Financial Mathematics

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LEVEL
High School and University

OBJECTIVES
In this paper, annuities are studied from different perspectives using the calculator.

Corresponding eActivity
finance.g1e

OVERVIEW
The future value, *FV* of an annuity is given by the formula

$$FV = PMT\frac{(1+i)^n - 1}{i}$$

where

PMT = regular payment or installment
i = rate per period
n = number of payments

We assume that payments are to be made at the end of each period.

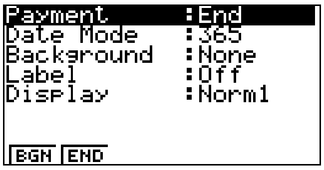
In this paper, we highlight the different possible ways to approach the study of annuities with the aid of the graphics calculator.

ACTIVITIES
Example 1. Lourdes started depositing \$2000 of her earnings each year into an account earning 8.5% compounded annually. What is the value of the annuity at the end of 30 years? How much interest did her deposits earn?

Solution:

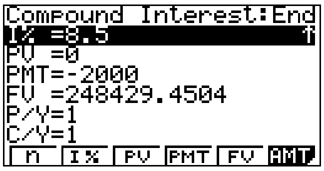
1.<u>Using the Financial Editor</u>

When using the financial editor, the user must specify [SHIFT SET UP], if payments are made during the beginning or end of the period. In the whole paper, we are going to assume that payments are made at the end of each period.



We access Compound Interest and enter the following values:

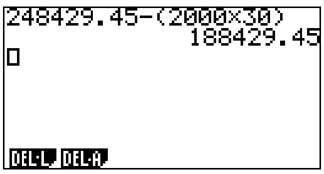
n = 30, *i* = 8.5, *PV*(present value) = 0,
PMT = -2000
P/Y(number of payments per year) = 1,
C/Y(number of compounding periods per year) = 1



Note that *PMT* is negative, since $2000 is being deposited in an account and is treated as an outgoing transaction from the viewpoint of Lourdes. The value of the annuity after 30 years is approximately $248,429.45.

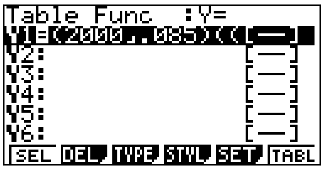
To find the amount of interest earned, we subtract the total amount deposited in the annuity (30 payments of $2,000) from the total value of the annuity after the 30^{th} payment. The interest earned would be $248,429.45 less ($2,000 x 30 years), which would be $188,429.45.

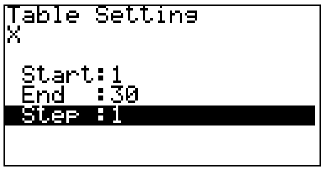
We have the following screen dump from the RUN editor:



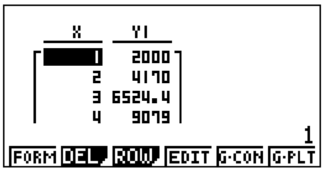
2.<u>Using the Table Editor</u>

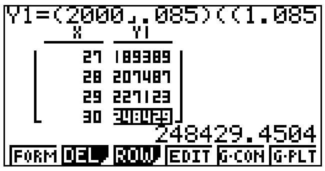
We enter the expression $\frac{2000}{0.085}((1.085)^x - 1)$ (which corresponds to the future value of the annuity) in the calculator. A table of values will be generated, after specifying the appropriate table settings.





In the table, the corresponding value at the end of each period is given. Note that after 30 years, the value of the annuity is $248,429.4504, the same value we obtained earlier. We give a partial list of the data as follows:



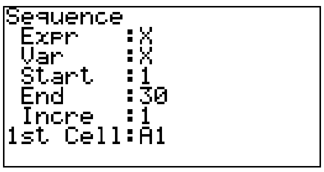


3.<u>Using the Spreadsheet</u>

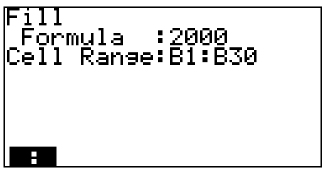
In the spreadsheet editor, we illustrate the growth of the account over 30 years.

We generate a spreadsheet consisting of four columns as follows: the first column(A) gives the payment period, the second column(B) reflects the deposits made, the third(C) and fourth(D) column give, respectively, the interest earned and the balance in the account as of the specified period.

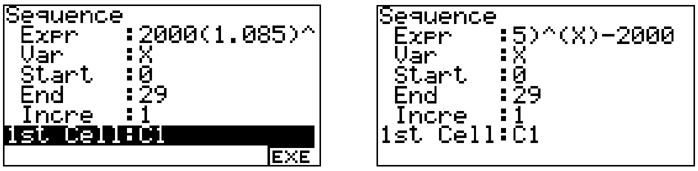
The command fill is used to generate the constant deposit of $2,000 made each period in column B. Meanwhile, sequence formulas are used to generate the values in columns A,C and D. We have



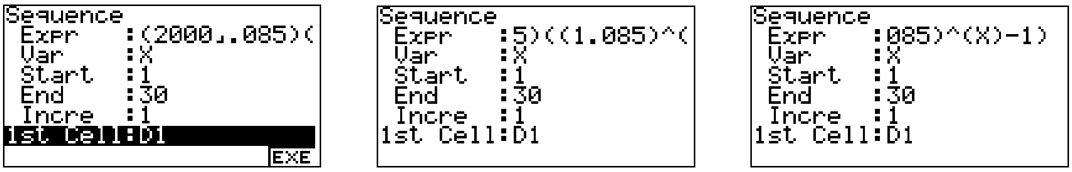
column A



column B

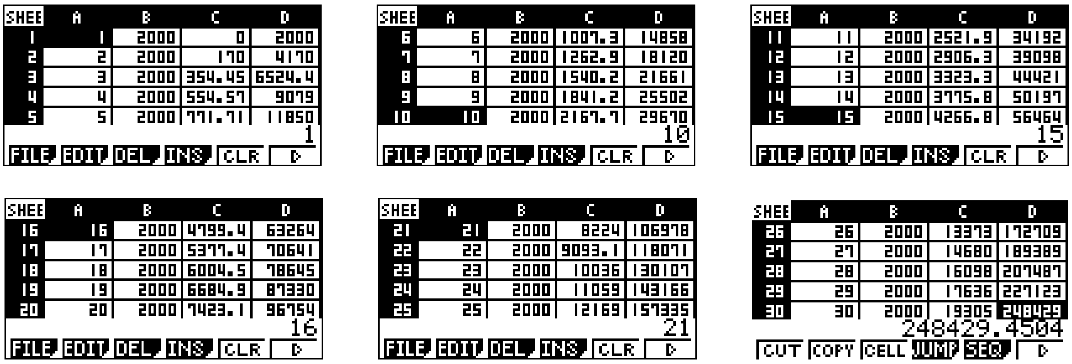


column C



column D

The spreadsheet is given as follows:



The spreadsheet serves as a “balance sheet”. The balance of the account is reflected in the fourth column: the first line shows that payment is made at the end of the period and no interest is earned. Each subsequent line in the fourth column is computed as follows:

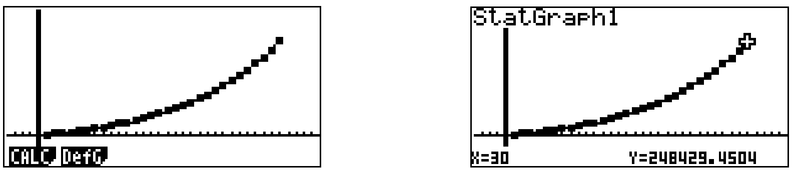
Payment + Interest + old balance = new balance

For instance, for period 2, we obtain:

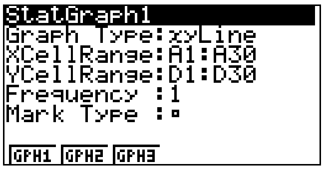
2,000 + 170 + 2,000 = 4,170

It can be seen from the last screen dump above that the value of the annuity at the end of 30 years is given to be approximately $248,429.45.

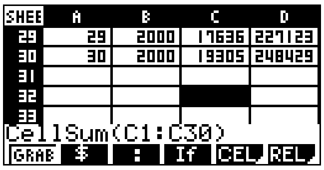
The period of growth can be illustrated graphically as follows:

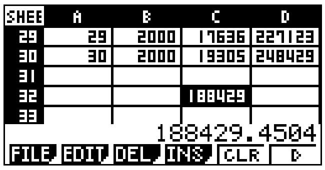


The following settings were used to generate the graph from the spreadsheet.



The third column reflects the interest earned during a given period. Summing all the entries in this column gives the total interest earned by the account during the 30 year period, $188,429.45.





Example 2. The Smiths have a newborn daughter. They would want to set up an account to accumulate money for her college education, in which they would like to have $200,000 after 17 years. If the account pays 4% interest per year compounded quarterly and the Smiths make deposits at the end of every quarter, how large must each deposit be for them to reach their goal?

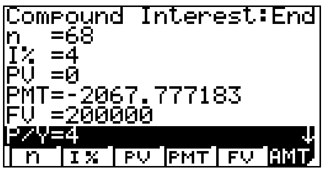
Solution:

1.<u>Using the Financial Editor</u>

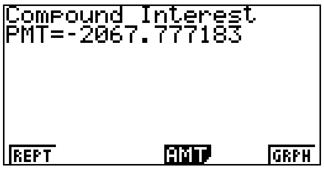
In the financial editor we enter the following values in the Compound Interest menu:

$$n = 68, \; i = 4, \; PV = 0,$$
$$FV = 200{,}000$$
$$P/Y = 4,$$
$$C/Y = 4$$

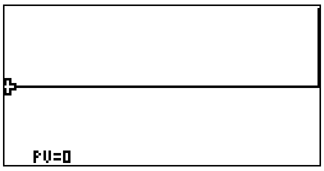
We have the following screen dump:

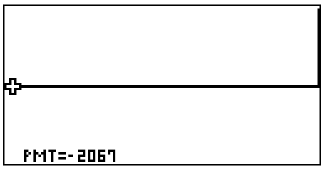


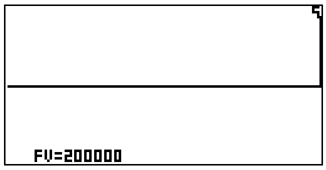
The answer we obtain, that is, the value for the periodic payments, *PMT* is approximately -2067.78. The value is negative, since it is considered an expense in the part of the Smiths. Thus, the Smiths would have to make deposits of $2,067.78 at the end of every quarter to be able to get $200,000 after 17 years.



The given problem, which is that of an increasing annuity, is illustrated more clearly with the graph generated by the calculator. The present value is zero, then constant payments of \$2,067 are made. At the end of the 17 year period, the value is \$200,000.







2.<u>Using the Solver</u>

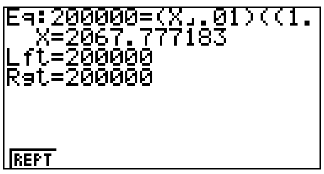
An alternative way to solve the problem is to solve the following equation in the Solver editor:

$$200{,}000 = x\frac{(1+\frac{.04}{4})^{68}-1}{\frac{.04}{4}}$$

$$200{,}000 = \frac{x}{.01}((1.01)^{68}-1)$$

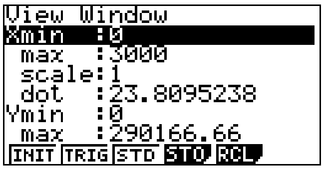
We are essentially finding the value of *PMT* here, represented as *x*, so that the future value after 68 payments(17 years x 4) is \$200,000.

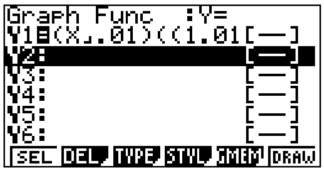
We obtain $x \approx 2{,}067.78$



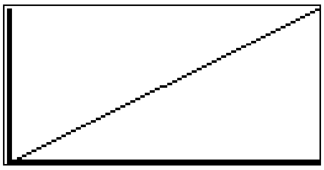
3. <u>Using the Graph Editor</u>

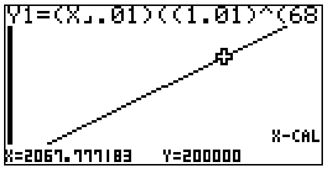
We enter the linear expression $\frac{x}{0.01}((1.01)^{68}-1)$ in the calculator.(This is the right hand expression of the equation given in the previous solution) We use the following view window:





The graph is generated. We specify the value of *y* as 200,000 and we get the corresponding approximate *x* value as 2,067.78.



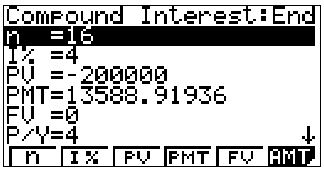


Example 3. (Referring to example 2 given earlier), suppose the Smiths have now accumulated $200,000 for their daughter's education. They would now like to make quarterly withdrawals over the next four years. How much money can they withdraw each quarter to draw the account to zero at the end of four years?

Solution:

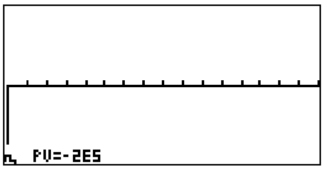
In the Financial Editor, we access Compound Interest and enter the following values:

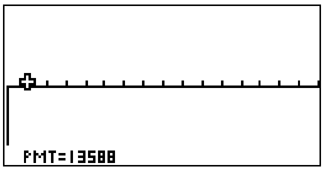
n = 16, *i* = 4, *FV*= 0,
PV = -200,000
P/Y = 4,
C/Y = 4

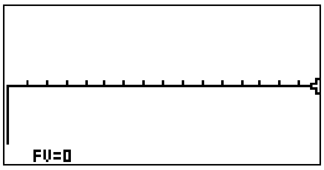


The calculations show that if the Smiths withdraw approximately $13,588.92 per quarter, their account balance will drop to zero at the end of four years.

The screen dumps below illustrate the timeline together with other information for this decreasing annuity problem. The present value is $200,000. There are 16 payments of approximately $13,588 and the future value is zero.







REFERENCE

[1] Waner, Stefan et al. *Finite Mathematics and Applied Calculus*,2nd Edition. Brooks and Cole, 2001.

