



Problem Solving Using Linear Functions

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LEVEL

Middle schools after students have studied linear functions.

OBJECTIVES

To engage real-life quantitative problem using linear function and solve the problems using graphical and table of values approaches.

CORRESPONDING eActivity

LINEAR.g1e

OVERVIEW

Quantitative problems usually can be modeled and expressed as functions. In this activity we discuss examples of problems which involve two variables: one dependent and the other independent. The graphics calculator is used to engage these problems from graphical and tabular perspective.

EXPLORATORY ACTIVITIES

[Note]

We shall use small letter x instead of capital X as shown on the calculator throughout the paper.

Activity 1: When starting a business, being able to track spending is very important. Marla plans to open a book shop where she estimates that the cost of setting up the shop with books would require \$15,000, and from her survey she found that she will need \$1,250 of monthly operation costs. The first task is to find a model which can help her tracks spending and plan for the shop's opening. Find this model, then

- (i) Find the capital needed to start and operate the shop for 4 months? 6 months?
- (ii) What must her minimum average monthly earning be in order to breakeven in 15 months?

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Solution:

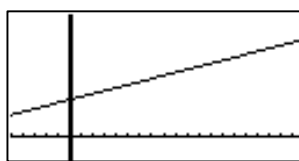
If y is the cost of setting up and operating the shop and x represents number of months, then a good model should be as follow:

$$y = 1250x + 15000$$

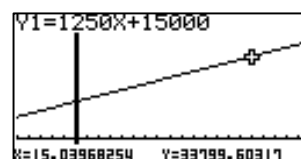
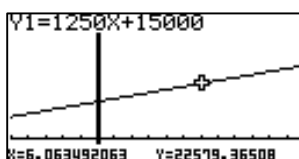
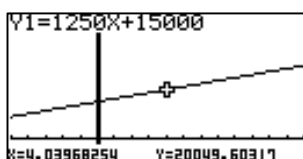
We begin by exploring this model graphically (Open eActivity LINEAR.g1e.)

A. Graphical Exploration

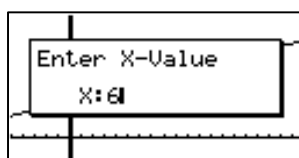
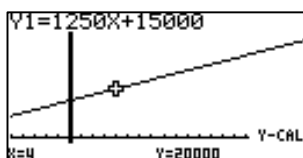
(a) Open the graph editor "Act1A" and graph the function Y1. Using appropriate View Window setting, we should see the graph at below right.



(b) With the graph still in display, tap **[F1]** (Trace) to trace the line graph. Scroll the cursor using **[◀]** **[▶]** to explore values when $x = 4$, 6 and 15 respectively.

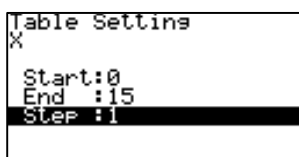


Then, tap **[F4]** **[F6]** **[F1]** to access [Y-CAL] and use it to accurately find the corresponding y -coordinates of $x = 4$, 6 and 15 .



B. Tabular Exploration

A table of values is great for exploring function and detecting exact solutions. Open the table editor "Act1B" and using the setting as suggested below, make the table of values for Y1.



Explore the table using **[▲]** **[▼]** and we shall see a table of values as shown below:

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Y1=1250X+15000	
X	Y1
0	15000
1	16250
2	17500
3	18750

16250

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Y1=1250X+15000	
X	Y1
4	20000
5	21250
6	22500
7	23750

23750

FORM DEL ROW EDIT G-COM G-PLT

X	Y1
12	30000
13	31250
14	32500
15	33750

15

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C. Discussion

The exploration exercises actually reveal the solution to the question on capital. The table indicates that the required capital to start and operate the business for 4 months and 6 months are \$20,000 and \$23,750 respectively.

And, to achieve breakeven status in 15 months, Marla would need a minimum average monthly sales of $\$33,750/15\text{months} = \2250 , as according to the table of values the cost to operate the shop for 15 months is \$33,750. □

EXERCISES A

Exercise 1.

What should Marla's minimum average monthly sales be if she wants to achieve breakeven in a year?

Exercise 2.

If Marla actually spent \$23,723 to set up the shop and the monthly operation cost remains at \$1,250, write a new function to model this case. Find the new minimum average monthly sales for this case if she still wants to breakeven in a year.

SOLUTIONS to EXERCISES A

Exercise 1

In the eActivity, open table editor strip "Act1C", and explore the table of values for $y = 1250x + 15000$ again.

X	Y1
12	30000
13	31250
14	32500
15	33750

15

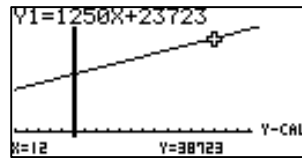
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From the table, to achieve breakeven in a year (12 months) the minimum average monthly sales should be $\$30,000/12 = \2500 .

Exercise 2

By comparison, the new function should be $y = 1250x + 23723$. Using the approach used in solving Activity 1, open the graph editor strip "Act1D" at the eActivity and key in this function. Graph the function and explore it. Then, calculate the exact coordinates when $x = 12$ with [Y-CAL].

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From the exploration clearly the total costs for the first year is \$38,723. Therefore the average monthly sales must be $\$38,723/12 = \3226.9 to breakeven in a year. You should check your observation using a table of values: open the table editor strip "**Act1E**" and make the table for Y1, then explore the table obtained. □

In Activity 1 we have discussed performing problem solving with function. The importance of using function grows as the problem becomes more complex. Let's look at this next activity which is an extension of Activity 1.

Activity 2: After six months in operation Marla finds that the average monthly sales is \$2,107, and believes this average sales trend will persist for at least 3 more years. A friend of Marla is interested to be a partner in this venture, and he wants to find out some information before he does join Marla.

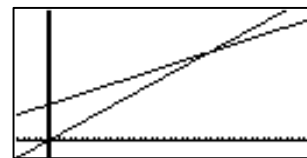
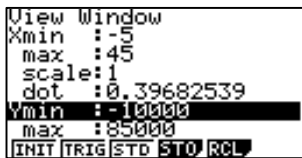
- (i) How long before the bookshop starts turning profit?
 - (ii) What is the projected profit after 2 years? 3 years?
- Recall that Marla actually spent \$23,723 to set up the shop instead of \$15,000.

Solution:

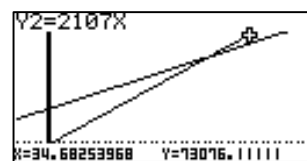
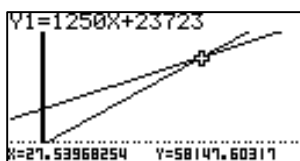
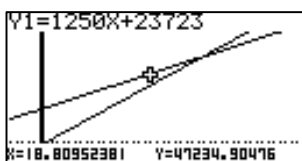
The revised cost function of starting and operating the shop is now $y = 1250x + 23723$. And, as the monthly sales is \$2,107, the income I , can be modelled by the function $I = 2107x$.

A. Graphical Exploration (Open eActivity LINEAR.g1e.)

(a) Open the graph editor "**Act2A**" and graph the function Y1, Y2 simultaneously. Using appropriate View Window setting, we should see the graph at below right.

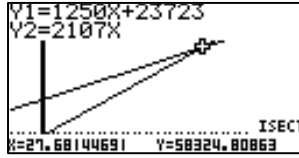


(b) With the graphs still in display, tap [F1] (Trace) to trace the line graph. Scroll the cursor using ◀ ▶ ▲ ▼ to explore both graphs especially where the two line graphs intersect.



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(c) Then use [ISCT] function in [G-SLV] to find the coordinates of the intersection point. This intersection point is the key to finding when the venture starts turning in profit.

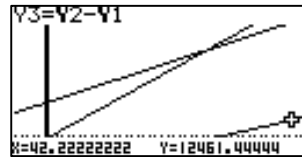


(d) If we are given x (month), we can make projection of the profit or loss of the venture. The difference between the two functions at a given value of x is the projected profit, or loss. This projection can be made using the function P defined as follow:

$$P = \text{Income} - \text{Cost}$$

$$= Y_2 - Y_1$$

Now open the graph editor "Act2B" and enter a new function Y2-Y1 to Y3 then graph it. Use [Trace] to explore the graph of Y3.



B. Tabular Exploration

(a) Open the table editor "Act2C" and using the setting below, make the table of values for Y1, Y2 and Y3.



Explore the table using \uparrow \downarrow \leftarrow \rightarrow and we shall see a table of values as shown below:

X	Y1	Y2	Y3
3	27473	6321	-21152
4	28723	8428	-20295
5	29973	10535	-19438
6	31223	12642	-18581

X	Y1	Y2	Y3
27	57473	58889	-584
28	58723	58996	273
29	59973	61103	1130
30	61223	63210	1987

X	Y1	Y2	Y3
41	74973	86387	11414
42	76223	88494	12271
43	77473	90601	13128
44	78723	92708	13985

C. Discussion

Both explorations play different roles in solving the problems. The graphical exploration projects that this venture will turn in profit after 27.7 months in business. The tabular exploration supports that: Y3 column shows loss when x=27 but profit when x=28.

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As this study of the venture is done 6 months after it started, hence the projected profit after 2 years is when $x = 6 + 2 \cdot 12 = 30$. Similarly the projected profit after 3 years is when $x = 6 + 36 = 42$.

The graph or the table of values tells us the projected profit after 2 years (at $x = 30$) is about \$1,987 and after 3 years (at $x = 42$), it is \$12,271. □

EXERCISES B

Exercise 1.

Marla has plans to add a new section in the shop which will cost \$8,000. She and her partner decided to use $\frac{2}{3}$ of the accumulated profit to pay for the new section. When will they have enough money to begin this addition?

SOLUTIONS to EXERCISES B

Exercise 1

In the eActivity, use the strips "Act2D" and "Act2E" provided after the exercise to explore then solve the problem. Note that we are looking for when $\frac{2}{3}$ of the accumulative income is \$8,000 or more, which means the accumulative income must at least be \$12,000.

Y3=Y2-Y1			
X	Y1	Y2	Y3
40	73723	84280	10557
41	74973	86387	11414
42	76223	88494	12271
43	77473	90601	13128
			12271

An efficient approach is to define a function which directly provides the answer. Since Y_3 is the function of profit and loss, we let $Y_4 = \frac{2}{3} Y_3$ which answers the question directly.

Table Func :Y=	
Y1	1250X+23723
Y2	2107X
Y3	Y2-Y1
Y4	2.3Y3
Y5:	
Y6:	

Y4=2.3Y3			
X	Y2	Y3	Y4
39	82173	9700	6466.6
40	84280	10557	7038
41	86387	11414	7609.3
42	88494	12271	8180.6
			8180.666667

The tabular exploration indicates that Marla can begin to install the new section after 42 months in business. The graphical exploration gives a more exact answer, although this does not mean the graphical approach is more advantages than the tabular approach.

