Trigonometric Identities

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LEVEL
High School and University

OBJECTIVES
To use the calculator to study trigonometric identities from several perspectives.

Corresponding eActivity
trigiden.g1e

OVERVIEW
A graphing utility can be a very helpful tool in investigating the existence of an identity. For instance, the graph feature of the calculator can be used to explore whether the left and right hand expressions in an identity to be verified, would give the same graph. If the graphs are identical, then this provides the evidence that both expressions are equal and the given equality is an identity. In the same manner, the table feature of the calculator can provide the functional values of the left and right hand expressions, and can be a means to explore the validity of the given equality.

EXPLORATORY ACTIVITIES
[Note] We shall use small letter x instead of capital X as shown on the calculator throughout the paper. Using SHIFT SET UP we specify that the angle will be in radian mode.

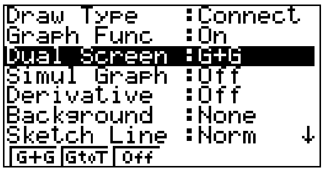
Activity: A function is even if $f(-x) = f(x)$ for all x in the domain of f. A function f is odd if $f(-x)= -f(x)$ for all x in the domain of f. Which of the trigonometric functions have the even property? the odd property?

Solution:

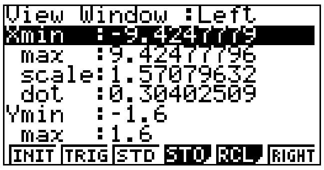
Let us illustrate the problem for the sine function. We verify whether the sine function is even, that is, we check if $\sin(-x) = \sin x$.

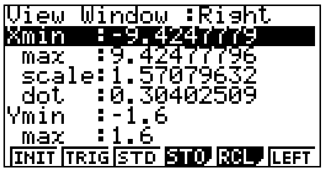
A. <u>Graphical exploration:</u>

a. We open the Graph Editor and use dual screens to explore the identity. We specify that the dual graph feature of the calculator will be used before we start graphing [SHIFT SET UP] as follows:

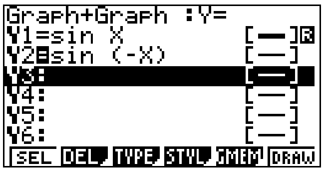


We make sure that the view settings are the same for the active screen(left screen) and the inactive screen(right screen):

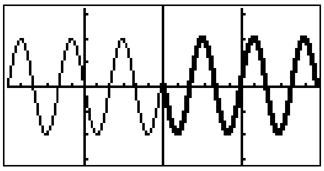




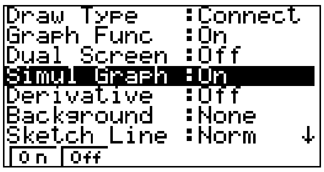
We select sin(*x*) to be graphed on the right screen and sin(-*x*) on the left screen.



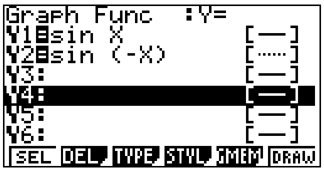
Both graphs are shown on the dual screens below. The learner can observe that the graphs are not identical, and represent two different functions. Thus, sin(-*x*) ≠ sin(*x*).

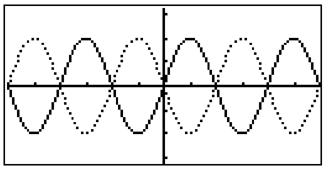


b. Another way to verify the identity is to graph both sin(-*x*) and sin(*x*) simultaneously. We set the Graph Editor to simultaneous mode[SHIFT SET UP] as follows:

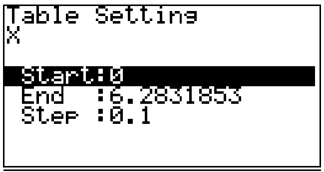


We obtain the following screen dumps which confirm that sin(-*x*) ≠ sin(*x*).

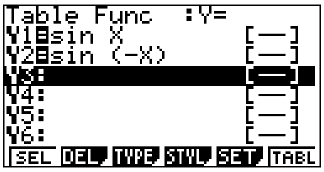


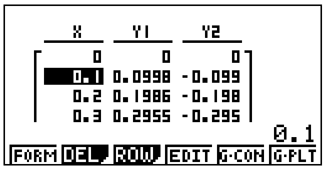


<u>B. Tabular exploration:</u> In the <u>Table Editor</u> we can explore the values of sin(*x*) and sin(-*x*). We use the following table settings:

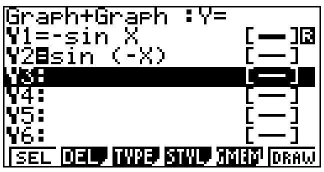


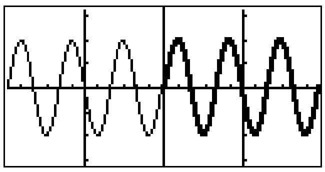
We obtain the following partial list of data that verifies that sin(-*x*) ≠ sin(*x*).



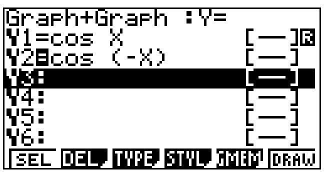


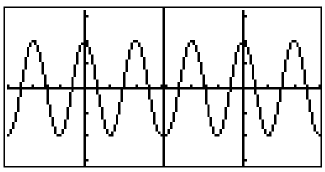
Thus, we can conclude that the sine function is not an even function. However, **the sine function is an odd function**, that is sin(-*x*) = -sin(*x*). Consider the following screen dumps:

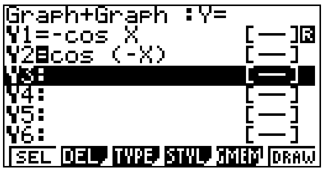


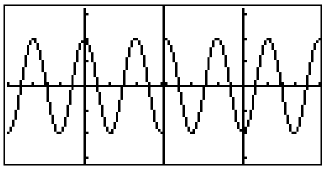


The cosine function is an even function and not an odd function as seen from the following results:

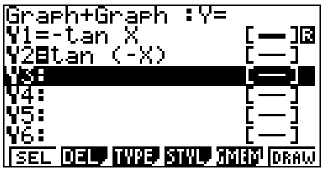


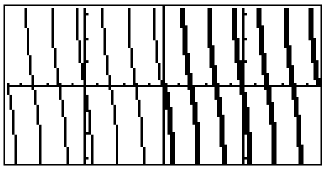


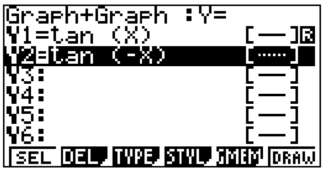




The tangent function is an odd function and not an even function as seen from the following results:







From the fact that $\csc x = \dfrac{1}{\sin x}$, $\sec x = \dfrac{1}{\cos x}$ and $\cot x = \dfrac{1}{\tan x}$ and the result obtained above we conclude that

$$\csc(-x) = \frac{1}{\sin(-x)} = -\frac{1}{\sin x} = -\csc x$$

$$\sec(-x) = \frac{1}{\cos(-x)} = \frac{1}{\cos x} = \sec x$$

$$\cot(-x) = \frac{1}{\tan(-x)} = -\frac{1}{\tan x} = -\cot x$$

These results imply that **cosine and secant are even functions and cosecant is an odd function.**

EXERCISES

Exercise 1. Simplify $\sin^2(-x) + \cos^2(-x)$

Exercise 2. Find the values of :

a. $\sin(-\frac{3\pi}{5})$

b. $\cos(-\frac{17\pi}{10})$

Exercise 3. Is the following expression and identity?

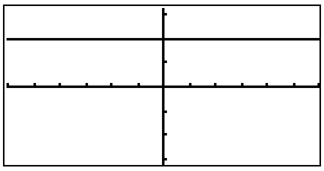
$$\frac{\sin^2(-x)-\cos^2(-x)}{\sin(-x)-\cos(-x)}=\cos x-\sin x$$

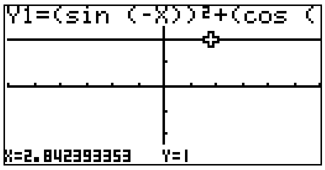
SOLUTIONS:

Exercise 1.

A. <u>Graphical exploration:</u>

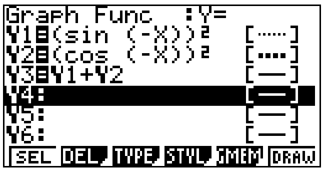
a. We graph the function $\sin^2(-x) + \cos^2(-x)$ in the <u>Graph Editor</u> as follows:

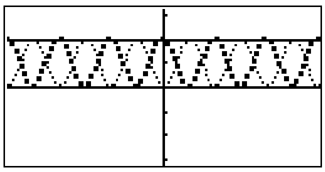




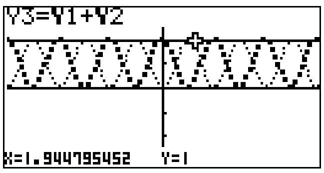
The graph is that of the horizontal line $y = 1$. This implies that the expression $\sin^2(-x) + \cos^2(-x) = 1$.

b. On approach would be to graph Y1= $\sin^2(-x)$, Y2= $\cos^2(-x)$ and then graph Y1 + Y2 as follows:



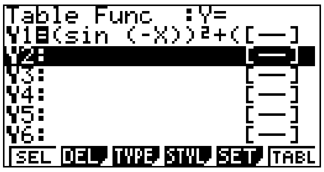


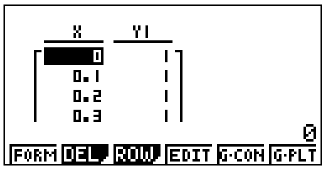
As we trace the graph representing the sum Y1 + Y2, we see that we obtain Y1+Y2=1.



B. Tabular exploration:

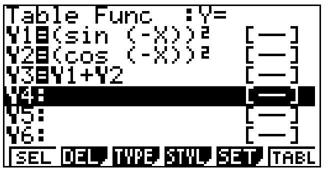
a. In the Table Editor, we can determine the functional values of $\sin^2(-x) + \cos^2(-x)$.

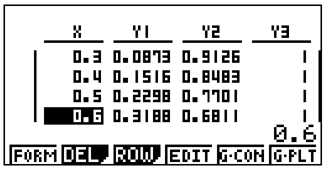




As we scroll down the data, we can verify that the value of $\sin^2(-x) + \cos^2(-x)=1$.

b. Another approach would be to examine the functional values of $Y1= \sin^2(-x)$ and $Y2 = \cos^2(-x)$. What would be the resulting functional values of Y1 +Y2? We obtain the following:





As seen in the table of values, the Y1+Y2 = 1.

C. Analytical solution:

We can establish the identity $\sin^2(-x) + \cos^2(-x)=1$ from the results we obtain in Activity 1.

Note that $\sin^2(-x) + \cos^2(-x) = [\sin(-x)]^2 + [\cos(-x)]^2$

$= (-\sin x)^2 + (\cos x)^2$

$= 1$

In fact, the identity $\sin^2(-x) + \cos^2(-x)=1$ or equivalently, the identity $\sin^2 x + \cos^2 x = 1$ is called the **Pythagorean identity.**

D. Graphical exploration in Parametric Mode:

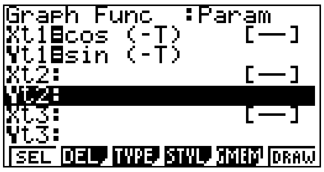
We can graph the following simultaneously in parametric mode:

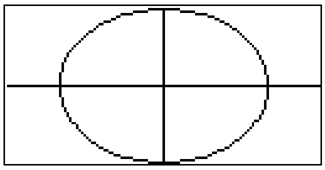
$$x = \cos(-t) \text{ and } y = \sin(-t) \text{ with } t \text{ in } [0,2\pi]$$

The equivalent rectangular equation can be obtained by squaring each parametric equation:

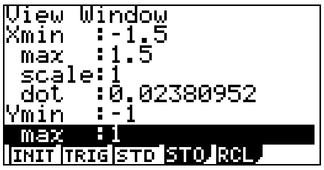
$$x^2 = [\cos(-t)]^2 \quad \text{and } y^2 = [\sin(-t)]^2$$

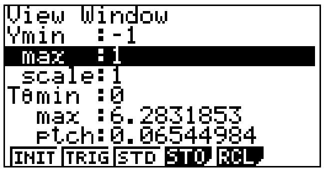
When we graph the equation, we get the unit circle, which confirms the identity $\sin^2(-x) + \cos^2(-x)=1$.





The viewing window used was:





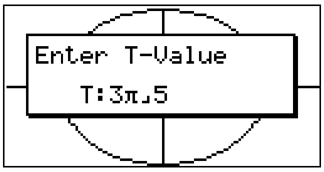
One advantage of graphing the unit circle parametrically is that it provides a method of finding trigonometric function values. Consider for instance, Exercise 2.

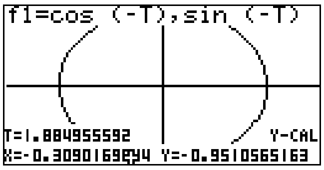
Exercise 2.

a. Find $\sin(-\frac{3\pi}{5})$

We can solve for this value while the parametric graph is on the screen. It can also give the learner an idea what quadrant is referred to when we consider $\sin(-\frac{3\pi}{5})$.

Enter [SHIFT GSolv Y-Cal] and specify the value of T:

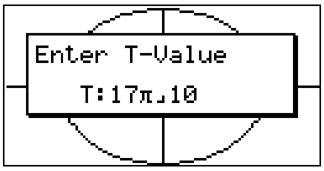


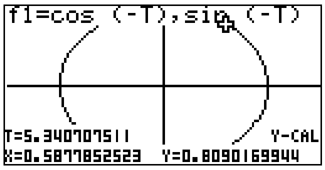


As seen from the graph the point on the unit circle is in the third quadrant. The *y* value represents $\sin(-\frac{3\pi}{5}) \approx$ -0.9510565163 and the *x* value represents $\cos(-\frac{3\pi}{5}) \approx$ -0.3090169044.

b. $\cos(-\frac{17\pi}{10})$

As seen from the graph the point on the unit circle is in the first quadrant. The *x* value represents $\cos(-\frac{17\pi}{10}) \approx$ 0.5877852523 and the *y* value represents $\sin(-\frac{17\pi}{10}) \approx$ 0.809016944.

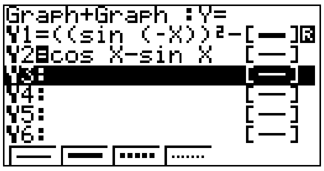


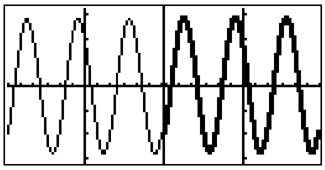


Exercise 3.

The expression $\frac{\sin^2(-x)-\cos^2(-x)}{\sin(-x)-\cos(-x)} = \cos x - \sin x$ is an identity.

Consider the following screen dumps:





REFERENCE

[1] Bittinger et al. *Algebra and Trigonometry*, *Graphs and Models*, 2nd Edition. Addison Wesley Verlag, 2001.

