow:

So, $10^{\circ}10'' + 2'25''$ or $10^{\circ}0'10'' + 0^{\circ}2'25'' = 10^{\circ}2'35''$ or 10.04305556° .

Guided Activity 5:

Convert 120° into radian measure.

Steps to Follow:

For this conversion, calculator should be in *Radian Mode*, so perform following actions.

Note: R at the top of the screen indicates that the calculator is in radian mode.

Now, to convert 120° into radian measure, press calculator's keys as follows:

So $0^{\circ}25'10''$ is equal to 0.4194444444° .

Guided Activity 3:

Convert $\frac{19\pi}{32}$ radian into sexagesimal system.

Steps to Follow:

First we ensure to convert the calculator in degree mode, we proceed as follows:

First we will convert
$$\frac{19\pi}{32}$$
 radian into the degree measures by Pressing

Guide Book for CASIO fx-991ES PLUS | 5. Trigonometry

Mathematics Association of Pakistan

 $\operatorname{So}\frac{19\pi}{32}$ radian is equivalent to $106^{\circ}52'30$.

Guided Activity 4:

Simplify $10^{\circ}10'' + 2'25''$.

Steps to Follow:

So, $10^{\circ}10'' + 2'25''$ or $10^{\circ}0'10'' + 0^{\circ}2'25'' = 10^{\circ}2'35''$ or 10.04305556° .

Guided Activity 5:

Convert 120° into radian measure.

Steps to Follow:

For this conversion, calculator should be in *Radian Mode*, so perform following actions.

Note: R at the top of the screen indicates that the calculator is in radian mode.

Now, to convert 120° into radian measure, press calculator's keys as follows:

Guided Activity 6:

Convert -25.5° into radian measure

Steps to Follow:

Hence -25.5° is equivalent to $-\frac{17}{120}\pi$ radian .

Guided Activity 7:

Convert $\frac{3\pi}{2}$ radian into degree measure.

Steps to Follow:

For this we need calculator in *Degree Mode*; therefore, we perform the following actions:

Note: D at the top of the screen indicates that the calculator is in degree mode.

To convert $\frac{3\pi}{2}$ radianinto degree measure press following sequence of calculator's keys.

Hence
$$\frac{3\pi}{2}$$
 radian is equivalent to 270 °.

General Solution of a Trigonometric Equation

The following table can help us in writing the general solution:

Table -I				
	Principle angle from calculator	Formula for second angle, if it exists	General Solution	
$\sin^{-1}(a) = ?$	b	$\pi - b$	${b+2n\pi} \cup {(\pi-b)+2n\pi}; n \in \mathbb{Z}$	
$\cos^{-1}(a) = ?$	b	$2\pi - b$	$\{b+2n\pi\}\cup\{(2\pi-b)+2n\pi\};n\in\mathbb{Z}$	
$\tan^{-1}(a) = ?$	b	$\pi + b$	$\{b+n\pi\}; n\in \mathbb{Z}$	

Guided Activity 8:

Steps to Follow:

To find the solution of this equation, calculator should be in the *Radian Mode*. Therefore, press the following sequence of keys to convert it into radian mode.

AC SHIFT MODE	1:MthIO 3:Deg 5:Gra	2:LineIO 4:Rad 6:Fix
	7:SCi	8:Norm

Guided Activity 6:

Convert -25.5° into radian measure

Steps to Follow:

Hence -25.5° is equivalent to $-\frac{17}{120}\pi$ radian .

Guided Activity 7:

Convert $\frac{3\pi}{2}$ radian into degree measure.

Steps to Follow:

For this we need calculator in *Degree Mode*; therefore, we perform the following actions:

Note: D at the top of the screen indicates that the calculator is in degree mode.

To convert $\frac{3\pi}{2}$ radianinto degree measure press following sequence of calculator's keys.

Hence
$$\frac{3\pi}{2}$$
 radian is equivalent to 270 °.

General Solution of a Trigonometric Equation

The following table can help us in writing the general solution:

Table -I				
	Principle angle from calculator	Formula for second angle, if it exists	General Solution	
$\sin^{-1}(a) = ?$	b	$\pi - b$	${b+2n\pi} \cup {(\pi-b)+2n\pi}; n \in \mathbb{Z}$	
$\cos^{-1}(a) = ?$	b	$2\pi - b$	$\{b+2n\pi\}\cup\{(2\pi-b)+2n\pi\};n\in\mathbb{Z}$	
$\tan^{-1}(a) = ?$	b	$\pi + b$	$\{b+n\pi\}; n\in \mathbb{Z}$	

Guided Activity 8:

Steps to Follow:

To find the solution of this equation, calculator should be in the *Radian Mode*. Therefore, press the following sequence of keys to convert it into radian mode.

AC SHIFT MODE	1:MthIO 3:Deg 5:Gra	2:LineIO 4:Rad 6:Fix
	7:SCi	8:Norm

Now to solve given equations we proceed as given below.

AC [SHF]
$$\sin(\sin^{-1})$$
] = 2 = $\sin^{-1}(\frac{1}{2}$

Hence $\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$ and to calculate the second value of x we may use the Table-I and press the keys as given below.

Therefore, $\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$ and $\frac{5\pi}{6}$.

The general solution of the equation is $\left\{\frac{\pi}{6} + 2n\pi\right\} \cup \left\{\frac{5\pi}{6} + 2n\pi\right\}; n \in \mathbb{Z}$

Activity Sheet (Trigonometry)

1. Convert
$$\frac{3\pi}{7}$$
 radian into sexagesimal system:

2. Simplify $2'1'' - 2^{\circ}50''$ and express the result in degree measure.

	0°2°1°-2°0°50°
Ans:	-1°58'49"

3. Convert -115.5° into radian measure.

	-115.5°	3	Math 🔺
Ans:		-	- <u>77</u> 120 ^π

Now to solve given equations we proceed as given below.

AC [SHF]
$$\sin(\sin^{-1})$$
] = 2 = $\sin^{-1}(\frac{1}{2}$

Hence $\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$ and to calculate the second value of x we may use the Table-I and press the keys as given below.

Therefore, $\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$ and $\frac{5\pi}{6}$.

The general solution of the equation is $\left\{\frac{\pi}{6} + 2n\pi\right\} \cup \left\{\frac{5\pi}{6} + 2n\pi\right\}; n \in \mathbb{Z}$

Activity Sheet (Trigonometry)

1. Convert
$$\frac{3\pi}{7}$$
 radian into sexagesimal system:

2. Simplify $2'1'' - 2^{\circ}50''$ and express the result in degree measure.

	0°2°1°-2°0°50°
Ans:	-1°58'49"

3. Convert -115.5° into radian measure.

	-115.5°	3	Math 🔺
Ans:		-	- <u>77</u> 120 ^π

Calculus

Calculus deals with motion and change and it is applicable to measure the rate of change e.g. velocity and acceleration, growth and slopes of curves.

The calculus with the help of calculator will save our time of calculation but one need to have a good command over the concepts of calculus to use the right functions and sequence of keys to be pressed.

Note: Your calculator should be in Computational Mode

Guided Activity 1:

Evaluate
$$\frac{d}{dx}\left(\frac{1}{1-x^2}\right)$$
 at $x = -2$

Steps to follow:

Guided Activity 2:

Evaluate
$$\frac{d}{dx}(\cos ec^2 x)$$
 at $x = \frac{\pi}{2}$ and at $x = \frac{\pi}{6}$

Steps to follow:

49

For this question, we have to convert our calculator into radian mode first. So enter

To evaluate the derivative of the function at $x = \frac{\pi}{6}$, pressing will allow us to edit the last performed calculation.

Guided Activity 3:

Evaluate
$$\frac{d}{dx}(e^{3x} - \ln x^2)$$
 at $x = 1$

<u>Steps to follow:</u>

As the calculation does not involve trigonometric function, it can be performed either in degree or radian mode.

Calculus

Calculus deals with motion and change and it is applicable to measure the rate of change e.g. velocity and acceleration, growth and slopes of curves.

The calculus with the help of calculator will save our time of calculation but one need to have a good command over the concepts of calculus to use the right functions and sequence of keys to be pressed.

Note: Your calculator should be in Computational Mode

Guided Activity 1:

Evaluate
$$\frac{d}{dx}\left(\frac{1}{1-x^2}\right)$$
 at $x = -2$

Steps to follow:

Guided Activity 2:

Evaluate
$$\frac{d}{dx}(\cos ec^2 x)$$
 at $x = \frac{\pi}{2}$ and at $x = \frac{\pi}{6}$

Steps to follow:

49

For this question, we have to convert our calculator into radian mode first. So enter

To evaluate the derivative of the function at $x = \frac{\pi}{6}$, pressing will allow us to edit the last performed calculation.

Guided Activity 3:

Evaluate
$$\frac{d}{dx}(e^{3x} - \ln x^2)$$
 at $x = 1$

<u>Steps to follow:</u>

As the calculation does not involve trigonometric function, it can be performed either in degree or radian mode.

Guided Activity 4:

Evaluate
$$\frac{d}{dx}(\log_5 x)$$
 at $x=3$

Steps to follow:

Instant Mathematics: As
$$\log_a b = \frac{\ln b}{\ln a}$$
, therefore $\frac{d}{dx}(\log_5 x) = \frac{d}{dx}\left(\frac{\ln x}{\ln 5}\right)$

Now its time to use the calculator,

Or

We can also evaluate the derivative of the function directly as

Guided Activity 5:

Steps to follow:

Because calculator does not have the direct feature for $\cot x$, we may use the relation $\cot x = \frac{1}{\tan x}$ to evaluate this question.

Guided Activity 4:

Evaluate
$$\frac{d}{dx}(\log_5 x)$$
 at $x=3$

Steps to follow:

Instant Mathematics: As
$$\log_a b = \frac{\ln b}{\ln a}$$
, therefore $\frac{d}{dx}(\log_5 x) = \frac{d}{dx}\left(\frac{\ln x}{\ln 5}\right)$

Now its time to use the calculator,

Or

We can also evaluate the derivative of the function directly as

Guided Activity 5:

Steps to follow:

Because calculator does not have the direct feature for $\cot x$, we may use the relation $\cot x = \frac{1}{\tan x}$ to evaluate this question.

Guided Activity 6:

Evaluate $\int_{1.5}^{2} \sec^{-1} x \, dx$

Steps to follow:

So
$$\int_{1.5}^{2} \sec^{-1} x \, dx = 0.478$$

Guided Activity 7:

Find the area under the curve $y = \sqrt{5 - x^2}$ between $-\sqrt{5}$ and $\sqrt{5}$.

Steps to follow:

Required Area =
$$\int_{-\sqrt{5}}^{\sqrt{5}} \sqrt{5 - x^2} dx$$

Note: Its time for your calculator tobe in Radian Mode

(Note: after pressing , there will be a very long delay before the calculator displays answer)

So, the required area under the curve is 7.854 sq. unit.

Guided Activity 8:

Find the equation of tangent to the curve $y = x^2 - 4$ at (2,0).

Steps to follow:

The slope of the tangent is $\frac{d}{dx}(x^2-4)|x=2$, therefore

So, the slope of the tangent is 4. Using the general form of straight line $y - y_1 = m(x - x_1)$, we get the required equation of tangent i.e. y = 4x - 8

Guided Activity 6:

Evaluate $\int_{1.5}^{2} \sec^{-1} x \, dx$

Steps to follow:

So
$$\int_{1.5}^{2} \sec^{-1} x \, dx = 0.478$$

Guided Activity 7:

Find the area under the curve $y = \sqrt{5 - x^2}$ between $-\sqrt{5}$ and $\sqrt{5}$.

Steps to follow:

Required Area =
$$\int_{-\sqrt{5}}^{\sqrt{5}} \sqrt{5 - x^2} dx$$

Note: Its time for your calculator tobe in Radian Mode

(Note: after pressing , there will be a very long delay before the calculator displays answer)

So, the required area under the curve is 7.854 sq. unit.

Guided Activity 8:

Find the equation of tangent to the curve $y = x^2 - 4$ at (2,0).

Steps to follow:

The slope of the tangent is $\frac{d}{dx}(x^2-4)|x=2$, therefore

So, the slope of the tangent is 4. Using the general form of straight line $y - y_1 = m(x - x_1)$, we get the required equation of tangent i.e. y = 4x - 8

Activity Sheet (Calculus)

1. Find derivative of $f(x) = \sin x - \cot x$ at $x = \frac{\pi}{3}$.

_告(cos¹(X)-√X)[ݤ

-1.86180732

Math 🔺

- 2. Evaluate $\frac{d}{dx} \left(\cos^{-1} x \sqrt{x} \right)$ at $x = \frac{1}{2}$.
- 3. Find the slope of the tangent to the curve $y = (x-3)^2 + 3$ at x = 2.

(Hint: The slope of the tangent to a curve is equal to first derivative of the function at the point of tangency)

Ans:

4. Evaluate
$$\int_{0}^{\frac{\pi}{6}} \sin^2 x dx$$
.

Ans:
$$0.04529303685$$

5. Find the area under the curve
$$\frac{(x-3)^2}{9} + \frac{y^2}{16} = 1$$
 for $x = 3$ to $x = 4$.
(Hint: $\frac{(x-3)^2}{9} + \frac{y^2}{16} = 1$ can also be written as $y = \frac{\sqrt{144 - 16(x-3)^2}}{3}$)

$$\int \frac{4\sqrt{144 - 16(x-3)^2}}{3} = \frac{\sqrt{144 - 16(x-3)^2}}{3}$$
Ans: $3 = 92463954$

Vectors

A vector is a mathematical entity which needs magnitude and direction to specify it.

A vector can be rendered on the page like this:

Guided Activity 1:

Find the magnitude of the vector $\vec{A} = 2\hat{i} - 3\hat{j} + \hat{k}$

Steps to follow:

First we need to bring our calculator to the Vector Mode as follows:

1:COMP 2:CMPLX STAT BASE-N AC MODE EQN IATRIX TABI F 8: VECTOR Vector? 2:VctB 1:VctA 8 3:VctC VctA(m) m? 2:2 1:3 1

It's time to define the dimension of the vector A and since A is a three dimensional vector, therefore we select or press 1.

To find the magnitude of the vector $\vec{A} = 2\hat{i} - 3\hat{j} + \hat{k}$ press the following sequence of button.

Guide Book for CASIO fx-991ES PLUS | Vectors 56 Mathematics Association of Pakistan

Activity Sheet (Calculus)

1. Find derivative of $f(x) = \sin x - \cot x$ at $x = \frac{\pi}{3}$.

_告(cos¹(X)-√X)[ݤ

-1.86180732

Math 🔺

- 2. Evaluate $\frac{d}{dx} \left(\cos^{-1} x \sqrt{x} \right)$ at $x = \frac{1}{2}$.
- 3. Find the slope of the tangent to the curve $y = (x-3)^2 + 3$ at x = 2.

(Hint: The slope of the tangent to a curve is equal to first derivative of the function at the point of tangency)

Ans:

4. Evaluate
$$\int_{0}^{\frac{\pi}{6}} \sin^2 x dx$$
.

Ans:
$$0.04529303685$$

5. Find the area under the curve
$$\frac{(x-3)^2}{9} + \frac{y^2}{16} = 1$$
 for $x = 3$ to $x = 4$.
(Hint: $\frac{(x-3)^2}{9} + \frac{y^2}{16} = 1$ can also be written as $y = \frac{\sqrt{144 - 16(x-3)^2}}{3}$)

$$\int \frac{4\sqrt{144 - 16(x-3)^2}}{3} = \frac{\sqrt{144 - 16(x-3)^2}}{3}$$
Ans: $3 = 92463954$

Vectors

A vector is a mathematical entity which needs magnitude and direction to specify it.

A vector can be rendered on the page like this:

Guided Activity 1:

Find the magnitude of the vector $\vec{A} = 2\hat{i} - 3\hat{j} + \hat{k}$

Steps to follow:

First we need to bring our calculator to the Vector Mode as follows:

1:COMP 2:CMPLX STAT BASE-N AC MODE EQN IATRIX TABI F 8: VECTOR Vector? 2:VctB 1:VctA 8 3:VctC VctA(m) m? 2:2 1:3 1

It's time to define the dimension of the vector A and since A is a three dimensional vector, therefore we select or press 1.

To find the magnitude of the vector $\vec{A} = 2\hat{i} - 3\hat{j} + \hat{k}$ press the following sequence of button.

Guide Book for CASIO fx-991ES PLUS | Vectors 56 Mathematics Association of Pakistan

Guided Activity 2:

For three vectors, $\vec{A} = \hat{i} - \hat{j} + 2\hat{k}$, $\vec{B} = 2\hat{i} + 3\hat{k}$ and $\vec{C} = -\hat{i} + 2\hat{j} + 3\hat{k}$ find:

- i. $\vec{A} \bullet \vec{B}$
- ii. $\vec{A} \times \vec{B}$
- iii. $\vec{A} \bullet (\vec{B} \times \vec{C})$
- iv. Angle between vectors A and B.

Steps to follow:

First we enter the three vectors A, B and C.

Entering vector

$$\vec{A} = \hat{i} - \hat{j} + 2\hat{k}$$

(The currently stored vector is shown on the calculator)

Enter the x, y and z component of vector A as given in this question

Entering vector

i. <u>Steps to follow to calculate $\vec{A} \cdot \vec{B}$ </u>

Guided Activity 2:

For three vectors, $\vec{A} = \hat{i} - \hat{j} + 2\hat{k}$, $\vec{B} = 2\hat{i} + 3\hat{k}$ and $\vec{C} = -\hat{i} + 2\hat{j} + 3\hat{k}$ find:

- i. $\vec{A} \bullet \vec{B}$
- ii. $\vec{A} \times \vec{B}$
- iii. $\vec{A} \bullet (\vec{B} \times \vec{C})$
- iv. Angle between vectors A and B.

Steps to follow:

First we enter the three vectors A, B and C.

Entering vector

$$\vec{A} = \hat{i} - \hat{j} + 2\hat{k}$$

(The currently stored vector is shown on the calculator)

Enter the x, y and z component of vector A as given in this question

Entering vector

i. <u>Steps to follow to calculate $\vec{A} \cdot \vec{B}$ </u>

2:Data 4:VctB

2:VctB

VCT 🖬

VCT 🖪

0

2 🔳

2:Data

4:VctB

2:Data 4:VctB

2:Data 4:VctB

6:VctAns

6:VctAns

VCT 🖬

VCT 🖪

6:VctAns

0)

0

3

0

0

E]

6:VctAns

2:Data 4:VctB

2:VctB

VCT 🖬

VCT 🖪

0

2 🔳

2:Data

4:VctB

2:Data 4:VctB

2:Data 4:VctB

6:VctAns

6:VctAns

VCT 🖬

VCT 🖪

6:VctAns

0)

0

3

0

0

E]

6:VctAns

iv. <u>Steps to follow in finding the angle between vectors A and B</u>

Instant Mathematics: If angle between vectors A and B is θ , then using formula

 $\theta = \cos^{-1} \left(\frac{\vec{A} \bullet \vec{B}}{\left| \vec{A} \right| \left| \vec{B} \right|} \right)$

First, finding $\frac{\vec{A} \cdot \vec{B}}{|\vec{A}||\vec{B}|}$ with the help of calculator

61 Guide Book for CASIO fx-991ES PLUS | Vectors Mathematics Association of Pakistan

The above answer is stored in the calculator's memory and can be recall through Ans key.

Now finding required angle as follows:

iv. <u>Steps to follow in finding the angle between vectors A and B</u>

Instant Mathematics: If angle between vectors A and B is θ , then using formula

 $\theta = \cos^{-1} \left(\frac{\vec{A} \bullet \vec{B}}{\left| \vec{A} \right| \left| \vec{B} \right|} \right)$

First, finding $\frac{\vec{A} \cdot \vec{B}}{|\vec{A}||\vec{B}|}$ with the help of calculator

61 Guide Book for CASIO fx-991ES PLUS | Vectors Mathematics Association of Pakistan

So, $\frac{\vec{A} \bullet \vec{B}}{\left|\vec{A}\right| \left|\vec{B}\right|} = 0.9058$

The above answer is stored in the calculator's memory and can be recall through Ans key.

Now finding required angle as follows:

As the calculator is in *Radian Mode*, the angle between two vectors A and B is 0.43748 radians.

To convert this answer in degrees, if needed, follow the sequence as below:

SHIFT MODE	1:MthIO 3:De9 5:Gra 7:Sci	2:LineIO 4:Rad 6:Fix 8:Norm
3 🗖	cos ⁻¹ (Ans	vcra B
	64.	.05671494

So, the angle between the two vectors A and B is 64.0567 degrees.

And to find this angle in sexagesimal system

i.e. the angle between two vectors A and B is $64^{\circ}3'24.17''$.

Activity Sheet (Vectors)

1. Find a unit vector in the direction of $\vec{A} = 2\hat{i} - 3\hat{j} + \hat{k}$.

(Hint: If \hat{a} is a unit vector in the direction of \vec{A} , then as per the definition of unit vector $\hat{a} = \frac{\vec{A}}{|\vec{A}|}$.)

	Verma Ans Di Museuus -0.80 1 0.2612)
Ans:	0.5345224838

- 2. Calculate the projection of $\vec{A} = 2\hat{i} 3\hat{j} + \hat{k}$ along $\vec{B} = \hat{i} + 2\hat{k}$. (Hint: Projection of \vec{A} along \vec{B} is equal to $\frac{\vec{A} \cdot \vec{B}}{|\vec{B}|}$) (VctA · VctB) ÷Abs Ans: 1.788854382
- 3. Are three vectors $\hat{i} 3\hat{j} + 5\hat{k}$, $3\hat{i} 2\hat{j} + \hat{k}$ and $2\hat{i} + \hat{j} 4\hat{k}$ form a right

angle?

(Hint: If three vectors \vec{A} , \vec{B} and \vec{C} form a right angle, then $\vec{A} \bullet (\vec{B} \times \vec{C}) = 0$)

	VctA•(VctB×VctC)
Ans:	0

4. Find a unit vector perpendicular to the plane containing $\vec{A} = \hat{i} - 3\hat{j} + 5\hat{k}$ and $\vec{B} = 3\hat{i} - 2\hat{j} + \hat{k}$.

(Hint: A unit vector perpendicular to the plane is $\frac{\vec{A} \times \vec{B}}{|\vec{A} \times \vec{B}|}$)

As the calculator is in *Radian Mode*, the angle between two vectors A and B is 0.43748 radians.

To convert this answer in degrees, if needed, follow the sequence as below:

SHIFT MODE	1:MthIO 3:De9 5:Gra 7:Sci	2:LineIO 4:Rad 6:Fix 8:Norm
3 🗖	cos ⁻¹ (Ans	vcra B
	64.	.05671494

So, the angle between the two vectors A and B is 64.0567 degrees.

And to find this angle in sexagesimal system

i.e. the angle between two vectors A and B is $64^{\circ}3'24.17''$.

Activity Sheet (Vectors)

1. Find a unit vector in the direction of $\vec{A} = 2\hat{i} - 3\hat{j} + \hat{k}$.

(Hint: If \hat{a} is a unit vector in the direction of \vec{A} , then as per the definition of unit vector $\hat{a} = \frac{\vec{A}}{|\vec{A}|}$.)

	Verma Ans Di Museuus -0.80 1 0.2612)
Ans:	0.5345224838

- 2. Calculate the projection of $\vec{A} = 2\hat{i} 3\hat{j} + \hat{k}$ along $\vec{B} = \hat{i} + 2\hat{k}$. (Hint: Projection of \vec{A} along \vec{B} is equal to $\frac{\vec{A} \cdot \vec{B}}{|\vec{B}|}$) (VctA · VctB) ÷Abs Ans: 1.788854382
- 3. Are three vectors $\hat{i} 3\hat{j} + 5\hat{k}$, $3\hat{i} 2\hat{j} + \hat{k}$ and $2\hat{i} + \hat{j} 4\hat{k}$ form a right

angle?

(Hint: If three vectors \vec{A} , \vec{B} and \vec{C} form a right angle, then $\vec{A} \bullet (\vec{B} \times \vec{C}) = 0$)

	VctA•(VctB×VctC)
Ans:	0

4. Find a unit vector perpendicular to the plane containing $\vec{A} = \hat{i} - 3\hat{j} + 5\hat{k}$ and $\vec{B} = 3\hat{i} - 2\hat{j} + \hat{k}$.

(Hint: A unit vector perpendicular to the plane is $\frac{\vec{A} \times \vec{B}}{|\vec{A} \times \vec{B}|}$)

