Fractions and Decimals

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LEVEL
High school or university students

OBJECTIVES
In this activity, we use the calculator to
a) investigate the relationship between decimals and fractions
b) study real-world applications of fractions and decimals

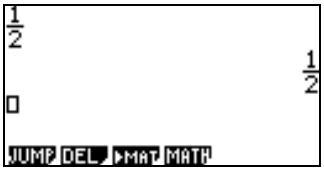
Corresponding eActivity
Fracdec.g1e

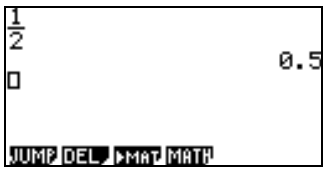
OVERVIEW
Students can understand and appreciate more the concept of fractions and decimals through activities carried out with the aid of technology. Technology facilitates the exploration of fractional/decimal representations and the application of these notions to solving real world problems.

EXPLORATORY ACTIVITIES
Calculations on the two activities that will follow will be done on the Run Editor or within the eactivity worksheet.

To determine the decimal representation of a given fraction, enter the fraction, example, $\frac{1}{2}$ as follows:

a) on the Run Editor:





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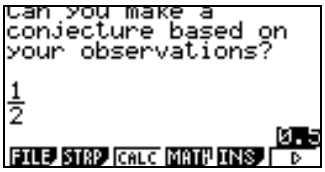
b) Insert a math calculation within an eactivity worksheet as follows:



Activity 1: Determine the decimal equivalents of the following fractions:

a) $\frac{1}{2}, \frac{3}{4}, \frac{1}{8}, \frac{3}{16}, \frac{3}{10}, \frac{1}{12}$

b) $\frac{1}{3}, \frac{2}{3}, \frac{4}{9}, \frac{4}{27}, \frac{1}{15}$

c) $\frac{1}{5}, \frac{1}{10}, \frac{7}{20.}, \frac{1}{25}, \frac{2}{125}$

d) $\frac{1}{7}, \frac{3}{49}, \frac{8}{11.}, \frac{1}{121}, \frac{3}{77}$

1. Determine whether the decimal forms of the given sets of fractions terminate or repeat.
2. Compare the denominators of the terminating fractions and repeating fractions. Can you make a conjecture based on your observations?

Solution:

We obtain the following decimal forms of the given fractions:

a) $\frac{1}{2} = 0.5$, $\frac{3}{4} = 0.75$, $\frac{1}{8} = 0.125$, $\frac{3}{16} = 0.1875$, $\frac{3}{10} = 0.3$, $\frac{1}{12} = 0.08\overline{3}$

The decimals obtained are terminating decimals except the last one, $\frac{1}{12}$ which is a repeating decimal. The terminating fractions $\frac{1}{2}, \frac{3}{4}, \frac{1}{8}, \frac{3}{16}, \frac{3}{10}$ have respective denominators 2, $4 = 2^2$, $8 = 2^3$, $16 = 2^4$ and $10 = 2x5$. The repeating fraction $\frac{1}{12}$ has denominator $12 = 2^2 x3$.

b) $\frac{1}{3}=0.\bar{3}$, $\frac{2}{3}=0.\bar{6}$, $\frac{4}{9}=0.\bar{4}$, $\frac{4}{27}=0.\overline{148}$, $\frac{1}{15}=0.0\bar{6}$

The decimals are repeating. The denominators are 3, $9=3^2$, $27=3^3$ and $15=3x5$

c) $\frac{1}{5}=0.2$, $\frac{1}{10}=0.1$, $\frac{7}{20}=0.35$, $\frac{1}{25}=0.04$, $\frac{2}{125}=0.16$

The decimals obtained are terminating. The denominators are $5, 10=2x5$, $20=2^2x5$, $25=5^2$ and $25=5^3$.

d) $\frac{1}{7}=0.\overline{142857}$, $\frac{1}{11}=0.\overline{09}$, $\frac{8}{11}=0.\overline{72}$, $\frac{1}{13}=0.\overline{076923}$, $\frac{3}{77}=0.\overline{038961}$

We obtain repeating decimals. The denominators are 7, 11, 13, $77=7x11$.

The following generalization is true for terminating and repeating decimals:

A fraction $\frac{a}{b}$ in lowest terms results in a **terminating decimal** if the only prime factor of the denominator is 2 or 5 (or both).

A fraction $\frac{a}{b}$ in lowest terms results in a **repeating decimal** if a prime other than 2 or 5 appears in the prime factorization of the denominator.

The justification of this rule is based on the fact that the prime factors of 10 are 2 and 5, and for the decimal system, 10 is used as the base.

Remarks: The learner can experiment on as many fractions as possible to appreciate fully the above generalization.

Activity 2. Determine the decimal representations of the following fractions:

a) $\frac{2}{9}, \frac{3}{9}, \frac{4}{9}$

b) $\frac{2}{99}, \frac{3}{99}, \frac{4}{99}$

c) $\frac{2}{999}, \frac{3}{999}, \frac{4}{999}$

What pattern do you observe? Can you make a conjecture regarding changing a repeating decimal to a fraction?

Solution:

a) $\frac{2}{9} = 0.\bar{2},\ \frac{3}{9} = 0.\bar{3},\ \frac{4}{9} = 0.\bar{4}$

b) $\frac{2}{99} = 0.\overline{02},\ \frac{3}{99} = 0.\overline{03},\ \frac{4}{99} = 0.\overline{04}$

c) $\frac{2}{999} = 0.\overline{002},\ \frac{3}{999} = 0.\overline{003},\ \frac{4}{999} = 0.\overline{004}$

The following rule applies for changing a pure repeating decimal to a fraction.

1. Write the repeating part as the numerator of the fraction.
2. The denominator consists of as many nines as there are digits in the part that repeats.

EXERCISES

Exercise 1. Without dividing, determine whether the decimal form of the given rational number terminates or repeats.

a) $\frac{7}{32}$ b) $\frac{13}{150}$ c) $\frac{10}{125}$

Exercise 2. Find quotients of two integers equal to the following:

a) $0.\bar{8}$ b) $0.\overline{85}$ c) $0.\overline{0001}$

SOLUTIONS

Solution to Exercise 1.

a) Since 32 factors as 2^5, the decimal equivalent will terminate. No primes other than 2 divide the denominator.

b) $150 = 2x3x5^2$. Since 3 appears as a prime factor of the denominator, the decimal form will repeat.

c) We first reduce the fraction to lowest terms. Now, $\frac{10}{125} = \frac{2}{25}$. Since $25 = 5^2$, the decimal form will terminate.

Solution to Exercise 2.

a) $0.\bar{8} = \frac{8}{9}$ b) $0.\overline{85} = \frac{85}{99}$ c) $0.\overline{0001} = \frac{1}{9999}$

APPLICATIONS.

1. The following chart appears on a package of Quaker Quick Grits.

MICROWAVE		**STOVE TOP**		
servings	1	1	4	6
water	¾ cup	1 cup	3 1/4 cups	4 1/4 cups
Grits	3 tbsp	2 ¾ tbsp	1 cup	1 cup
salt	-	-	¼ tsp	½ tsp

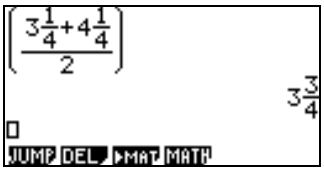
How many cups of water and tablespoons of grits would be needed for 5 microwave and 5 stove top servings?

Solution

For 5 microwave servings, we calculate in the Run Editor the answer either in decimal, mixed fraction or improper fraction form as follows:

Thus, there will be $3\frac{3}{4}$ cups of water. Moreover, 5(3)=15 tablespoons of grits will be required.

For 5 stove top servings, we obtain $(\frac{3\frac{1}{4}+4\frac{1}{4}}{2}) = 3\frac{3}{4}$ cups of water and 1 cup of grits. Note that we average the number of cups for 4 and 6 servings.



2. After 9 games at a softball league, the following statistics were obtained.

PLAYER	AT-BATS	HOME RUNS	HITS
Larry Santos	40	3	8
Robynne Lopez	36	3	12

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Mary Thomas	11	1	5
Bong Olpoc	16	0	8
Cathy Yu	20	2	10

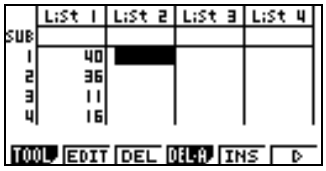
1) Which player got a hit in just less than $\frac{1}{2}$ of his or her at-bats?

2) Which player got a home run just less than $\frac{1}{10}$ of his or her at-bats?

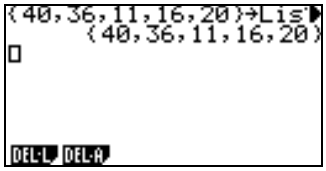
3) Which players got a hit in exactly $\frac{1}{3}$ of his or her at-bats?

Solution

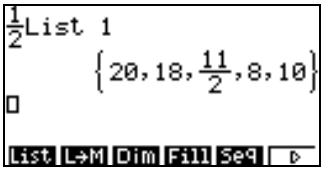
Enter the data pertaining to at-bats in list 1 of the Stat Editor as follows:

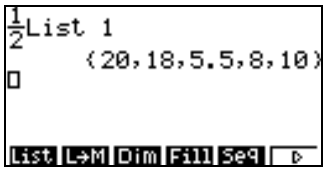


However, when working within an eactivity worksheet, the data pertaining to at-bats is entered in the RUN strip:



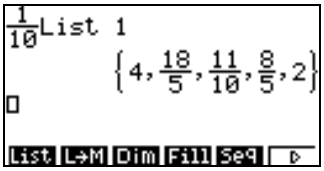
1) To answer the first question, enter the following command in the Run Editor. Then $\frac{1}{2}$ of each of the players' at-bats will be computed as follows:

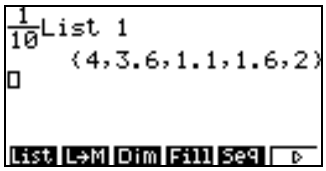




Note that the players with hits less than $\frac{1}{2}$ of his or her at-bats are Larry- 8 out of 20; Robynne- 12 out of 18 and Mary-5 out of 5.5.

2) We compute $\frac{1}{10}$ of players' at bats as follows:





The players with home runs less than $\frac{1}{10}$ of his or her at-bats are Larry- 3 out of 4; Robynne-3 out of 3.6; Mary- 1 out of 1.1; Bong- 0 out of 1.6.

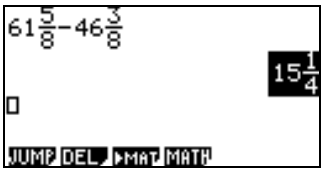
Following the steps given in 1) and 2) above we have the following answer for 3:

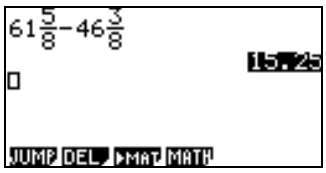
3) Robynne has 12 hits, which is exactly $\frac{1}{3}$ of her at-bats.

3. In a recent year, the price per share of stock for Hershey Foods was $\$46\frac{3}{8}$. A year later, the price per share rose to $\$61\frac{5}{8}$. If a stockholder bought 100 shares and sold the stock later, what is the percentage growth of his investment?

Solution:

The net increase is given by $61\frac{5}{8}-46\frac{3}{8}=15\frac{1}{4}$ or equivalently,$15.25.





The yield or percent increase during this one-year period is found by dividing the difference in the selling price and original price by the original price. We have $\frac{15\frac{1}{4}}{46\frac{3}{8}}\approx 33\%$.

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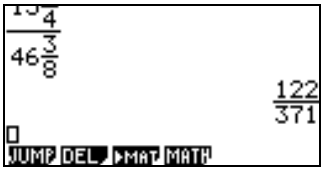

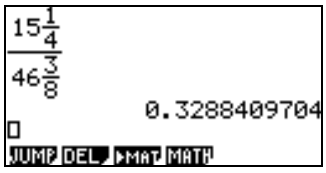

REFERENCE

[1] Bello, Ignacio. *Basic College Mathematics a Real World Approach*, 2nd Edition. Mc Graw Hill,, 2006, New York.

