# 

**Unit**

**8**

**VECTORS**

# KEY CONCEPTS

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**Vector notation**

Vectors are quantities that possess both magnitude and direction.

A vector is represented by a directed line segment, with the direction indicated by the arrow drawn on the line. The magnitude of the vector is represented by the length of the line segment.

Magnitude of a vector **a** is denoted by |**a**|. The vector below can be denoted by *AB* or **a**.

*B*

*A*

**a**

# Equal vectors

Two vectors are equal if they have the same magnitude and the same direction.

**Example** A parallelogram *PQRS* has *PQ* = *SR* and *SP* = *RQ*.

*P Q*

*S R*

# Negative vectors

Negative vectors have the same magnitude but are in opposite direction.

# Zero vector

The zero vector or null vector is a vector with zero magnitude and no direction.

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# Addition of vectors (Triangle Law of Vector Addition)

When two vectors are represented by two sides of a triangle in magnitude and direction taken in same order then the resultant of the vectors can be represented by the third side of that triangle.

*C*

**a** + **b**

**b**

*A* **a** *B*

Notice how the direction flows from point *A* (initial point) to point *B* and then to point *C* (terminal point). This means that the shortest pathway to get from point *A* to point *C* is *AC*.

From the diagram, the resultant vector can be written as *AC* = **a** + **b**

# Addition of vectors (Parallelogram Law of Vector Addition)

If two vectors acting simultaneously at a point can be represented both in magnitude and direction by the adjacent sides of a parallelogram drawn from a point, then the resultant vector is represented both in magnitude and direction by the diagonal of the parallelogram passing through that point.

*D C*

**a**

**a** + **b**

**b**

*B*

*A*

Notice how the direction of both vectors are acting on the point *A*. Hence, we draw out the dotted lines

*DC* and *BC* to show the parallelogram. The diagonal of this parallelogram will be the resultant vector. From the diagram, the resultant vector can be written as *AC* = **a** + **b**.

* Commutative Law: **a** + **b** = **b** + **a**
* Associative Law: (**a** + **b**) + **c** = **a** + (**b** + **c**)

Notice that point *C* is a connecting point for the addition of the vector.

# Subtraction of vectors

**a** – **b** = **a** + (–**b**)

A negative vector has the opposite direction from the positive vector.

Magnitude is not affected by the negative sign.

Unit 8 **Vectors**

# Scalar multiplication of a vector

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When a vector **a** is multiplied by a scalar *k*, the magnitude becomes *k*|**a**| and the direction will be affected only if *k* is negative.

If **a** and **b** are vectors and *m* and *n* are real numbers,

* *m*(*n***a**) = *n*(*m***a**) = (*mn*)**a**
* (*m* + *n*)**a** = *m***a** + *n***a**
* *m*(**a** + **b**) = *m***a** + *m***b**
* *AB* = *k BC* , if points *A*, *B* and *C* lie on a straight line.

# Column vectors

Column vectors are written in the form of where *x* is the horizontal change and *y* is the vertical change.

Laws of column vectors: Given **a** = and **b** = .

If **a** = **b**, then *p* = *r* and *q* = *s*.

**a** + **b** =

**a** – **b** =

*h***a** =

*h***a** + *k***b** = + =

Magnitude of a 2-dimensional column vector: Given **a** = , |**a**| = .

Magnitude of a 3-dimensional column vector: Given **c** = , ||= .

# Position vectors

Position vectors are vectors of a point drawn from the origin to the point itself. Position vectors are made with reference to the Cartesian plane and the origin *O*.

Given *OA* = , *x* is the change in *x* units from origin and *y* is the change in *y* units from origin.

# Dot product

The dot product is a scalar quantity and is commutative: **a∙b = b∙a**.

Geometric definition: **a∙b =**  cos *θ*, where *θ* is the angle formed between the two vectors.

Algebraic definition: Given **a** = and **b** = , **a∙b** = **mp** + **nq**

Algebraic definition: Given **a** = and **b** = , **a∙b** = **mp** + **nq** + **lr**

# Cross product

The resultant vector of a cross product is a 3-dimension vector and is perpendicular to the plane containing the two vectors.

Geometric definition: **a∙b =** |**a**| |**b**| sin *θ*, where *θ* is the angle between the two vectors.

Algebraic definition: Given **a** = and **b** = , **a**  **b** =.

Not commutative: **a**  **b** ≠ **b**  **a**.

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# WORKED EXAMPLES

**Level 1**

## Worked Example 1

Find the value of the unknown constants *h* and *k* for 3**a** + 2*k***b** = *h***a** + 6**b**.

Solution

By comparison, *h* = 3 and *k* = 3.

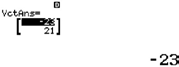
## Worked Example 2

Find the value of the unknown constants *a* and *b* in =

Solution

By comparison, *a* = 2 and *b* = 3.

***Worked Example 3***



**ClassWiz steps**

Solution using ClassWiz steps

Press w5 and select 1 followed by 2 to define vector *A* as a 2-dimensional vector.

Press T and select 1, 2 followed by 2 to define vector *B*

as a 2-dimensional vector.

Press T and select 1, 3 followed by 2 to define vector *C*

as a 2-dimensional vector.

Press W and 3(T3 + T4)pT5

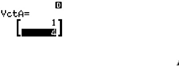
Press = and the calculator will show

Evaluate the column vector

3.

Solution

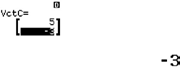
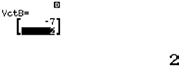
3



= 3

=

=



Unit 8 **Vectors**

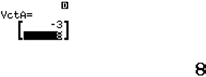
## Worked Example 4

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Evaluate the magnitude of the following column vectors. You may use your ClassWiz calculator. (a)

(b)

Solution



Press W to go back to the main screen and press

q(for the f button.

Press T3 and press =. The calculator will display 8.544003745.

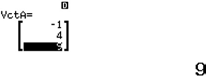
**ClassWiz steps**

Solution using ClassWiz steps

Press w5 and select 1 followed by 2 to define vector *A* as a 2-dimensional vector.

(a) = =

(b) = =



Press W to go back to the main screen and press

q(for the f button.

Press T3 and press =. The calculator will display 9.899494937.

**ClassWiz steps**

Solution using ClassWiz steps

Press w5 and select 1 followed by 3 to define vector *A* as a 3-dimensional vector.

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# Level 2

## Worked Example 5

In the diagram, *ABCD* is a parallelogram and the diagonals intersect at point *E*. It is given that *DA* = 2**p** + **q**

and *DC* = –**p** + 4**q**.

*B*

1. Find *AB* .
2. Find *DB*.
3. Find *AC* .
4. Find *CE* .

2**p** + **q**

*D*

*A*

E

*C*

–**p** + 4**q**

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Solution

1. *AB* = –**p** + 4**q**.

Note that *AD* = –*DA*.

1. *DB* = 2**p** + **q** – **p** + 4**q** = **p** + 5**q**
2. *AC* = *AD* + *DC* = –2**p** – **q** – **p** + 4**q** = –3**p** + 3**q** (d) *CE* = – *AC* = – (–3**p** + 3**q**) = + **p** – **q**

## Worked Example 6

Find the value(s) of the unknown constants *p* and *q* in the following vectors.

1. 4*p***a** + 5**b** = (*p*2 + 3)**a** + *q***b**

Solution

(a) 4*p***a** + 5**b** = (*p*2 + 3)**a** + *q***b**

By comparison, 4*p* = *p*2 + 3

*p*2 – 4p + 3 = 0

*(p – 3)(p – 1) = 0*

*p* = 3 or 1 By comparison, *q* = 5

(b)

By comparison, 3*p* = 9

*p* = 3

By comparison, – *q* + 2*q*2 = 1

2*q*2 – *q* – 1 = 0 (2*q* + 1)(*q* – 1) = 0

*q* = – or 1

Unit 8 **Vectors**

## Worked Example 7

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Find the value of the angle, in degrees, formed by the vector against the positive *x*-axis.

Solution

Gradient of *OA* = –

Since gradient is negative, the angle formed by the vector against the positive *x*-axis is going to be obtuse.

tan *θ* = –

Basic angle = tan–1 = 38.7°

Angle formed by the vector against the positive *x*-axis = 180° – 38.7° = 141.3°

## Worked Example 8

1. In the diagram, 2*AB* = 3*AX*. Find the ratio of the area of ∆*ABC* to the area of ∆*XBC*.

*X*

*A*

*C*

*B*

1. In the diagram, *BC* is parallel to *DE* and *AC* = 5*CE*. Find the ratio of the area of ∆*ABC* to the area of ∆*ADE*.

*D*



*B*

*A C* E

Solution

1. Given 2*AB* = 3*AX*, and *AX* : *AB* = 2 : 3,

Since triangle *CAB* and triangle *CXB* share the same height,

Note that the base in reference to the height is *AB* and *XB.*

= =

1. Since ∆*ABC* *is* similar to *ΔADC,*

= = =

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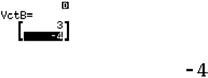
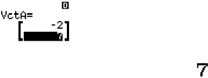
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# Level 3

## Worked Example 9

Evaluate the dot product of **a** = and **b** = using the ClassWiz calculator and hence find the angle

that is formed between the two vectors.



**ClassWiz steps**

Solution using ClassWiz steps

Press w5 and select 1 followed by 2 to define vector *A* as a 2-dimensional vector.

Press T and select 1, 2 followed by 2 to define vector *B*

as a 2-dimensional vector.

Press WT3TR2T4 = .

Unit 8 **Vectors**

## Worked Example 10

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1. By finding the magnitude of the cross product of **a** = and **b** = with the use of ClassWiz

calculator, find the area of the triangle that is formed by the two vectors.

1. By finding the magnitude of the cross product of **a** = and **b** = with the use of ClassWiz

calculator, find the area of the triangle that is formed by the two vectors.

Solution

1. The area of the triangular plane is 6.5 square units.

The magnitude of the cross product gives the area of the parallelogram plane.

To find the area of the triangular plane, we need to divide the area by 2.



as a 2-dimensional vector .

Press WeT3OT4 = .

Press T and select 1, 2 followed by 2 to define vector *B*

a 2-dimensional vector .

.

Press w5 and select 1 followed by 2 to define vector *A* as

**ClassWiz steps**

Solution using ClassWiz steps

1. The area of the triangular plane is 20.5 square units. (3 s.f.)

|  |  |
| --- | --- |
|  | The magnitude of |
|  |
|  | the cross product |
|  | gives the area of the |
|  | parallelogram plane. |
|  | To find the area of the |
|  | triangular plane, we  need to divide the area |
|  | by 2. |

Press w5 and select 1 followed by 3 to define vector *A* as



**ClassWiz steps**

Solution using ClassWiz steps

a 3-dimensional vector

.

Press T and select 1, 2 followed by 3 to define vector *B*

as a 3-dimensional vector .

Press WeT3OT4 = .

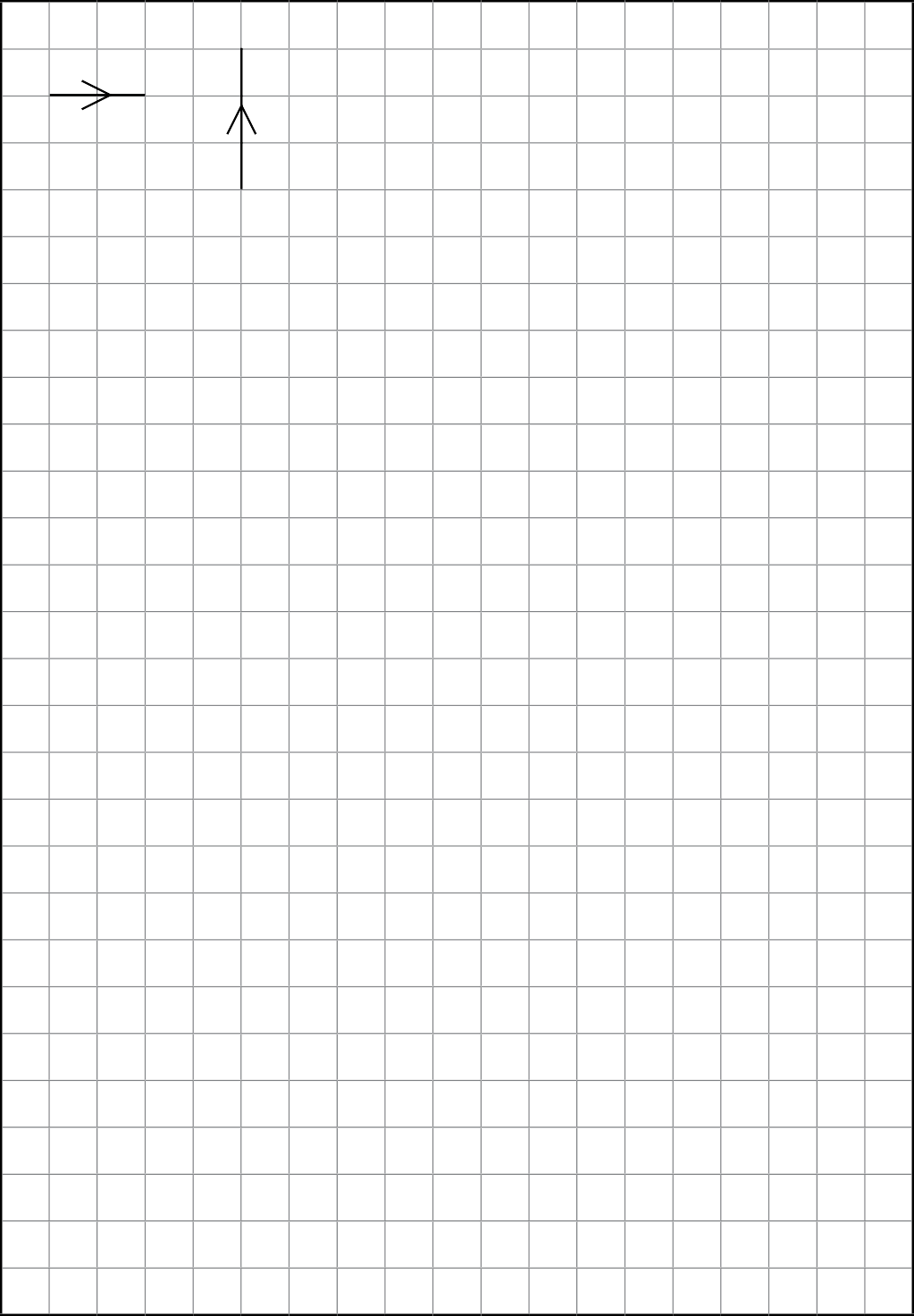
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**CLASSWIZ WORKSHEETS**

# CLASSWIZ WORKSHEETS

**Level 1**

1. In the grids, vectors **a** and **b** are given. Draw the following vectors. (a) 2.5**a** (b) –3**a**
2. 2**b** (d) 4**b**

(e) –**b**

**a**

**b**

Unit 8 **Vectors**

1. Simplify the following vectors.

**CLASSWIZ WORKSHEETS**

|  |  |
| --- | --- |
| (a) –(**a** + **b**) | (b) 2**a** – (**b** – 2**a**) |
| (c) 0**b** – 3**a** + 7**b** | (d) 4**b** – (0**a** + 2**b**) |

1. Find the resultant vector *AB* in the following diagrams.

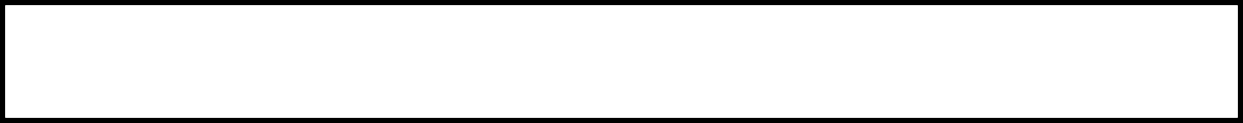
|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| (a) *A* |  |  | **a** | *O* |  |  | (b) 3**q**  *O* |  | *B* |  |  |
|  |  |  |  |  |  |  | –2**p** |  |  |  |  |
|  |  |  | 2**b** |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  | *A* |  |  |  |
| *B* |  |  |  |  |  |  |  |  |  |  |  |
| (c) *A* |  |  |  |  | *B* |  | (d) *O* | –5**g** |  |  |  |
| 2**h** |  | *O* | 3**k** |  |  |  | 2**h** | *B* |  |  |  |
|  |  |  |  |  |  |  | *A* | | | | |
| (e) *A* |  |  |  |  |  | *B* | (f ) *A* | 5**m** |  |  |  |
|  | 1\_ **i**  2 |  |  |  | 3**j** |  | *O* | | | | |
|  |  |  | *O* |  |  |  |  | | | | |
|  |  |  |  |  |  |  | –2**n** | | | | |
|  |  |  |  |  |  |  | *B* | | | | |
| (g) *A*  *C* 2**a**  5\_ **c**  3 – 1\_ **b**  2  *O*  *B* | | | | | | | (h) *O* | –2**k** |  |  |  |
|  |  |  |  | *C* |
|  |  |  | 1\_ **l** |  |
| –3**j** | *B* |  | 2 |  |
| *A* |  |  |  |  |

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**CLASSWIZ WORKSHEETS**

1. In the grids, vectors **a** and **b** are given. Draw the following resultant vectors.

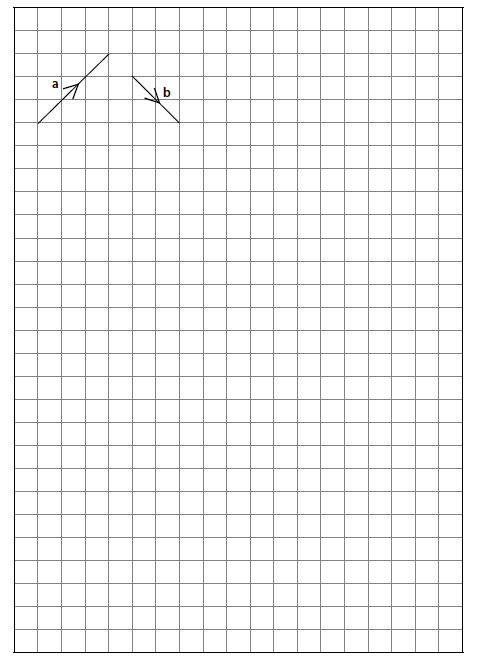
(a) 2**a** + **b** (b) **a** + **b**



(c) **a** + 2**b**

(d) 2**b** +**a**

Teachers to note that vectors are commutative. Hence resultant vectors are the same.



Unit 8 **Vectors**

1. Evaluate the following scalar multiplication.

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|  |  |
| --- | --- |
| (a) 2(3**a**) | (b) 3(**a** – 4**b**) |
| (c) 5(2**a** – **b**) + 3**a** | (d) –4(–3**a** + 7**b**) + 2(5**a** – 3**b**) |

1. By comparison, find the value of the unknown constants.

|  |  |
| --- | --- |
| (a) 2**a** + 5**b** = *p***a** + *q***b** | (b) – 3**a** + 7**b** = *h***a** – *k***b** |
| (c) – 4**a** – 6**b** = *p***a** – **a** + *q***b** | (d) 3(**b** – *h***a**) = 5**a** + *k***b** |
| (e) *k***a** + 2**b** = 5(**a** + *h***b**) – 3**b** | (f ) –5(2**a** – *p***b**) = 3(**a** – 4**b**) + *q***a** – 3**b** |

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1. Write down the column vector for the following vectors in the Cartesian plane.

**i**

**h**

**g**

**f**

**e**

**d**

**b**

**a**

**c**

* 1. **a** = (b) **b** =

(c) **c** = (d) **d** =

(e) **e** = (f) **f** =

(g) **g** = (h) **h** =

(i) **i** =

Unit 8 **Vectors**

 8. Evaluate the following column vectors, given that

**ClassWiz set-up**

Press w5 and select 1 followed by 2 to

define vector *A* as a 2-dimensional vector.

Press T and select 1, 2 followed by 2 to

define vector *B* as a 2-dimensional vector.

Press T and select 1, 3 followed by 2 to

define vector *B* as a 2-dimensional vector.

Press T and select 1, 4 followed by 2 to

define vector *B* as a 2-dimensional vector.

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*OA* = , *OB* = , *OC* = and *OD* = .

|  |  |  |
| --- | --- | --- |
| (a) *OA* + *OB* |  |  |
| (b) *OA* – *OB* | (c) *OA* + *OB* + *OC* |  |
| (d) *OC* + 2*OD* | (e) 2*OD* + *OC* |  |
| Teacher to note: Vectors are commutative. | |  |
| (f ) 3*OA* – 5*OD* | (g) 4(*OD* – *OB*) |  |
| (h) (*OA* + *OB*) + *OD* | (i) *OA* + (*OB* + *OD*) |  |
| Teacher to note: Vectors are associative. | |  |

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**CLASSWIZ WORKSHEETS**

1. By comparison, find the value of the unknown constants *a* and *b* in the following vectors.

|  |  |
| --- | --- |
| (a) = | (b) = |
| (c) = | (d) -2 = |
| (e) = | (f ) 2= |

1. Evaluate the magnitude of the following column vectors.

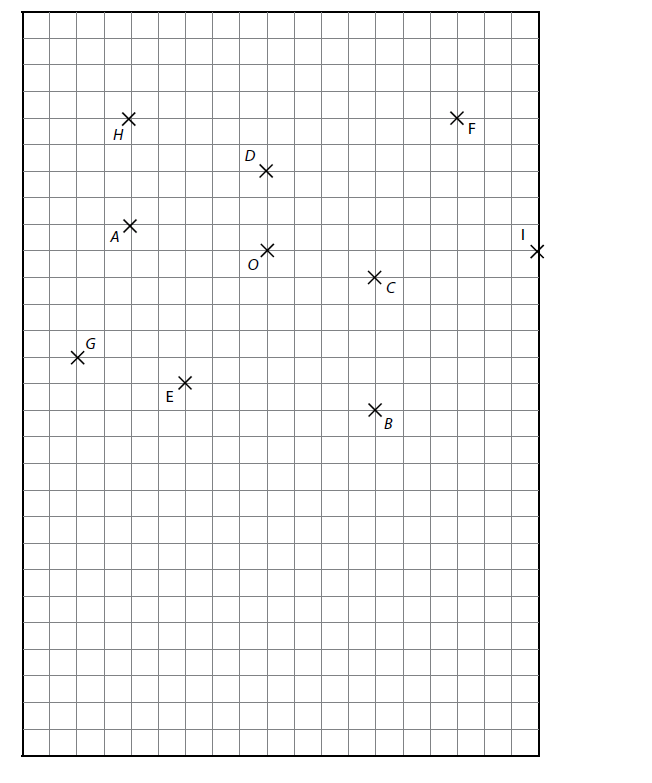
You may use the ClassWiz calculator by pressing the f function.

|  |  |
| --- | --- |
| (a) |  |
| (b) | (c) |
| (d) | (e) |
| Teacher to note: Is the magnitude of going to be twice of ? Since 2= , the scalar  is 2, and the magnitude will be twice of . | |
| (f ) | (g) |
| Teacher to note: Will these 2 vectors give the same magnitude? Since = -, the two  vectors are in the opposite direction. They will have the same magnitude. | |

Unit 8 **Vectors**

1. Write down the position vectors of the following vectors.

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* 1. *OA* = (b) *OB* =

(c) *OC* = (d) *OD* =

(e) *OE* = (f) *OF* =

(g) *OG* = (h) *OH* =

1. *OI* =

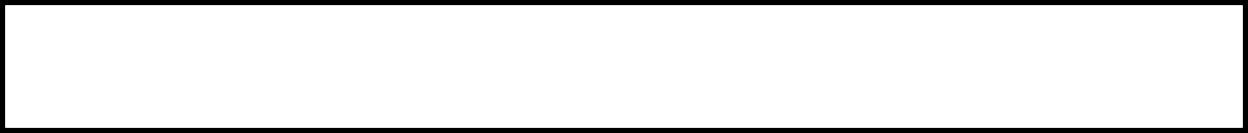
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**CLASSWIZ WORKSHEETS**

# Level 2

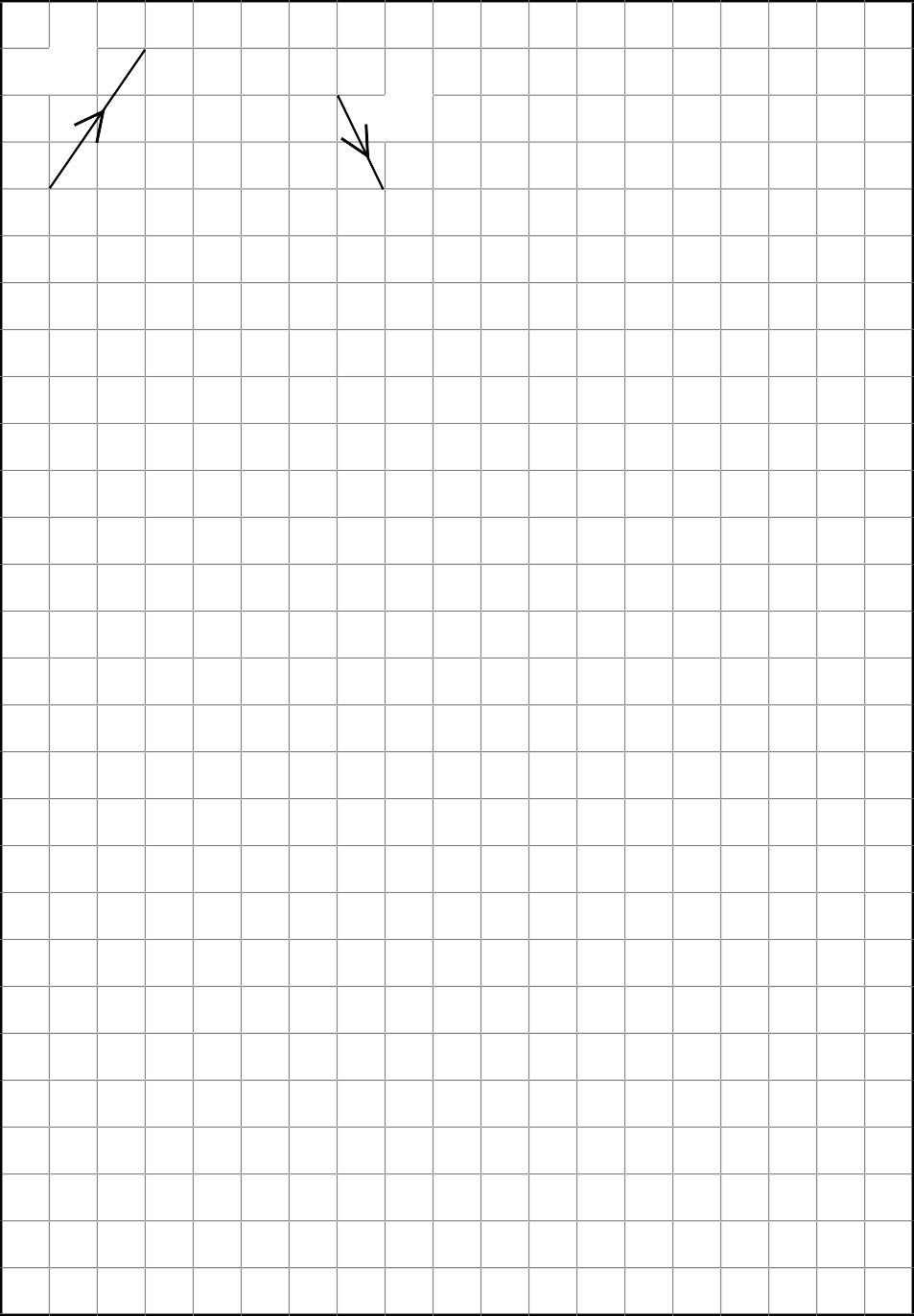
1. In the grids, vectors **a** and **b** are given. Draw the following resultant vectors.

(a) 2**a** + **b** (b) –2**a** – 2**b**



(d) – **b** – **a**

(c) **a** + **b**

(e) 3**a** – 2.5**b**

Teachers to note that two vectors are in opposite direction, and help them understand why.

**b**

**a**

Unit 8 **Vectors**

1. On the grid, *OX* = **a** and *OY* = **b**. There are several points marked on the grid. Write down in terms of **a**

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and/or **b** for the following vectors.

E

*D*

F

*O*

*C*

*Y*

*X*

**a**

**b**

*A*

*B*

* 1. *OA* (b) *OB*

(c) *OC* (d) *OD*

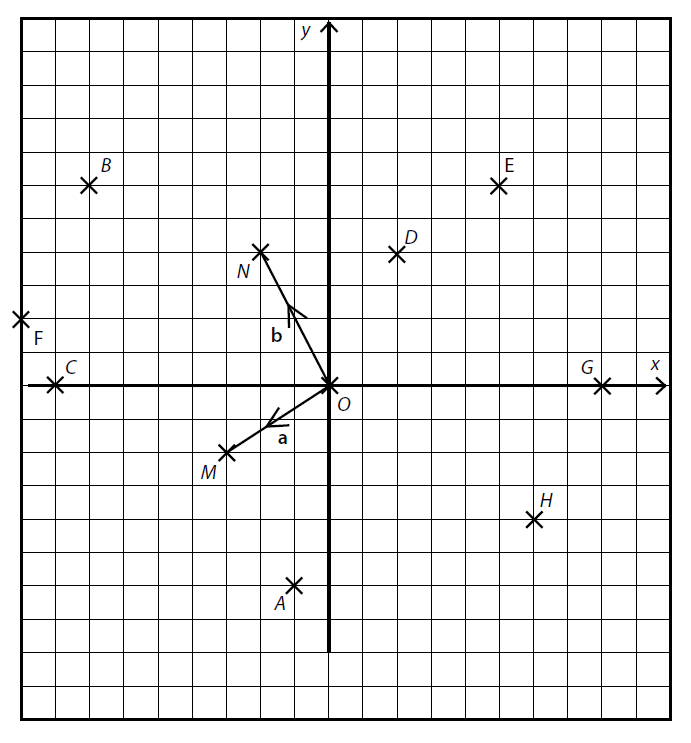
(e) *OE* (f) *OF*

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1. On the grid, *OM* = **a** and *ON* = **a**. There are several points marked on the grid. Write down in terms of **a**

and/or **b** for the following vectors.



* 1. *OA* (b) *OB*

(c) *OC* (d) *OD*

(e) *OE* (f) *OF*

(g) *OG* (h) *OH*

Unit 8 **Vectors**

1. In the diagram, *ABCD* is a parallelogram and the diagonals intersect at point *E*. Find

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4**a** *B*

*A*

E

*C*

**b**

*D*

* 1. *DC* (b) *AC*

(c) *BD* (d) *CE*

1. In the diagram, triangle *ABC* is similar to triangle *DEC*, where the corresponding lengths are in the ratio of 2 : 1 respectively. *AD* and *BE* are straight lines that intersect at *C*. Given that *AC* = 4**h** and *EC* = **k**, find the following vectors in terms of **h** and **k**.

*A B*

E

*D*

**k**

*C*

4**h**

* 1. *CD* (b) *BC*

Teacher to note the parallel property of ED and AB. Can you find the scalar multiple? Since triangle ABC is similar to triangle DEC, angle EDC = angle BAC. By the property of alternate angles, ED is parallel to AB.

(d) *BA*

(c) *ED*

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1. The diagram shows a quadrilateral *ABCD* where *AD* = 1.5 *BC* . Given that *AB* = 2**q** and *AD* = 3**p**, find

*A*

*D*

2**q**

3**p**

*B*

*C*

* 1. *BC* (b) *DC*

(c) *DB* (d) *AC*

1. The diagram shows a trapezium *ABCDEF* where *AB* is parallel to *FC* and *FE* : *ED* : *DC* are in the ratio of 1 : 3 : 4. Given that *FE* = **a** and *AE* = **b**, find

*A B*

F *C*

**b**

**a**

E

*D*

* 1. *FD* (b) *AB*

(c) *FB* (d) *CE*

(e) *BC* (f) *AC*

Unit 8 **Vectors**

1. By comparison, find the value(s) of the unknown constants.

You may use your Classwiz calculator for solving the quadratic equations.

**CLASSWIZ WORKSHEETS**

(a) 3**a** – 2**b** = (*p*2 – 2*p*)**a** + *q***b**

(b) 7*p***a** – 5**a** – 6**b** = 2*p*2 **a** + *q***b**

(c) 10*q***a** + (*p*2 – 1)**b** = 2(2*q*2 – 3)**a** + 3**b**

1. By comparison, find the value of the unknown constants *a* and *b* in the following vectors.

|  |  |
| --- | --- |
| You may use your Classwiz calculator for solving the | quadratic equations and simultaneous equations. |
|  |  |
| (a) – 2 = | (b) + = |
| (c) 4 – 3 = | (d) – 5 = |
| (e) 2 = + | (f ) = + 5 |

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1.  It is given that **p** = and **q** = and **r** =.
   1. Express as a column vector
      1. 3**p** – **q**.
      2. –4**p** + 2**q**.
   2. Find

(i) |–2**p**|.

1. |3**p** – **q**|.



**CLASSWIZ WORKSHEETS**

* 1. Given that 2**p** + **q** = **r**, find the value of *a* and *b*.
  2. Given that –2**p** – 3**q** = 2**s**, express **s** as a column vector.

1. It is given that **a** = and **b** = and **r** = .
   1. Express as a column vector
      1. 4**a** – 3**b**.
      2. –**a** + 2**b**.
   2. Find

(i) |4**a** – 3**b**|.

(ii) |3**a** – **b**|.

* 1. Given that 2**a** + **b** is parallel to **r**, find the value of *m.*

Unit 8 **Vectors**

1. It is given that *OA* = , *OB* = and *OC* = .

**CLASSWIZ WORKSHEETS**

* 1. Express *AB* as a column vector.
  2. Express *BC* as a column vector.
  3. Find | *BC* |.



13. It is given that *AB* = and *BC* = .

1. Express *AC* as a column vector.
2. Find | *AC* |.
3. Given that *CD* = *CB*, express *CD* as a column vector.
4. Given that *C* is the point (4, 6), find coordinates of point *D*.



14. The coordinates of points *A* and *B* are (5, –7) and (8, 11) respectively.

1. Express *OA* and *OB* as column vectors.
2. Find *AB*.
3. Given that *AB* = 3*BC*, find the coordinates of *C*.

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**CLASSWIZ WORKSHEETS**

1. It is given that *OD* = and *OE* .
   1. Find *DE*.
   2. Given that *OP* = and *OP* is parallel to *DE*, find the value of *m*.
2. It is given that *PQ* = and *QR* = .
   1. Find *OQ* given that the coordinates of point *R* is (8, –5).
   2. Find the coordinates of *X* given that *QX* = *PQ*.
3. It is given that *AB* = and *AD* = .
   1. Express *BD* as a column vector.
   2. Given that *ABCD* is a parallelogram, find the column vector *DC*.
   3. A point *X*(6, t) is such that *DX* = *DC*. Given that the coordinates of *D* are (*k*, *k*), find the value of *k*

and of *t*.

Unit 8 **Vectors**

1. The Cartesian plane shows the points *O, P, Q, R*.

**CLASSWIZ WORKSHEETS**

*R*

*O*

*x*

*P*

*Q*

*y*

* 1. Express, as a column vector,
     1. *PQ*.
     2. *QR*.
  2. Using the vector method, find the coordinates of *S*, given that *PQRS* is a parallelogram. Mark out the point *S* in the diagram.
  3. Given that *XY* = – *OQ* and the coordinates of *Y* are (–5, –3), find the coordinates of *X*.

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**CLASSWIZ WORKSHEETS**

1. Given the following column vectors, find the value of the angle in degrees, against the positive *x*-axis.

|  |  |
| --- | --- |
| (a) | (b) |
| (c) | (d) |
| (e) | (f) |

multiples and are parallel to each other. Will the angle formed against the positive *x*-axis be the same?

Teacher to note: ( ) = –2( ). Hence, they are scalar

Unit 8 **Vectors**

1. Find the ratio of the area of the triangles in the form of area of triangle *ABC* : area of triangle *XBC*.

**CLASSWIZ WORKSHEETS**

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| (a) *A*  *X*  8  *B* | 8 | *C* | | (b)  *B* | *A*  3 |  |  | *C* | *X* | |
| (c) *B*  *A*  4 *C* | 23 |  | *X* | (d) *B*  *A*  b | *X* | |  | 1.5b | | *C* |
| (e) *X* | | | | (f) |  | | *X* |  |  | |
|  | | | | 5 | | | | | | |
| *A* |  |  | 2h | *B* |  | |  | *C* |  | |
| h | | | | 12 | | | | | | |
| *B* | *C* |  |  | *A* | | | | | | |
| (g) 2b | *X* |  |  | (h) 7p  *C*  *B*  4p | | | | | | *A*  *X* |
| *C* |  | *B* |  |
| 3b | | | |
| *A* | | | |

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**CLASSWIZ WORKSHEETS**

1. For questions (a) to (h), either the magnitude or vector of the sides of the triangle is given.
   1. Given that triangle *ABE* is similar to triangle *DBC*. Find the ratio of area of triangle *ABE* : area of triangle *DBC* .

*C*

2p 6p

*A*

*B*

E

*D*

* 1. Given that triangle *ABE* is similar to triangle *ADC*. Find the ratio of area of triangle *ABE* : area of triangle *ADC*.

*C*

*A* E *D*

*B*

10

5

Unit 8 **Vectors**

* 1. Given that triangle *ABC* is similar to triangle *EDC*. Find the ratio of area of triangle *ABC* : area of triangle *EDC*.

**CLASSWIZ WORKSHEETS**

E

*A*

4

*C*

6

*B*

*D*

* 1. Given that triangle *ADC* is similar to triangle *ABE* and *BCDE* is a parallelogram. Find the ratio of area of triangle *ADC* : area of triangle *ABE*.

*B*

E

*A*

Teachers to prompt students to think further: Are triangles ADC and ABE similar or congruent?

*D C*

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**CLASSWIZ WORKSHEETS**

* 1. Given that triangle *ABE* is similar to triangle *ACD*. Find the ratio of area of triangle *ABE* : area of triangle *ACD* .

*A*

Teachers to prompt students to think further: Can you find the ratio of area of triangle ABE to the area of quadrilateral BECD?

*C D*

3

*B*

E

1

* 1. Given that triangle *ABE* is similar to triangle *CDE*. Find the ratio of area of triangle *ABE* : area of triangle *CDE*.

*A*

Teachers to prompt students to think further: Are lines AB and DC parallel? How can you prove it?

*D*

3p

E

p

*B C*

Unit 8 **Vectors**

* 1. Given that triangle *ACB* is similar to triangle *BDC*. Find the ratio of area of triangle *ACB* : area of triangle *BDC* .

**CLASSWIZ WORKSHEETS**

*A*

4.5

E

*B*

3

*C*

Teachers to prompt students to think further: Can you find the length of DC?

*D*

* 1. Given that triangle *ABE* is similar to triangle *ACD*. Find the ratio of area of triangle *ABE* : area of triangle *ACD*.

*C*

Teachers to prompt students to think further: Can you describe the property of line BE with line CD?

*A D*

*B*

E

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# Level 3

1. Given that **a** = , **b** = and **c** = find the following dot product. You may use the ClassWiz

calculator.

|  |  |  |  |
| --- | --- | --- | --- |
| (a) **a∙b** | | (b) **b∙a** | |
|  | Teacher to note: Dot pro | oduct is commutative. |  |
| (c) **a∙c** | | (d) **b∙c** | |
| (e) (**a** + **c**)**∙b** | | (f ) **b∙**(**a** + **c**) | |
|  | Teacher to note: Dot pro | oduct is commutative. |  |

1. Given that **a** = , **b** = and **c** = , find the following dot product. You may use the

ClassWiz calculator.

**CLASSWIZ WORKSHEETS**

|  |  |  |  |
| --- | --- | --- | --- |
| (a) **a∙b** | | (b) **b∙a** | |
|  | Teacher to note: Dot pro | oduct is commutative. |  |
| (c) **a∙c** | | (d) **b∙c** | |
| (e) (**a** + **c**)**∙b** | | (f ) **b∙**(**a** + **c**) | |
|  | Teacher to note: Dot pro | oduct is commutative. |  |

Unit 8 **Vectors**

**CLASSWIZ WORKSHEETS**

3. Given that **a** = , **b** = and **c** = , find the following cross product and its magnitude.

You may use the ClassWiz calculator.

|  |  |  |  |
| --- | --- | --- | --- |
| (a) **a**  **b** | | (b) **b**  **a** | |
|  | Teacher to note: Cross pro | duct is NOT commutative. |  |
| (c) **a**  **c** | | (d) **b**  **c** | |

4.Given that **a** = , **b** = and **c** = , use the ClassWiz angle

function to find the angle formed between vectors. You may use

**ClassWiz set-up**

Press TR3 to use the angle function.

Select a vector, input a comma and select another vector. Press T3 q) T4 = to get the angle between Vct *A* and Vct *B*.

Press q) for the comma (,) function.

your ClassWiz calculator.

* 1. **a** and **b**
  2. **a** and **c**
  3. **b** and **c**

5. Given that **a** = , **b** = and **c** = , use the dot product to find the angle formed between

the vectors. You may use your ClassWiz calculator.

1. **a** and **b**
2. **a** and **c**
3. **b** and **c**

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**CLASSWIZ WORKSHEETS**

6. In the diagram, the coordinates of points *P*, *Q* and *R* are given as (2, 7), (9, 9) and (6, 2) respectively. You may use your ClassWiz calculator.

*y*

*x*

*Q* (9, 9)

*P* (2, 7)

*θ*1

*θ*2

*θ*3

*R* (6, 2)

*O*

1. Express as column vectors
   * 1. *PQ*.
     2. *QR*.
     3. *PR* .

(b) Find the magnitude of

1. *PQ*,
2. *QR*,
3. PR, leaving your answer in the exact form.

(c) By using the ClassWiz angle function, find the angle, to 2 decimal places,

1. *θ*1 formed between vectors *PQ* and *QR*.
2. *θ*2 formed between vectors *PQ* and *PR* .
3. *θ*3 formed between vectors *PR* and *QR*.
   1. Find the area of triangle by evaluating each of the following formulas.
      1. | *PQ*| |*QR*| sin

Teacher to note: This method is to find

the area of the triangular plane formed by points *A*, *B* and *C*. By using any 2 vectors, we can obtain the same answer. This is

a longer method as compared to using cross product in question 7.

* + 1. | *PQ*| | *PR* | sin *θ*2
    2. | *PR* | |*QR*| sin *θ*3

Unit 8 **Vectors**

7. In the diagram, the coordinates of points *A*, *B* and *C* are given as (2, 2), (6, 9) and (10, 4) respectively. You may use your ClassWiz calculator.

area of triangle = sin *θ*.

the triangle. This is much faster

than the formula in question 6:

to multiply to get the area of

The magnitude of the cross product of 2 vectors,

|**a** × **b**|, gives us the area of the

parallelogram. Hence, we need

**CLASSWIZ WORKSHEETS**

*y*

*B*(6,9)

*C*(10,4)

*A*(2,2)

0

*x*

(a) Express as column vectors

**ClassWiz set-up**

Press 0.5fT3OT4=)

to find the area of the triangle formed by Vct *A* and Vct *B*.

1. *AB* .
2. *BC* .
3. *AC* .

(b) Find half the magnitude of the cross product of

Teacher to note: This method is to find

the area of the triangular plane formed by points *A*, *B* and *C*. By using any 2 vectors, we can obtain the same answer.

1. *AB* and *AC* .
2. *AB* and *BC* .
3. *BC* and *AC* .

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1. Given that **a** = , **b** = and **c** = , use cross product to find the following vectors. You may

use your ClassWiz calculator.

Formula for finding area of triangle = |**a** × **b**|

(a) **a** and **b**

**ClassWiz set-up**

Press 0.5fT3OT4=)

to find the area of the triangle formed by Vct *A* and Vct *B*.

(b) **a** and **b**

(c) **b** and **c**

1. Given that **a** = , **b** = and **c** = , use cross product to find the area of the triangle

formed by the following vectors. You may use your ClassWiz calculator.

**CLASSWIZ WORKSHEETS**

1. **a** and **b**
2. **a** and **c**
3. **b** and **c**

Unit 8 **Vectors**

1. In the diagram, *OPQR* is a parallelogram. It is given that coordinates of point *P* and *Q* are (2, 5) and (7, 8) respectively. You may use your ClassWiz calculator.

**CLASSWIZ WORKSHEETS**

*y*

*x*

*Q*(7,8)

*P*(2,5)

*R*

*O*

* 1. Express as column vectors

1. *OP*
2. *RQ*
3. *PQ*
4. *OR*

(b) Given that point *D* lies on the Cartesian plane such that *ODPR* is a parallelogram, find the coordinates of *D*.

(c) Find

1. angle *POR*.
2. the area of *OPQR* by using cross product.
3. the area of *ODPR* by using cross product.

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# Level 4

1. In the diagram, *SP* = –2**p** + **q**, *SR* = 9**p** + 3**q** and *QR* = –4**p** – 3**q**.

*Q*

*P*

–2**p** + **q**

–4**p** – 3**q**

*O*

*R*

**CLASSWIZ WORKSHEETS**

9**p** + 3**q**

*S*

* 1. Express *PQ* in terms of **p** and **q**.
  2. Show that *PQRS* is a trapezium.
  3. Find the ratio of *PQ* : *SR*.
  4. Find the numerical value of

1. .
2. .

Unit 8 **Vectors**

1. In the diagram, *OABC* is a parallelogram. It is given that *OA* = 2**m** – **n** and *OC* = –**m** + 2**n**.

**CLASSWIZ WORKSHEETS**

*D C*

*B*

–**m** + 2**n**

2**m** – **n**

*O*

*A*

* 1. Express in the simplest form in terms of **m** and **n**,
     1. *AC* .
     2. *OB*.
  2. Given that *OD* = – 2**m** + 2**n**,
     1. explain why *OD* is parallel to *AC* .
     2. find the ratio of *OD* : *AC*.
  3. Given that the area of triangle *OCA* is 30 units2, find the area of triangle *ODA*.

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**CLASSWIZ WORKSHEETS**

1. In the diagram, *OPQR* is a parallelogram. *X* is the midpoint of *QR* and *Y* is the midpoint of *OX*. *S* is a point on *OR* such that *OS* = 2*SR*. It is given that *OR* = **r** and *OP* = **p**.

*P Q*

**p** *Y X O* **r** *S R*

* 1. Express in the simplest form in terms of **p** and **r**,
     1. *OX*.
     2. *PY* .
     3. *OS* .
     4. *YS* .
  2. Show that *PY* = *k YS* and find the value of *k*.
  3. Find the numerical value of .

Unit 8 **Vectors**

1. In the diagram, *OAB* is a triangle and *X* is the midpoint of *AB* and *Y* is the midpoint of *OX*. *Z* is a point on *OA* such that *OA* = 3 *OZ*. It is given that *OZ* = **a** and *BX* = **b**.

**CLASSWIZ WORKSHEETS**

*A*

*O*

*Z*

*X*

**a**

*Y*

**b**

*B*

* 1. Express in the simplest form in terms of **a** and **b**,
     1. *OA*.
     2. *AB* .
     3. *OB*.
     4. *OX*.
     5. *ZY* .
     6. *YB* .
  2. Show that *YB* = *k ZY* and find the value of *k*.
  3. Find the numerical value of .

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**CLASSWIZ WORKSHEETS**

1. In the diagram, *OABE* is a parallelogram and *D* is produced from *AE. OEC* and *BCD* are straight lines, *E* is the midpoint of *AD* and 2*OC* = 3*OE*. It is given that *OA* = **a** and *OE* = **b**.

*D*

*C*

E

**b**

*B*

*O* **a** *A*

* 1. Express in the simplest form in terms of **a** and **b**,
     1. *ED* .
     2. *CD*.
     3. *BC* .
  2. Find the numerical value of

1. *.*
2. .
3. .
4. .