



Financial Mathematics

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LEVEL

High School and University

OBJECTIVES

In this paper, annuities are studied from different perspectives using the calculator.

Corresponding eActivity

finance.g1e

OVERVIEW

The future value, FV of an annuity is given by the formula

$$FV = PMT \frac{(1+i)^n - 1}{i}$$

where

PMT = regular payment or installment

i = rate per period

n = number of payments

We assume that payments are to be made at the end of each period.

In this paper, we highlight the different possible ways to approach the study of annuities with the aid of the graphics calculator.

ACTIVITIES

Example 1. Lourdes started depositing \$2000 of her earnings each year into an account earning 8.5% compounded annually. What is the value of the annuity at the end of 30 years? How much interest did her deposits earn?

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Solution:

1. Using the Financial Editor

When using the financial editor, the user must specify [SHIFT SET UP], if payments are made during the beginning or end of the period. In the whole paper, we are going to assume that payments are made at the end of each period.

```
Payment      :End
Date Mode    :365
Background   :None
Label        :Off
Display      :Norm1
┌BGN└┌END└
```

We access Compound Interest and enter the following values:

$n = 30$, $i = 8.5$, $PV(\text{present value}) = 0$,
 $PMT = -2000$
 $P/Y(\text{number of payments per year}) = 1$,
 $C/Y(\text{number of compounding periods per year}) = 1$

```
Compound Interest:End
I% =8.5
PV =0
PMT=-2000
FV =248429.4504
P/Y=1
C/Y=1
┌n└┌I%└┌PV└┌PMT└┌FV└┌AMT└
```

Note that PMT is negative, since \$2000 is being deposited in an account and is treated as an outgoing transaction from the viewpoint of Lourdes. The value of the annuity after 30 years is approximately \$248,429.45.

```
Compound Interest
FV =248429.4504
┌REPT└┌AMT└┌GRPH└
```

To find the amount of interest earned, we subtract the total amount deposited in the annuity (30 payments of \$2,000) from the total value of the annuity after the 30th payment. The interest earned would be \$248,429.45 less $(\$2,000 \times 30)$, which would be \$188,429.45.

We have the following screen dump from the RUN editor:

```
248429.45-(2000×30)
188429.45
┌
└
┌DEL└┌DEL-R└
```

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2. Using the Table Editor

We enter the expression $\frac{2000}{0.085}((1.085)^x - 1)$ (which corresponds to the future value of the annuity) in the calculator. A table of values will be generated, after specifying the appropriate table settings.



In the table, the corresponding value at the end of each period is given. Note that after 30 years, the value of the annuity is \$248,429.4504, the same value we obtained earlier. We give a partial list of the data as follows:

X	Y1
1	2000
2	4170
3	6524.4
4	9079

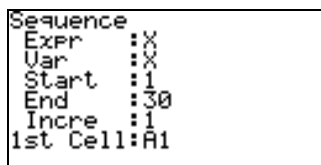
X	Y1
27	189389
28	207487
29	227123
30	248429.4504

3. Using the Spreadsheet

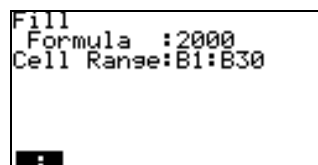
In the spreadsheet editor, we illustrate the growth of the account over 30 years.

We generate a spreadsheet consisting of four columns as follows: the first column(A) gives the payment period, the second column(B) reflects the deposits made, the third(C) and fourth(D) column give, respectively, the interest earned and the balance in the account as of the specified period.

The command fill is used to generate the constant deposit of \$2,000 made each period in column B. Meanwhile, sequence formulas are used to generate the values in columns A,C and D. We have



column A



column B

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```
Sequence
Expr :2000(1.085)^
Var :X
Start :0
End :29
Incr :1
1st Cell:C1
```

```
Sequence
Expr :5^(X)-2000
Var :X
Start :0
End :29
Incr :1
1st Cell:C1
```

column C

```
Sequence
Expr :(2000(.085)^
Var :X
Start :1
End :30
Incr :1
1st Cell:D1
```

```
Sequence
Expr :5)((1.085)^
Var :X
Start :1
End :30
Incr :1
1st Cell:D1
```

```
Sequence
Expr :.085^(X)-1)
Var :X
Start :1
End :30
Incr :1
1st Cell:D1
```

column D

The spreadsheet is given as follows:

SHEE	A	B	C	D
1	1	2000	0	2000
2	2	2000	170	4170
3	3	2000	354.45	6524.4
4	4	2000	554.57	9079
5	5	2000	771.71	11850

SHEE	A	B	C	D
6	6	2000	1007.3	14858
7	7	2000	1262.9	18120
8	8	2000	1540.2	21661
9	9	2000	1841.2	25502
10	10	2000	2167.7	29670

SHEE	A	B	C	D
11	11	2000	2521.9	34192
12	12	2000	2906.3	39098
13	13	2000	3323.3	44421
14	14	2000	3775.8	50197
15	15	2000	4266.8	56464

SHEE	A	B	C	D
16	16	2000	4799.4	63264
17	17	2000	5377.4	70641
18	18	2000	6004.5	78645
19	19	2000	6684.9	87330
20	20	2000	7423.1	96754

SHEE	A	B	C	D
21	21	2000	8224	106978
22	22	2000	9093.1	118071
23	23	2000	10036	130107
24	24	2000	11059	143166
25	25	2000	12169	157335

SHEE	A	B	C	D
26	26	2000	13373	172709
27	27	2000	14680	189389
28	28	2000	16098	207487
29	29	2000	17636	227123
30	30	2000	19305	248429

The spreadsheet serves as a "balance sheet". The balance of the account is reflected in the fourth column: the first line shows that payment is made at the end of the period and no interest is earned. Each subsequent line in the fourth column is computed as follows:

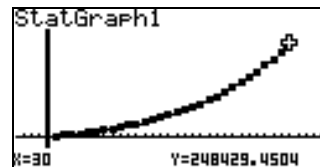
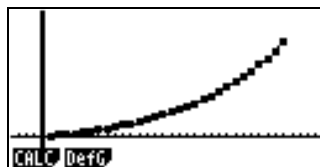
$$\text{Payment} + \text{Interest} + \text{old balance} = \text{new balance}$$

For instance, for period 2, we obtain:

$$2,000 + 170 + 2,000 = 4,170$$

It can be seen from the last screen dump above that the value of the annuity at the end of 30 years is given to be approximately \$248,429.45.

The period of growth can be illustrated graphically as follows:



The following settings were used to generate the graph from the spreadsheet.

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```

StatGraph1
Graph Type:xyLine
XCellRanse:A1:A30
YCellRanse:D1:D30
Frequency :1
Mark Type :•
|GPH1 |GPH2 |GPH3
  
```

The third column reflects the interest earned during a given period. Summing all the entries in this column gives the total interest earned by the account during the 30 year period, \$188,429.45.

SHEET	A	B	C	D
29	29	2000	17636	227123
30	30	2000	19305	248429
31				
32				
33				
CellSum(C1:C30)				

SHEET	A	B	C	D
29	29	2000	17636	227123
30	30	2000	19305	248429
31				
32			188429	
33				
			188429.4504	

Example 2. The Smiths have a newborn daughter. They would want to set up an account to accumulate money for her college education, in which they would like to have \$200,000 after 17 years. If the account pays 4% interest per year compounded quarterly and the Smiths make deposits at the end of every quarter, how large must each deposit be for them to reach their goal?

Solution:

1. Using the Financial Editor

In the financial editor we enter the following values in the Compound Interest menu:

$$\begin{aligned}
 n &= 68, & i &= 4, & PV &= 0, \\
 FV &= 200,000 \\
 P/Y &= 4, \\
 C/Y &= 4
 \end{aligned}$$

We have the following screen dump:

```

Compound Interest:End
n =68
I% =4
PV =0
PMT=-2067.777183
FV =200000
P/Y=4
| n | I% | PV | PMT | FV | AMT |
  
```

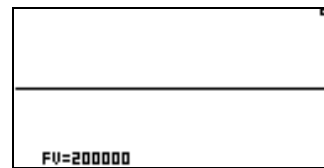
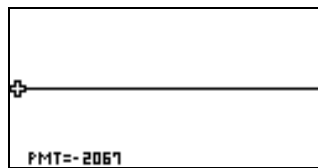
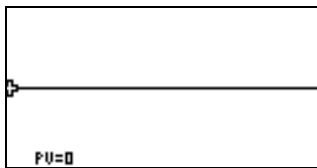
The answer we obtain, that is, the value for the periodic payments, *PMT* is approximately -2067.78. The value is negative, since it is considered an expense in the part of the Smiths. Thus, the Smiths would have to make deposits of \$2,067.78 at the end of every quarter to be able to get \$200,000 after 17 years.

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```
Compound Interest
PMT=-2067.777183

[REPT] [AMT] [GRPH]
```

The given problem, which is that of an increasing annuity, is illustrated more clearly with the graph generated by the calculator. The present value is zero, then constant payments of \$2,067 are made. At the end of the 17 year period, the value is \$200,000.



2. Using the Solver

An alternative way to solve the problem is to solve the following equation in the Solver editor:

$$200,000 = x \frac{(1 + \frac{.04}{4})^{68} - 1}{\frac{.04}{4}}$$

$$200,000 = \frac{x}{.01} ((1.01)^{68} - 1)$$

We are essentially finding the value of PMT here, represented as x , so that the future value after 68 payments (17 years \times 4) is \$200,000.

We obtain $x \approx 2,067.78$

```
E4: 200000=(X,.01)((1.
X=2067.777183
Lft=200000
Rst=200000

[REPT]
```

3. Using the Graph Editor

We enter the linear expression $\frac{x}{0.01}((1.01)^{68} - 1)$ in the calculator. (This is the right hand

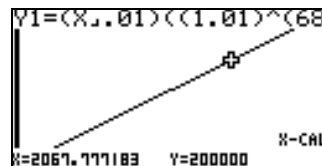
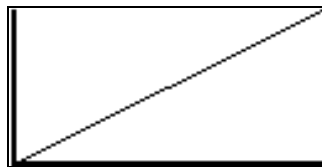
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expression of the equation given in the previous solution) We use the following view window:

```
View Window
Xmin : 0
max : 3000
scale: 1
dot : 23.8095238
Ymin : 0
max : 290166.66
INIT TRIG STD STO RCL
```

```
Graph Func : Y=
Y1:(X(.01))((1.01)^(68))
Y2:
Y3:
Y4:
Y5:
Y6:
SEL DEL TYPE STVL ZMEM DRAW
```

The graph is generated. We specify the value of y as 200,000 and we get the corresponding approximate x value as 2,067.78.



Example 3. (Referring to example 2 given earlier), suppose the Smiths have now accumulated \$200,000 for their daughter's education. They would now like to make quarterly withdrawals over the next four years. How much money can they withdraw each quarter to draw the account to zero at the end of four years?

Solution:

In the Financial Editor, we access Compound Interest and enter the following values:

$$\begin{aligned} n &= 16, & i &= 4, & FV &= 0, \\ PV &= -200,000 \\ P/Y &= 4, \\ C/Y &= 4 \end{aligned}$$

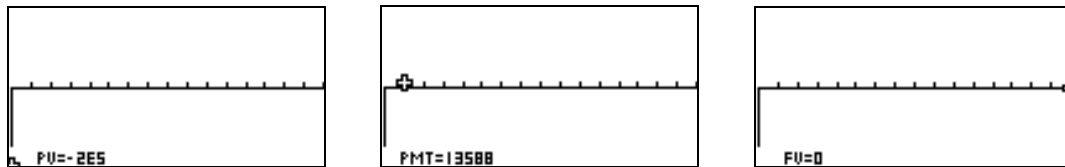
```
Compound Interest:End
n = 16
I% = 4
PV = -200000
PMT = 13588.91936
FV = 0
P/Y = 4
n I% PV PMT FV AMT
```

The calculations show that if the Smiths withdraw approximately \$13,588.92 per quarter, their account balance will drop to zero at the end of four years.

```
Compound Interest
PMT = 13588.91936
REPT AMT GRPH
```

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The screen dumps below illustrate the timeline together with other information for this decreasing annuity problem. The present value is \$200,000. There are 16 payments of approximately \$13,588 and the future value is zero.



REFERENCE

[1] Waner, Stefan et al. *Finite Mathematics and Applied Calculus*, 2nd Edition. Brooks and Cole, 2001.