



## Higher Order Equations

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### LEVEL

High School or University

### OBJECTIVES

In this paper, we study and investigate the solutions of certain types of equations with the aid of the graphics calculator.

### Corresponding eActivity

highord.g1e

### OVERVIEW

Recall that an equation is a statement equating two expressions. If at least one of the expressions is an algebraic expression in a variable  $x$ , the statement is called an equation in  $x$ . To solve an equation in  $x$  means to find all the real values of  $x$  for which the equation is true.

In this paper, we illustrate how to explore and determine solutions to equations in  $x$  using the numeric and graphical capabilities of the graphics calculator.

### EXPLORATORY ACTIVITIES

**Example 1.** Which of the following equations will yield an infinite number of solutions?

1.  $x^2 - 3 = \sqrt{x}$
2.  $x(x-25)(x-35) = 3000$
3.  $|x| + |x-3| = 6$
4.  $|x-2| + |x-3| = 1$

#### Solution:

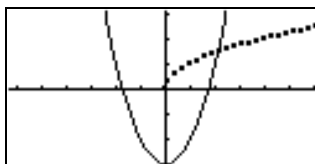
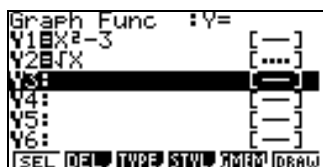
1.

Method (i): Using the [Graph Editor](#).

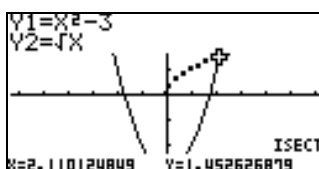
To solve the equation  $x^2 - 3 = \sqrt{x}$ , our approach would be to graph the expression on the

## Higher Order Equations

left hand side of the equation, given by  $x^2-3$ , and graph the expression on the right hand side, which is  $\sqrt{x}$ , simultaneously in one coordinate system:

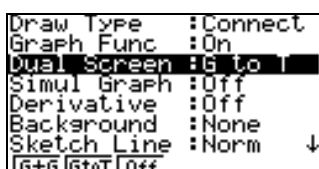


The x value of the point of intersection of both curves will satisfy the equation and serves as the solution to the equation. Using [GSolve ISCT] we obtain the solution as follows:

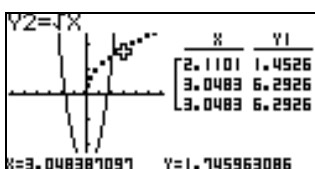
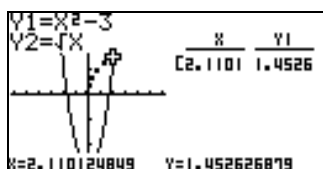


The solution to the equation  $x^2-3 = \sqrt{x}$  is not exact. The approximate solution is given by  $x = 2.1101$ .

It is sometimes helpful to use a table alongside the graph to assist us in our exploration. To show a graph with a given table, we enter [SHIFT SET UP] and set the calculator to Dual Screen mode, specifically, the "graph to table" mode. [G to T].



As we trace along the graph, or solve a point of intersection, the same values appear on the table when we press EXE:



Method (ii): Using the Solver.

We select [Solver], then enter the equation.

## Higher Order Equations

```
Equation
Select Type
F1:Simultaneous
F2:Polynomial
F3:Solver
SIML POLY SOLV
```

```
Eq: X^2-3=JX
```

The solution is immediately given, together with the left and right hand values of the equation. We can check that the equation has approximate solution  $x= 2.1101$ , the same solution we obtained earlier:

```
Eq: X^2-3=JX
X=2.110124849
Lft=1.452626879
Rst=1.452626879
REPT
```

Method (iii): Using the Run Editor.

The problem may be solved by pressing [OPTN CALC Solve], and entering the equation as follows:

```
Solve(X^2-3=JX,X)
2.110124849
Solve d/dx J/dx J/dx J/dx
```

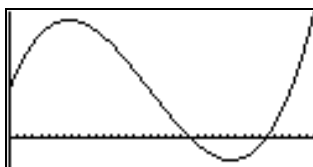
2.

Method (i): Using the Graph Editor.

One way to solve an equation of the type  $x(x-25)(x-35) = 3000$ , where we have a constant in one side of the equation, is to graph the expression on the left hand side, given by  $x(x-25)(x-35)$ , and from the graph, determine the value of  $x$  that will give the value of the constant, in this case, 3000.

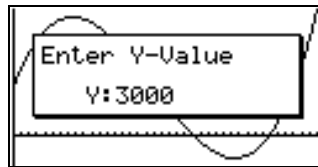
In the Graph Editor, we graph  $x(x-25)(x-35)$  using an appropriate View Window:

```
View Window
Xmin :2
max :42
scale:1
dot :0.31746031
Ymin :-1000
max :4000
INIT TRIG STD STO RCL
```

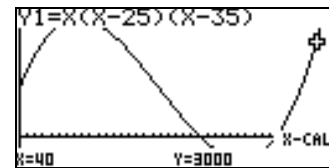
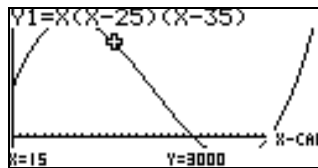
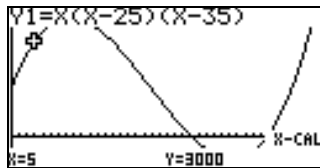


We press [SHIFT GSolv X-CAL], enter the  $y$  value to determine the corresponding  $x$  value:

## Higher Order Equations

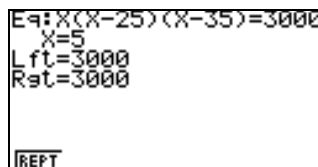


We obtain the answers  $x = 5, 15$  and  $40$ .

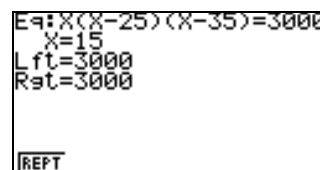
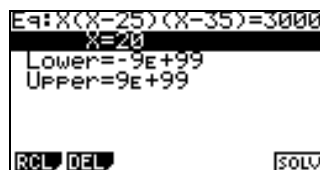


Method (ii): Using the Solver.

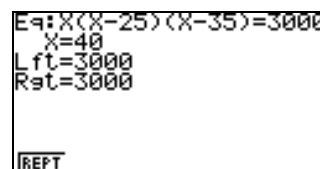
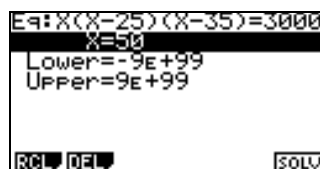
When using the Solver to solve an equation with more than one solution, the calculator provides the solution one at a time. Consider, for instance solving the equation  $x(x-25)(x-35) = 3000$ . The calculator gives one particular solution first:



To determine the second solution, we press [REPT]. We enter a value, a "guess" to what the next solution would be. The "guess" serves as a starting point for the calculator to look for answers. Although, one can enter any value he wishes as his "guess", a close value will give the solution more quickly, and a value which is quite far from a solution may not give a satisfactory result. Consider for instance inputting the value  $x = 20$  as our "guess". This gives rise to the second solution  $x = 15$ .



Next, we enter 50 as our "guess". Consequently, we obtain the third solution,  $x = 40$ .



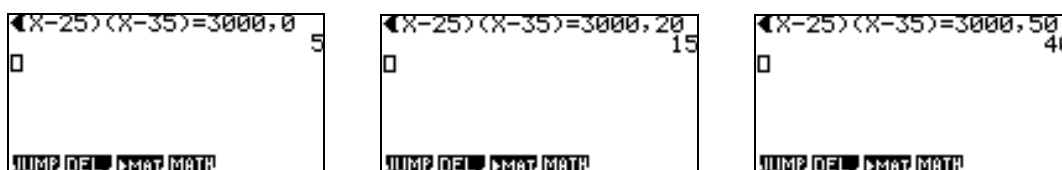
If after inputting a value as one's "guess", the calculator gives an answer where the lower and upper values are not equal, the process is continued by pressing EXE to produce the

## Higher Order Equations

correct answer. When using the solver, one has to be careful to obtain all the solutions when there is more than one. One also has to be ready providing the calculator with starting points to look for solutions.

Method (iii): Using the Run Editor.

Solving an equation with more than one solution in the Run Editor works almost the same way as in the Solver. One provides an estimate or "guess" to the solution. This process is needed to guide the calculator in finding the solution. Consider for instance:

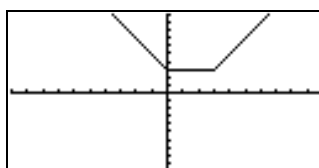


The first estimate is 0, which gives rise to the solution  $x = 5$ . The next estimate provided is 20, which gives the solution  $x = 15$ , and the third estimate is 50, which gives the solution  $x = 40$ .

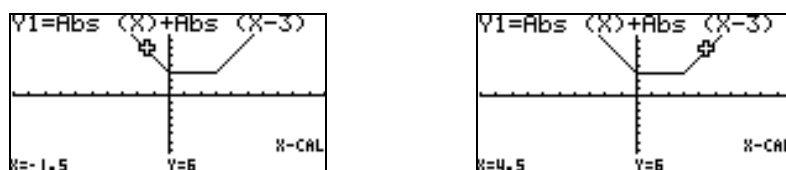
3.

Method (i): Using the Graph Editor.

We solve the equation  $|x| + |x-3| = 6$  in the Graph Editor. We first graph  $|x| + |x-3|$ :



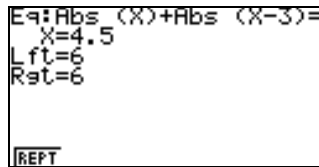
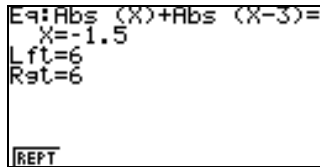
By entering [SHIFT GSolv X-CAL] while the graph is on the screen, we obtain two values of  $x$  which will give a  $y$  value 6, namely,  $x = -1.5$  and  $x = 4.5$ .



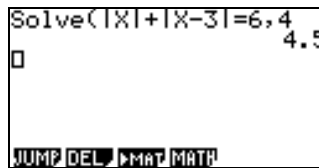
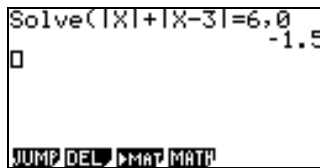
The following screen dumps below illustrate that we obtain the same results when using the Solver or the Run Editor.

Method (ii): Using the Solver.

**Higher Order Equations**



Method (iii): Using the Run Editor.

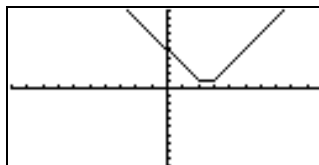


The first estimate is 0, which gives rise to the solution  $x = -1.5$ . The next estimate provided is 4, which gives the solution  $x = 4.5$ .

There are advantages of using the Graph Editor over the Solver and the Run Editor. One can tell from the graph how many solutions to the equation there are. There are also cases when there are infinitely many solutions to the equation. In which case, the Graph Editor provides the only systematic way to solving the equation. This is apparent in number 4:

4.

To solve the equation  $|x - 2| + |x - 3| = 1$  graphically, we follow the same process used in numbers 2 and 3. We graph the left hand side of the equation, the expression  $|x - 2| + |x - 3|$ :



We press [SHIFT GSolv X-CAL] while the graph is on the screen and solve for the value of  $x$  when  $y = 1$ . We note that the solution is not a discrete number of points, but the interval  $[2, 3]$ :



We use the Table Editor to explore further the situation. We enter the expression  $|x - 2| + |x - 3|$  and generate the table of values by giving an appropriate table setting:

## Higher Order Equations

Y1	Y2	Y3	Y4	Y5	Y6
Table Func :Y=					
Abs (X-2)+Abs					
Y1:					
Y2:					
Y3:					
Y4:					
Y5:					
Y6:					
[SEL DEL TYPE STVL SET TABL					

Table Settings	
X	
Start:	2
End :	3
Step :	0.01

A partial list is given below which shows that the value of the expression anywhere in the interval  $[2,3]$  is 1.

X	Y1
2	1
2.01	1
2.02	1
2.03	1

FORM DEL ROW EDIT G-COM G-PLT 2

X	Y1
2.13	1
2.14	1
2.15	1
2.16	1

FORM DEL ROW EDIT G-COM G-PLT 2.16

X	Y1
2.97	1
2.98	1
2.99	1
3	1

FORM DEL ROW EDIT G-COM G-PLT 3

One may explore further into the interval by specifying smaller step values like 0.001. It can be verified that the solution to the equation is the interval  $[2,3]$ .

Hence, the equation  $|x-2| + |x-3| = 1$  yields an infinite number of solutions. In this exercise, the first given equation has one solution; the second equation, which is cubic in nature has three solutions. (Other cubic equations yield one real solution). Equations involving absolute values, in general, yield an empty set, a finite set or an infinite set as its solution set. The equation in number 3 has a solution set consisting of two elements.

**Example 2.** Do the given equations have the same solution? Why or why not?

1.  $x^3 - 10x^2 + x + 50 = 0$

2.  $\frac{x^3 - 10x^2 + x + 50}{x - 2} = 0$

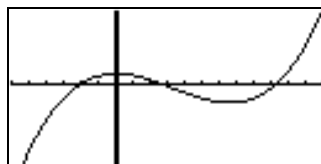
3.  $\frac{x^3 - 10x^2 + x + 50}{x + 2} = 0$

**Solution:**

1.

Method (i): Using the Graph Editor.

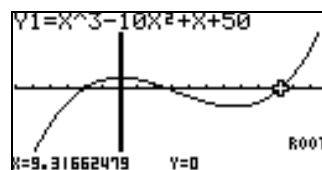
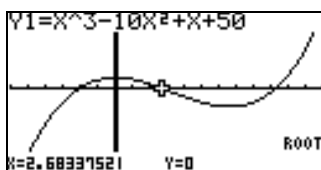
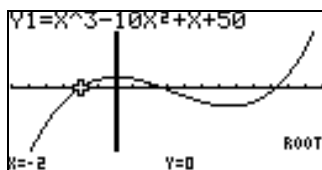
We graph the cubic  $x^3 - 10x^2 + x + 50$  using an appropriate View Window:



View Window	
Xmin	-6
max	12
scale	1
dot	0.14285714
Ymin	-400
max	350
[INIT TRIG STD STO RCL]	

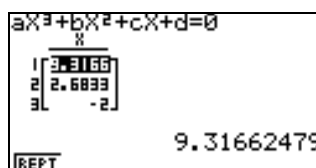
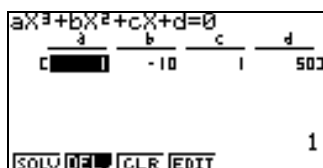
Using the command [SHIFT GSolve ROOT], we solve for the values of  $x$  that will make the equation 0. We obtain  $x = 2$ ,  $x \approx 2.68$  and  $x \approx 9.32$ .

## Higher Order Equations



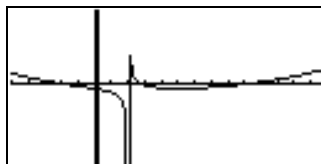
Method (ii): Using the Equation Editor.

The cubic equation can also be solved in the Equation Editor by selecting [Polynomial]. (When in eactivity mode, we insert the Poly Equation strip.) We enter the degree of the polynomial and enter the coefficients of the equation and press [Solv] to obtain the answers:

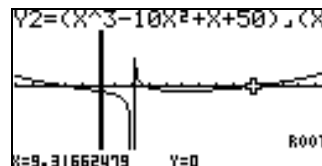
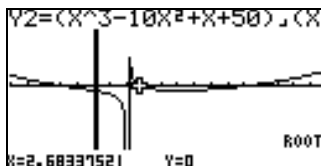
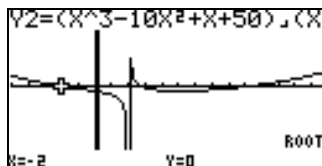


We obtain the same results given earlier, namely,  $x = 2$ ,  $x \approx 2.68$  and  $x \approx 9.32$ .

2. The graph of the rational expression  $\frac{x^3 - 10x^2 + x + 50}{x - 2}$  is given below. A vertical asymptote of the graph is  $x = 2$ :



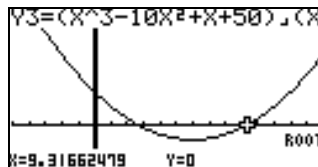
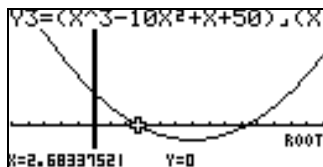
The rational expression  $\frac{x^3 - 10x^2 + x + 50}{x - 2}$  has the same roots as the cubic  $x^3 - 10x^2 + x + 50$ , namely,  $x = 2$ ,  $x \approx 2.68$  and  $x \approx 9.32$ .



3. In solving the equation  $\frac{x^3 - 10x^2 + x + 50}{x + 2} = 0$ , we graph the left hand expression  $\frac{x^3 - 10x^2 + x + 50}{x + 2}$  and solve for its roots:



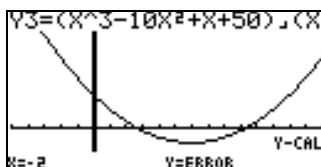
## Higher Order Equations



The roots obtained are  $x \approx 2.68$  and  $x \approx 9.32$ .

Observe that  $x=-2$  is not part of the domain of the rational expression  $\frac{x^3 - 10x^2 + x + 50}{x + 2}$ .

Thus, if we ask for the value of  $y$  when we specify  $x = -2$ , the calculator gives an error message:



Thus, the equation  $\frac{x^3 - 10x^2 + x + 50}{x + 2} = 0$  does not have the same solution set as the two

other equations,  $x^3 - 10x^2 + x + 50 = 0$  and  $\frac{x^3 - 10x^2 + x + 50}{x - 2} = 0$ .

A lesson to be learned in this activity is that one has to take extra caution in solving equations involving rational expressions such as  $\frac{x^3 - 10x^2 + x + 50}{x + 2} = 0$ , one cannot simply ignore the denominator, and assume that this equation has the same solution as  $x^3 - 10x^2 + x + 50 = 0$ .

**Remarks:** In the previous exercise, the equations (1)-(3) are also solvable using the Run Editor; the equations (2)-(3) are also solvable using the Solver. We do not anymore show the solutions here.

## REFERENCE

[1] *Mathematics with a Graphics Calculator*, Kissane, Barry, 2003, Australia, ISBN: 1-876583 24 X.