# Activity 1: Let’s Be Rational and Get to the Root

#### CATEGORY: NUMBER & QUANTITY

#### DOMAIN: THE REAL NUMBER SYSTEM

**Extend the properties of exponents to rational exponents**

1. Explain how the definition of the meaning of rational exponents follows from extend-

ing the properties of integer exponents to those values, allowing for a notation for

radicals in terms of rational exponents. For example, we define $5^{\frac{1}{3}}$ to be the cube root

of 5 because we want ($5^{\frac{1}{3}}$ )3 = $5^{\left(\frac{1}{3}\right)·3}$ to hold, so ($5^{\frac{1}{3}}$ )3must equal 5.

1. Rewrite expressions involving radicals and rational exponents using the properties of exponents.

#### LEARNING OBJECTIVES

The idea of extension is extremely important in algebra. Students will be able to

extend the properties of integer exponents to rational exponents in order to make

sense of the definition of rational exponents. Students will also be able to rewrite expressions involving rational exponents.

#### BACKGROUND KNOWLEDGE

Students are expected to recognize the graph of the square root function:

e.g. *f* (*x*) = $\sqrt{x}$ or y = $\sqrt{x}$

#### Work with radicals and integer exponents.

1. Know and apply the properties of integer exponents to

generate equivalent numerical expressions. *For example,* 32 × 3–5 = 3–3 = $\frac{1}{3^{3}}$ = $\frac{1}{27}.$

1. Use square root and cube root symbols to represent solutions to equations of

the form *x*2 = *p* and *x*3 = *p*, where *p* is a positive rational number. Evaluate square

roots of small perfect squares and cube roots of small perfect cubes. Know that

$\sqrt{2}$ is irrational.

#### Activity 1: Getting Started

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#### GETTING STARTED

Consider the meaning of $8^{\frac{1}{3}}$. The power rule for exponents states ($a^{b}$)*c*= $a^{b·c}$. Suppose you

apply this rule to $8^{\frac{1}{3}}$. Try cubing it so that ($ 8^{\frac{1 }{3 }}$)3 = $8^{\frac{1}{3}}$·3 = $8^{1}$ = 8. So, if $8^{\frac{1}{3}}$ means anything at

all, it has to be a number with a cube that is 8. There is only one real number with a cube

that is 8. Therefore it makes sense to define:

 $8^{\frac{1}{3}}$= 2, because 23 = 8

1. Create a TABLE using the values {1 – 10} for the function *f* (*x*)= $x^{\frac{1}{2}}$ , then use

graph paper to plot the ordered pairs. This graph should look familiar to you. Make

conjectures about what other function is equivalent to *f* (*x*)= $x^{\frac{1}{2}}$ . Enter your guess as a

second equation to verify your conjecture. Make comparisons with another classmate.

1. Use r to substitute values into $x^{\frac{1}{2}}$ from the list {1,4,9 ,16,25,36}. Write a conjecture

about the result when raising a base to the one-half power.

1. Use r to substitute values into $x^{\frac{1}{3}}$ from the list {1,8,27,64,125,216} and

write a conjecture about the result when raising a base to the one-third power.

Discuss your conjecture with a classmate that has a different conjecture.

1. Use r to substitute values into $x^{\frac{2}{3}}$ from the list {1,8,27,64,125,216} and write a conjecture about the result when raising a base to the two-thirds power.

#### Activity 1: Getting Started

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#### UNDERSTAND

1. Create a TABLE for *f* (*x*) = $25^{x}$ [for values of *x* = 0 to 5] with incrementing by $\frac{1}{2}$ .

What value corresponds to $25^{\frac{3}{2}}$ ?

1. Study your TABLE from the previous question and explain any relationships you see.
2. How could you find the value of $49^{\frac{3}{2}}$ without a calculator? Check your answer using a

calculator.

1. How could you find the value of $27^{\frac{2}{3}}$ without a calculator? Check your answer using a

calculator and then test your strategy on $8^{\frac{5}{3}}$. Check your answer. Discuss your results

with a classmate.

1. Describe what it means to raise a number to a rational exponent, and generalize

a procedure for simplifying$a^{\frac{b}{c}}$.

#### PRACTICE

1. Solve each equation by rewriting the expression with a rational exponent and using the properties of exponents to find a positive solution.

= 14

4

*x*

9 *x* 5 = 26

3 *x* 8 = 47

1. Simplify each expression in two different ways: a) evaluate the rational exponents, and

b) use the laws of exponents.

a. $27^{\frac{2}{3}}$・ $27^{\frac{1}{3}}$

b. ($64^{\frac{1}{2}}$)($ 64^{\frac{1}{3}}$)

c. ($16^{\frac{3}{4}}$)2

#### EXTEND

1. Use the properties of exponents to show that $a^{\frac{b}{c}}$ = $\sqrt[c]{a^{b}}$ = $\left(\sqrt[c]{a}\right)$*b*
2. Approximate answers to the nearest hundredth.

a. $2\sqrt[5]{x}$+ 5 =18

b. $\sqrt[7]{x^{3}}$= 40

c. $2\sqrt[3]{x^{2}}$ = $\sqrt{17}$