Cost, Revenue and Profit Functions

Jozef Hvorecky
Vysoká škola manažmentu / City University
Bratislava, Slovakia

LEVEL

High school or university students starting with Functions and Calculus.

OBJECTIVES

To comprehend basic business notions of profitability from a calculation point of view. Perform and visualize the relevant calculation methods.

Corresponding eActivity

C06PARTY.g1e (for Activity 1), C06NECKL.g1e (for Activity 2)

OVERVIEW

The cost, revenue and profit belong among basic business calculations. They express principal applications of Mathematics in Business and Economy. By means of the following eActivities we demonstrate how cost, revenue and profit can be modeled using functions and how to read the graphs that represent them.

[Note]
We shall use small letter *x* instead of capital X as shown on the calculator throughout the paper.

EXPLORATORY ACTIVITIES

Activity 1 (C06PARTY.g1e):

A student body discusses a possibility to organize a fund-raising dinner. The presumed price of a ticket is $24. One member of the organizing committee has found an appropriate space for the event which can be rented for $350. Another one knows a company providing chairs for $1.50 per night plus free tables.
The committee needs to know the minimum number of people which has to come for covering all these expenses.

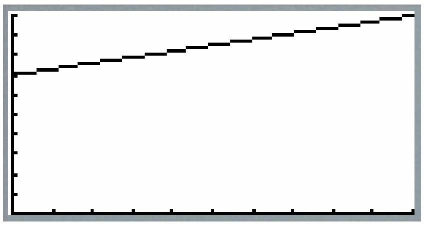
Solution:

(a) (Refer to Cost) The expenses grow with the number of people. The calculation of costs consists of two parts:

- The fixed cost (the room rental) is $350;
- The variable cost depends on *x* – the number of visitors. As everyone needs a chair, the needed budget is 1.5*x*.

$$C(x) = 1.5x + 350$$

The graph in eActivity shows the growth of costs for the number of visitors (*x*) between 0 and 100 and the cost (*y*) from $0 to $500.

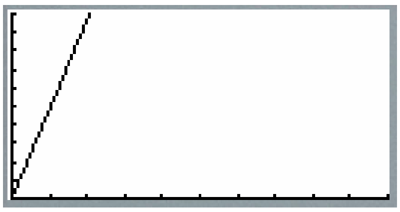


The scale of x-axis is 10, the scale of y-axis is 50.

(b) (Refer to Revenue) The revenue depends on the number of visitors. Everyone pays $24. The calculation is therefore simple:

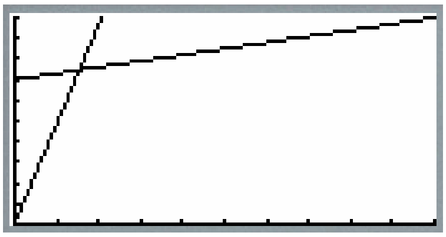
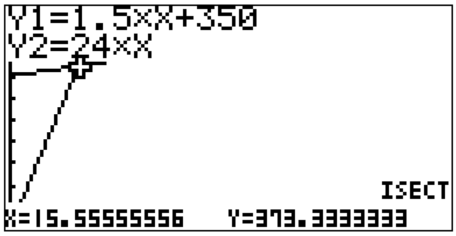
$$R(x) = 24x$$

The graph in eActivity shows the growth of costs for the number of visitors (*x*) between 0 and 100 and the cost from $0 to $500.



As above, the scale of x-axis is 10, the scale of y-axis is 50.

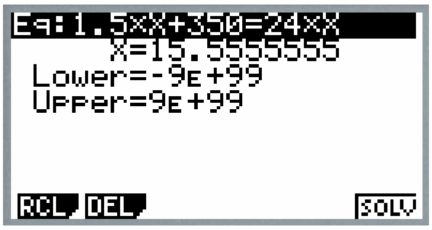
(c) (Refer to Cost and Revenue) From the graphs we see that the revenue function grows much faster than the cost function. This is even better observable when both graphs are displayed at the same screen as shown below and we use G-Solve ([Shift] + [F5]) to find the intersection at about x=15.56.

(d) (Refer to Break-even point) As the scale of the x-axis is 10, somewhere between 10 and 20 visitors are sufficient for the revenue line to intersect the cost line. Their intersection is called the break-even point. To the left of it, our costs are higher than our revenue. To the right of it, our revenue exceeds the cost and we become to profit. To calculate it means to solve the equation

$$1.5x + 350 = 24x$$

The calculator solver gives us the answer:



The solution is x = 15.5555555. As it not a whole number, we have to use the nearest higher value. The minimum number of visitors necessary for making the fund-raising party profitable is 16.

(e) (Refer to Profit $500) It is quite easy to see that the profit depends on our revenue and cost. To calculate it, we have to subtract the cost from the revenue. When *x* visitors come to our party, our profit will be

$$P(x) = R(x) - C(x)$$
$$P(x) = 24x - (1.5x + 350) = 22.5x - 350$$

The function P(x) is called the profit function. Now we can easily formulate the equations helping us to respond various questions on profitability. The question: "How many do visitors have to arrive to make the profit $500?" is equivalent to the equation:

$$22.5x - 350 = 500$$

The answer is 37.77, i.e. 38 people.

EXERCISES A

***Exercise 1*.**
How many visitors have to come to the party to make the profit $1000?

***Exercise 2*.**
Graph the profit function P(x). Use an appropriate scale to view its values for x between 0 and 60. In what point does it intersect the x-axis?

SOLUTIONS to EXERCISES A

***Exercise 1*.**
60 visitors

***Exercise 2*.**
The function has the form y = 22.5x – 350. An appropriate scale for y is -350 to 1000. The graph intersects the x-axis at the value equal to the break-even point.

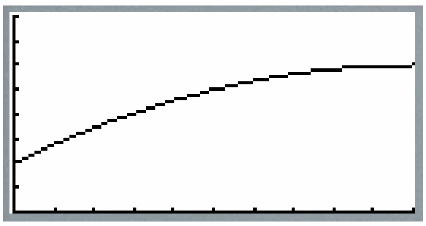
Activity 2 (C06NECKL.g1e):

Two girls want to make money to buy Christmas gifts for their relatives and friends. They see their opportunity in making and selling necklaces from glass beans. They realized that first they have to invest $50 to various tools. For each necklace they also need a set of beans. The supplier offers them for the basic price $2, but the price declines by 1 cent per set.

(a) (Refer to Cost function) The cost consists of the fixed cost ($50) and the variable cost per set is 2–0.1*x where x is the number of sets. For x sets it means x(2–0.1*x). Thus,

$$C(x) = 50 + x(2 - 0.1x) = 50 + 2x - 0.01x^2$$

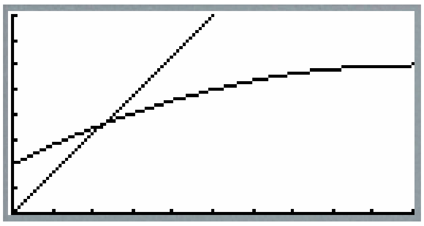
Notice that the cost function is quadratic so the growth is not linear.



(b) (Refer to Revenue function) The girls presume that $3.99 per necklace could be their price. The revenue depends on the number of sold necklaces:

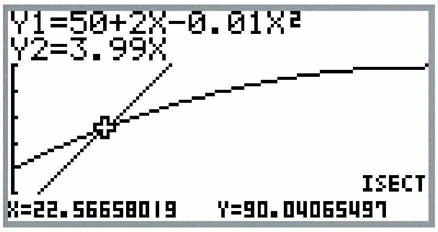
$$R(x) = 3.99x$$

(c) (Refer to Cost and Revenue) Drawing the both functions to the same screen gives us an estimation of the break-even point.



As the x-scale is from 0 to 100 with a step 10, the break-even point is between 20 and 30. The y-scale is between 0 and 200 with the step 25.

(d) (Refer to Using G-Solve) Using the G-Solve tool helps us to find the break-even point quickly. To activate it, press SHIFT F5 (G-Solv). Then again F5 (ISCT – "intersection"). The screen will show the result.

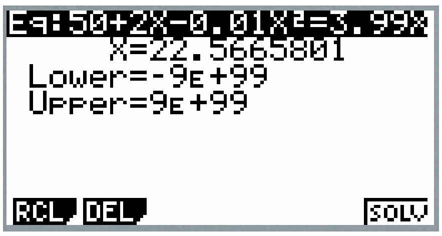


At the top, the formulas of both functions are shown, at the bottom the coordinates of their intersection. Its x-coordinate (22.56) is of a special meaning for us. It say that to break the profitability, the girls have to make at least 23 necklaces.

(e) (Refer to Using Solver) The same result can be achieved using the solver. The equation is simple and logical:

$$50 + 2x - 0.01x^2 = 3.99x$$

Typing it into Solver and pressing F6 (SOLV) produces the same value.



EXERCISES B

***Exercise 1*.**

Express the profit function P(x).

SOLUTIONS to EXERCISES B

***Exercise 1*.**

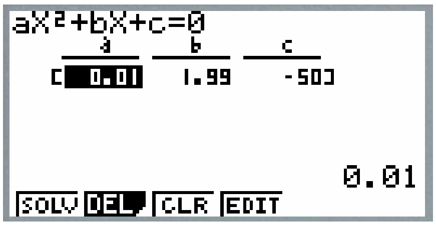
The function has the form

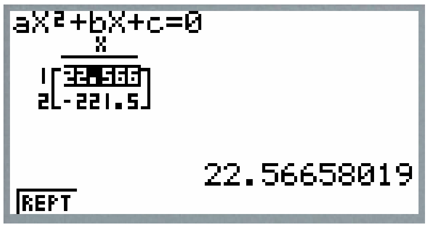
$$P(x) = R(x) - C(x) = 3.99x - (50x + 2x - 0.01x^2) = 0.01x^2 + 1.99 - 50$$

(f) (Refer to <u>Break-even point</u>) Using the polynomial solver, we can apply another method of the calculation of the break-even point. It is easy because it is the break-even point is the root of this function (i.e. the point in which the negative values are changing to positive ones). From above Exercise 1, we know that

$$R(x) = 0.01x^2 + 1.99 - 50$$

This is a quadratic polynomial so its degree is 2. We fill in its parameters (a = 0.01, b =1.99, c = -50). The solution looks as follows:





Notice that the equation has two roots: 22.566 and -221.5. Naturally, in the context of our problem only the positive one has meaning. So, to be profitable, the girls must complete at least 23 necklaces.

EXERCISES B

***Exercise 2*.**

How many necklaces have the girls to produce to make their total profit $150?

SOLUTIONS to EXERCISES B

***Exercise 1*.**

The equation must have the form P(x) = 150 i.e.

$$0.01x^2 + 1.99 - 200 = 0 .$$

The polynomial solver gives the results 73.4 and -272.4. The positive root is our solution.

REFERENCE

[Tan] S. T. Tan, *College Mathematics*, PWS-KENT Publishing Company, 1998. ISBN 0-534-91791-7.

