Radian Measure

Ma. Louise De Las Penas, Phd
Ateneo de Manila University
Philippines

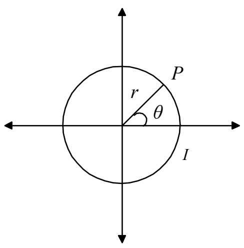
LEVEL
High School and University

OBJECTIVES
To illustrate how to use the calculator to

a) convert from degree measure to radian measure and vice versa
b) solve real world applications involving radian measure

Corresponding eActivity
radian.g1e

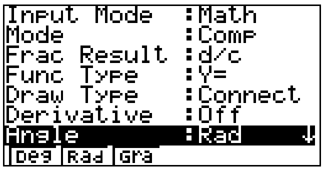
OVERVIEW
Given an angle whose vertex is at the center of the circle, a **radian** is defined to be the angle that intercepts an arc equal to the radius of the circle. In the figure below, the angle θ is subtended by the arc *IP* of the circle. If the length of *IP* is equal to the radius *r* of the circle, then θ has a measure of 1 radian.

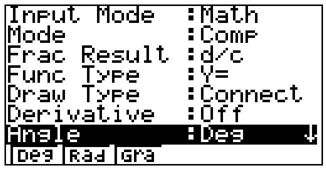


A calculator can be a very helpful tool in carrying out calculations involving radian, degree measure and handling angle conversions. Some real world applications of radian measure such as the length of a circular arc, the area of a circular sector and latitude problems will be solved in this paper using the calculator.

ACTIVITIES

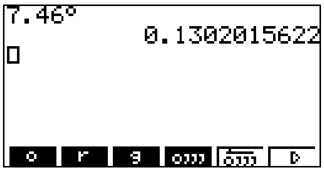
To obtain angle calculations in radians(respectively, degrees), we first specify using [SHIFT SET UP] that the angle will be in radians(respectively, degrees).



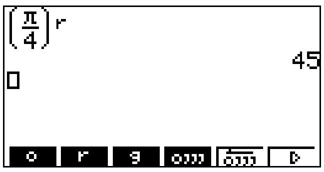


Angular calculations can be performed in the Run Editor. We press [OPTN ANGL] and enter the angle.

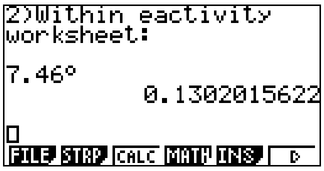
When in radian mode, angles in degrees are converted to radians by entering the degree symbol "o" after the angle measurement. For instance, 7.46 degrees in radians, is approximately 0.1302015622.

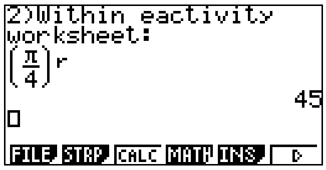


On the other hand, when in degree mode, angles in radians are converted to degrees by entering the radian symbol "r". For example $\pi/4$ in degrees is 45.



When in **eactivity mode**, the conversion can be carried out by converting from "Text" mode to "Calc" mode and entering the angle within the eactivity worksheet as follows:





Be careful when working within the eactivity worksheet. One must specify always using SHIFT SET UP the desired output, whether in radians or degrees. The left screen dump above shows that the eactiviy is in radian mode, while the right screen dump shows that the eactivity is in degree mode.

The solutions of the examples that follow illustrate calculations carried out in the RUN editor. However, the same calculations can be carried out within the worksheet when in **eactivity** mode, as illustrated above.

Example 1. Two concentric circles have radii of 15.7 meters and 29.2 meters. Using a central angle of 84^0 , find the area of the sector between the two circles.

Solution:

Radian measure is used in finding the area of a sector of a circle. The area *A* is of a sector is proportional to its central angle *θ*. That is, if *r* is the radius of the circle then

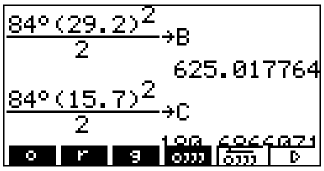
(area of sector *A*)/(area of circle) = (central angle of sector)/(central angle of circle)

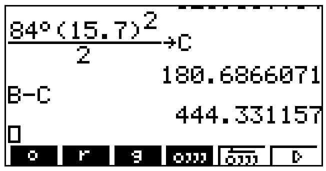
$$A/(\pi r^2) = \theta/2\pi$$

or equivalently,

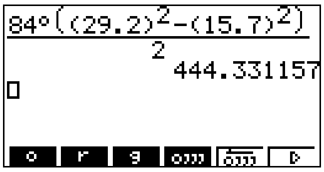
$$A = (\theta r^2)/2$$

We first solve the area *B* of the sector with radius 29.2 meters, then the area *C* of the sector with radius 15.7 meters and get the difference of the two areas, *B-C*.





Another way is to solve for the difference of the two areas directly, that is, the area of the sector between the two circles is given by $\frac{84^0((29.2)^2-(15.7)^2)}{2}$.



Thus, the area of the sector between the two circles is 444.331157 square meters.

Note that in the expression for the area, we assume that 84^0 is in radians. When we enter 84^0 in the calculator, we obtain the radian equivalence of 84 degrees.

Example 2. The end of a pendulum of length 40.2 cm travels an arc length of 5.6 cm as it swings through an angle θ. Find the measurement of the angle in

a. radians
b. degrees

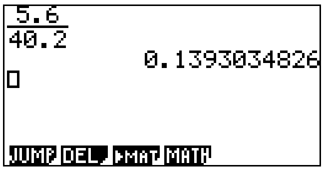
Solution:

If r is the radius of a circle and θ is the radian measure of a central angle that intercepts on the circle an arc of length s then

$$s = r\theta$$

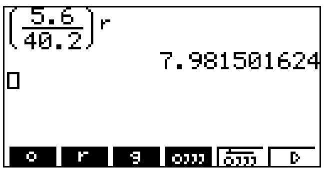
The measurement of the angle can be computed using the formula $\theta = s/r$ where $s = 5.6$ and $r = 40.2$. That is $\theta = 5.6/40.2$. We have the following screen dumps from the Run Editor.

a. The answer is approximately 0.1393034829 radians.



(Here, the calculator is set in radian mode)

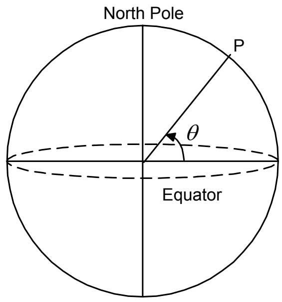
b. The answer is approximately 7.98 degrees.



(Here the calculator is set in degree mode)

Example 3. The latitude of a location point P, is the angle θ formed by a ray drawn from the center of the earth to the equator and from the center of the earth to P. Assume that the radius of the earth is about 4000 miles.

Radian Measure



a. Seattle is 47^0 north latitude. How far north of the equator is Seattle?
b. Part of the US Canadian border is 49^0 north latitude. How far south of the border is Seattle?
c. Glasgow, Montana is due north of Albuquerque, Mexico. Find the distance between Glasgow (48 degrees, 13 minutes, $48^0\ 13^l$ north latitude) and Albuquerque (35 degrees, 7 minutes, $35^0\ 7^l$ north latitude).

Solution:

a. We use the formula $s = r\theta$, where we express 47^0 in radians. We have

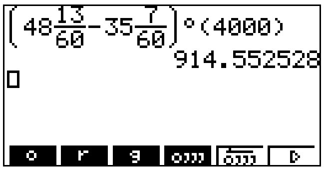
$$s = (4000 \text{ miles})\ (47^0) \approx 3281.218994.$$

Thus, Seattle is approximately 3281 miles north of the equator.

b. In the formula $s = r\theta$, we calculate s =(4000 miles) $(49^0) \approx 3420.845334$.

The US- Canadian border is about 3421 miles north of the equator. Since 3421-3281 is 140 miles, then Seattle is 140 miles south of the border.

c. We first get the radian equivalence of $48^0\ 13^l$ - $35^0\ 7^l$, the measure of the central angle between the two cities. Then we enter this value as θ together with the radius r= 4000 in the formula $s = r\theta$. We obtain the ff:



The distance between the two cities is approximately 915 miles.

EXERCISES

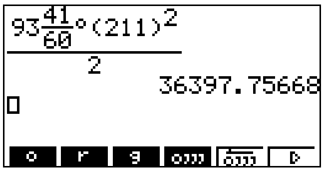
Exercise 1. Part of a baseball field and diamond is to be planted with grass seed. Determine the area of the field if the angle between the first and third baselines is $93^0\ 41^I$ and the radius of the sector to be seeded is 211 feet from home plate.

Exercise 2. Charleston, West Virginia, is due north of Jacksonville, Florida. Find the distance between Charleston(latitude $38^0 21^I$), Jacksonville(latitude $30^0 20^I$). Assume that the radius of the earth is about 4000 miles.

SOLUTIONS:

Exercise 1. The area of the sector is

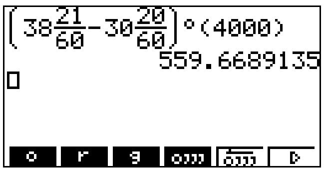
$A = (\theta r^2)/2$
$A = ((93^0\ 41^I \text{ in radians})(211 \text{ feet})^2/2$



The area of the sector is approximately 36,398 square feet.

Exercise 2.

Following the solution in Example 3, item 3, we obtain approximately 560 miles.



REFERENCE

[1] Sullivan, Michael et al. *Pre-Calculus*, *Graphing and Data Analysis*, 2nd Edition. Prentice Hall, 2001.

