**TEACHER’S GUIDE**

**Unit**

**7**

**MATRICES**

#### KEY CONCEPTS

A *m* × *n* matrix *A* is a rectangular array of numbers, called elements, in the form of

**TEACHER’S GUIDE**

*A* =$ \left(\begin{matrix}a\_{11}&a\_{12}&\cdots &a\_{1n}\\a\_{21}&a\_{22}&\cdots &a\_{2n}\\\vdots &\vdots &\cdots &\vdots \\a\_{m1}&a\_{m2}&\cdots &a\_{mn}\end{matrix}\right)$ *m* rows

*n* columns A matrix is used to store data and display information.

The number of rows and columns constitute the **order** of the matrices. Example $\left(\begin{matrix}2&6&-8\\5&-11&3\end{matrix}\right)$ is a 2 × 3 matrix.

(The order of the matrix is 2 by 3, it has 2 rows and 3 columns)

Example $\left(\begin{matrix}\begin{matrix}0.3\\5\end{matrix}&\begin{matrix}6\\-7\end{matrix}\\\begin{matrix}-4\\0\end{matrix}&\begin{matrix}2\\1\end{matrix}\end{matrix}\right)$ is a 4 × 2 matrix.

(The order of the matrix is 4 by 2, it has 4 rows and 2 columns)

A **row matrix** has exactly one row. Its order is given by 1 × *n*, where *n* is the number of columns. Example (4 –9 0) is a 1 × 3 row matrix.

A **column matrix** has exactly one column. Its order is given by *m* × 1, where *m* is the number of rows.

Example $\left(\begin{matrix}\begin{matrix}6\\1\end{matrix}\\\begin{matrix}7\\ -3 \end{matrix}\end{matrix}\right)$ is a 4 × 1 column matrix.

A **square matrix** has the same number of rows and columns. Example (8) is a 1 × 1 square matrix.

Example $\left( \begin{matrix}1&7&0 \\9&4&7 \\2&6&6 \end{matrix} \right)$ is a 3 × 3 square matrix.

A **zero matrix** or **null matrix** is a matrix of any order, with all its elements equal to zero.

Example $\left( \begin{matrix}0&0 \\0&0 \\0&0 \end{matrix} \right) $is a 3 × 2 zero matrix.

Example (0 0) is a 1 × 2 zero matrix.

**TEACHER’S GUIDE**

**Casio ClassWiz Mathematics Workbook**

An **identity matrix** is a square matrix where the elements in the leading diagonal are equal to 1 and the rest of the elements are equal to 0.

Example $\left( \genfrac{}{}{0pt}{}{1 0}{0 1} \right)$ is a 2 × 2 identity matrix.

Example $\left( \genfrac{}{}{0pt}{}{1 0 0}{\begin{array}{c}0 1 0\\0 0 1\end{array}} \right)$ is a 3 × 3 identity matrix.

Two matrices are **equal** if and only if they have the same order and their corresponding elements are equal.

Hence, given that *A* = $\left( \genfrac{}{}{0pt}{}{a c}{b d} \right)$ and B = $\left( \genfrac{}{}{0pt}{}{w y}{x z} \right)$ ,

If **A** = **B**, then *a* = *w*, *b* = *x*, *c* = *y*, *d* = *z*.

#### Addition and Subtraction of matrices

1. If both matrices are of the **same order**, you can add or subtract them. We add or subtract the corresponding elements.

Example $\left( \genfrac{}{}{0pt}{}{ 2 7}{-5 8 } \right)$ + $\left( \genfrac{}{}{0pt}{}{-3 4}{ 9 0} \right)$ = $\left( \genfrac{}{}{0pt}{}{2+\left(-3\right) 7+4}{ -5+9 8+0 } \right)$ = $\left( \genfrac{}{}{0pt}{}{-1 11}{ 4 8} \right)$

1. Matrix addition is **commutative**. **A** + **B** = **B** + **A**

Example $\left( \genfrac{}{}{0pt}{}{ 2 7}{-5 8 } \right)$ + $\left( \genfrac{}{}{0pt}{}{-3 4}{ 9 0} \right)$ = $\left( \genfrac{}{}{0pt}{}{-3 4}{ 9 0} \right)$ + $\left( \genfrac{}{}{0pt}{}{ 2 7}{-5 8 } \right)$

1. Matrix addition is associative.

(**A** + **B**) + **C** = **A** + (**B** + **C**) = **A** + **B** + **C**

Example [(2 5) + (1 0)] + (3 –1) = (3 5) + (3 –1) = (6 4)

(2 5) + [(1 0) + (3 –1)] = (2 5) + (4 –1) = (6 4)

(2 5) + (1 0) + (3 –1) = (6 4)

#### Multiplication of matrix by a scalar quantity

When multiplying a real number scalar to a matrix, multiply the scalar quantity to all elements in the matrix.

Example *M* = $\left( \genfrac{}{}{0pt}{}{6 -7}{0 -2} \right)$

 2*M* = 2$\left( \genfrac{}{}{0pt}{}{6 -7}{0 -2} \right)$ = $\left( \genfrac{}{}{0pt}{}{12 -14}{0 -4} \right)$

$\frac{ 1 }{2}$*M* = $\frac{ 1 }{2}\left( \genfrac{}{}{0pt}{}{6 -7}{0 -2} \right)$ = $\left( \genfrac{}{}{0pt}{}{3 \frac{-7}{2}}{0 -1} \right)$

#### Multiplication of matrices

1. If **A** is a matrix of order *p* × *n* and **B** is a matrix of order *n* × *q*, then the product **AB** is a matrix of order

*p* × *q*.

The number of columns in the first matrix MUST BE equal to the number of rows in the second matrix for the multiplication to be possible.

This is because the elements in the rows of the first matrix is multiplied to the elements in the columns of the second matrix.

Multiplication of matrices is not commutative, i.e. **AB** ≠ **BA**

Multiplication of matrices is associative, i.e. **ABC** = **(AB)C** = **A(BC)**

Unit 7 **Matrices**

**TEACHER’S GUIDE**

Example $\left( \genfrac{}{}{0pt}{}{5 -1}{\begin{array}{c}0 -2\\1 4\end{array}} \right)\left( \genfrac{}{}{0pt}{}{a}{b} \right)$ = $\left( \genfrac{}{}{0pt}{}{5 \left(a\right)+(-1)(b)}{\begin{array}{c}\left(0\right)\left(a\right)+(-2)(b)\\\left(1\right)\left(a\right)+(4)(b)\end{array}} \right)$ = $\left( \genfrac{}{}{0pt}{}{5a-b}{\begin{array}{c}-2b\\a+4b\end{array}} \right)$

1. Matrices with power

 $\left( \genfrac{}{}{0pt}{}{ a b }{c d} \right)^{2}$ ≠ $\left( \genfrac{}{}{0pt}{}{a^{2} b^{2}}{c^{2} d^{2}} \right)$

 $\left( \genfrac{}{}{0pt}{}{ a b }{c d} \right)^{2}$ = $\left( \genfrac{}{}{0pt}{}{ a b}{ c d} \right)$ $\left( \genfrac{}{}{0pt}{}{a b}{c d} \right)$ ­= $\left( \genfrac{}{}{0pt}{}{a^{2}+bc ab+bd}{ca+dc cb+d^{2}} \right)$

1. If A is a square matrix of order n × n and I is the identity matrix of the same order, then

**AI** = **IA** = **A**.

Example $\left( \genfrac{}{}{0pt}{}{a b}{c d} \right)$ $\left( \genfrac{}{}{0pt}{}{1 0}{0 1} \right)$ = $\left( \genfrac{}{}{0pt}{}{a b}{c d} \right)$

 $\left( \genfrac{}{}{0pt}{}{1 0}{0 1} \right)$ $\left( \genfrac{}{}{0pt}{}{a b}{c d} \right)$ = $\left( \genfrac{}{}{0pt}{}{a b}{c d} \right)$

#### Determinant of matrices

The determinant of matrix **A** is denoted by |**A**|. Determinant only exists for square matrices.

For a 2 × 2 matrix of $\left( \genfrac{}{}{0pt}{}{a b}{c d} \right)$, the determinant is *ad* – *bc*.

#### Inverse of matrices

The inverse of matrix **A** is denoted by **A**–1.

Inverse matrices only exist for square matrices. But not all square matrices have an inverse.

A matrix that has a value of 0 for the determinant (and hence does not have an inverse) is called a singular matrix.

When we multiply a matrix by its inverse (if the inverse exists), we get an identity matrix.

**A**–1 × **A** = **I** or **A** × **A**–1 = **I**

To find the inverse of matrix A (*a b*//*c d*):

 $Ａ^{-1}$ **=** $\left[\frac{1}{(ad-bc)}\right]$$\left( \genfrac{}{}{0pt}{}{ d -b}{-c a} \right)$

Example Given **XA** = **B**, where **X** is an unknown matrix, we can take inverse of **A** on both sides of the equation. Note that we place **A**–1 on the *right* side of the expressions.

**XA**(**A**–1) = **BA**–1 **XI** = **BA**–1

**X** = **BA**–1

Example Given **AX** = **B**, where **X** is an unknown matrix, we can take inverse of **A** on both sides of the equation. Note that we place **A**–1 on the *left* side of the expressions.

(**A**–1)**AX** = **A**–1**B IX** = **A**–1**B**

**X** = **A**–1**B**

**Casio ClassWiz Mathematics Workbook**

**TEACHER’S GUIDE**

#### Transpose of matrices

**TEACHER’S GUIDE**

The transpose of a matrix is a new matrix whose rows are the columns of the original, and the columns of the new matrix are the rows of the original.

A matrix has an order of 5 x 2. The transpose of this matrix will have an order of 2 × 5.

Example $\left(\begin{matrix}5&1\\2&-3\\-4&2\end{matrix}\right)^{T}$= $\left(\begin{matrix}5&2&-4\\1&-3&2\end{matrix}\right)$

#### WORKED EXAMPLES

**Level 1**

###### Worked Example 1

Evaluate $\left(\begin{matrix}4\\-3\\8\end{matrix}\right)$ － $\left(\begin{matrix}-5\\7\\6\end{matrix}\right)$

8

Solution

 $\left(\begin{matrix}4\\-3\\8\end{matrix}\right)$ － $\left(\begin{matrix}-5\\7\\6\end{matrix}\right)$ = $\left(\begin{matrix}4-\left(-5\right)\\-3-7\\8-6\end{matrix}\right)$ = $\left(\begin{matrix}9\\-10\\2\end{matrix}\right)$

 **ClassWiz steps**

Press w.

Press 4 for Matrix. You will see this:

Press 1 to define Matrix A. You will see this.

Select 3 for the number of rows.

Unit 7 **Matrices**

**TEACHER’S GUIDE**

 **ClassWiz steps**

Select 1 for the number of columns. You will see this.

Key in the matrix $\left(\begin{matrix}4\\-3\\8\end{matrix}\right).$

Next, press T and you will see this.

# Press 1 to Define Matrix. You will see the same page as you have seen earlier.

Press 2 to define matrix B. Select 3 for rows and 1 for columns. Next, key in the matrix $\left(\begin{matrix}-5\\7\\6\end{matrix}\right)$.

Press T again. Select 3 for MatA. You will see this.

Press pT4 to subtract MatB. You will see this.

Press = and the answer will be displayed.

Of course, such simple matrix addition and subtraction would be easier done by arithmetic calculation instead of the calculator. However, given larger matrices with more rows and columns, the calculator would be very useful. Also, in matrices multiplication, arithmetic calculation might be tedious. Hence it would be useful to use the calculator too.

These are the steps to define matrices and in the ClassWiz fx-991EX, we can store up to 4 matrices. In the following ClassWiz solutions or ClassWiz steps, I will refer to these steps by asking you to define the matrix using MatA, MatB, MatC and MatD.

**Casio ClassWiz Mathematics Workbook**

**TEACHER’S GUIDE**

###### Worked Example 2

Evaluate $\left( \genfrac{}{}{0pt}{}{5 -4}{0 3} \right)$ + $\left( \genfrac{}{}{0pt}{}{2}{-1} \right)$.

Solution

No solution because the order of the matrices is not the same. For addition and subtraction of matrices, the order of the matrices must be the same.

###### Worked Example 3

###### Evaluate 4$\left( \genfrac{}{}{0pt}{}{ 3 -1}{\frac{-3}{2} 0} \right)$.

Solution

4$\left( \genfrac{}{}{0pt}{}{3 -1}{\frac{-3}{2} 0} \right)$ = $\left(\genfrac{}{}{0pt}{}{ 4(3) 4(-1)}{4(\frac{-3}{2}) 4(0)}\right)$

|  |  |
| --- | --- |
|  | **ClassWiz steps** Press w4.Difine MatA to be $\left( \genfrac{}{}{0pt}{}{3 -1}{\frac{-3}{2} 0} \right)$.Press WPress 4T3=. The calculator willdisplay the answer $\left( \genfrac{}{}{0pt}{}{12 -4}{-6 0} \right).$ |
|  |

 = $\left( \genfrac{}{}{0pt}{}{12 -4}{-6 0} \right)$

###### Worked Example 4

Evaluate the inverse function of $\left( \genfrac{}{}{0pt}{}{3 5}{-2 1} \right)$.

Solution

$\left( \genfrac{}{}{0pt}{}{ 3 5 }{-2 1} \right)^{-1}$ = $\frac{1}{[3(1) – 5(–2)]}\left( \genfrac{}{}{0pt}{}{1 -5}{2 3} \right)$

= $\frac{1}{13}\left( \genfrac{}{}{0pt}{}{1 -5}{2 3} \right)$

 = $\left( \genfrac{}{}{0pt}{}{\frac{1}{13} \frac{-5}{13}}{\frac{2}{13} \frac{3}{13}} \right)$

 **ClassWiz steps**

Press w4.

Define MatA to be (3 5//–2 1). Press W

Press T3/=. The calculator will display the following.

Although you may not see all the elements displayed as fractions, you may move over each element and the fraction can be seen.

Unit 7 **Matrices**

###### Worked Example 5

**TEACHER’S GUIDE**

Evaluate $\left( 3 -5 \right)$ $\left( \genfrac{}{}{0pt}{}{2}{-1} \right)$ $\left( 7 \right)$ .

Solution

$\left( 3 -5 \right)$ $\left( \genfrac{}{}{0pt}{}{2}{-1} \right)$ $\left( 7 \right)$ = (3(2) + (–5)(–1))(7)

 **ClassWiz steps**

Press w4.

Define MatA to be $\left( 3 -5 \right)$,

= (11)(7)

= (77)

MatB to be $\left( \genfrac{}{}{0pt}{}{2}{-1} \right)$

and MatC to be

$\left( 7 \right)$.

Press W

Press T3OT4OT5=. The calculator will display the answer $\left( 77 \right)$ .

###### Worked Example 6

Evaluate $\left(\begin{matrix}0&-1&3\end{matrix}\right)\left(\begin{matrix}5&1\\2&-3\\-4&2\end{matrix}\right)$.

Solution

 $\left(\begin{matrix}0&-1&3\end{matrix}\right)\left(\begin{matrix}5&1\\2&-3\\-4&2\end{matrix}\right)$ = (0(5)+(–1)(2)+3(–4) 0(1)+(–1)(–3)+3(2))

= (–14 9)

|  |  |
| --- | --- |
|  |  **ClassWiz steps** Press w4.Define MatA to be (0 –1 3) and MatB to be$\left(\begin{matrix}5&1\\2&-3\\-4&2\end{matrix}\right)$.Press WPress T3OT4=. The calculator will display the answer (–14 9). |
|  |

###### Worked Example 7

Evaluate $\left( \genfrac{}{}{0pt}{}{ 4 -1 }{2 3} \right)^{2}$.

Solution

.

Press T3d=.

The calculator will display the answer $\left(\genfrac{}{}{0pt}{}{ 14 -7 }{14 7}\right)$

.

Press w4.

Define MatA to be $\left( \genfrac{}{}{0pt}{}{ 4 -1 }{2 3}\right)$

Press W

 $\left( \genfrac{}{}{0pt}{}{ 4 -1 }{2 3} \right)^{2}$ = $\left( \genfrac{}{}{0pt}{}{ 4 -1 }{2 3} \right)$ $\left( \genfrac{}{}{0pt}{}{ 4 -1 }{2 3} \right)$

 = $\left( \genfrac{}{}{0pt}{}{4\left(4\right)+\left(–1\right)\left(2\right) 4(–1)+(–1)(3) }{2\left(4\right)+3\left(2\right) 2(–1)+3(3) } \right)$

 = $\left( \genfrac{}{}{0pt}{}{ 14 -7 }{14 7} \right)$

**ClassWiz steps**

**Casio ClassWiz Mathematics Workbook**

###### Worked Example 8

Find the value of the unknown constants *a* and *b* in $\left( \genfrac{}{}{0pt}{}{ 2 1 }{5 -3}\right)\left( \genfrac{}{}{0pt}{}{ a 5 }{-2 1}\right)$ = $\left(\genfrac{}{}{0pt}{}{ 8 b }{31 c}\right)$ .

Solution

$\left( \genfrac{}{}{0pt}{}{ 2 1 }{5 -3}\right)\left( \genfrac{}{}{0pt}{}{ a 5 }{-2 1}\right)$ = $\left(\genfrac{}{}{0pt}{}{ 8 b }{31 c}\right)$

$\left( \genfrac{}{}{0pt}{}{ 2a+1(–2) 2(5)+(1)(1) }{5a+(–3)(–2) (5)(5)+(–3)(1)}\right)$ = $\left(\genfrac{}{}{0pt}{}{ 8 b }{31 c}\right)$

$\left( \genfrac{}{}{0pt}{}{ 2a–2 11 }{5a+6 22}\right)$ = $\left(\genfrac{}{}{0pt}{}{ 8 b }{31 c}\right)$

2*a* – 2 = 8

*a* = 5

*b* = 11

Check: 5*a* + 6 = 5(5) + 6 = 31

*c* = 22

#### Level 2

###### Worked Example 9

A cake shop sells chocolate buns and butter buns in two packet sizes, small packet and large packet. The selling price of a small packet of buns is $3.90 and the selling price of a large packet is $6.90. The quantities sold in a particular week are as listed.

|  |  |  |
| --- | --- | --- |
| Type of bun/Packet size | Small | Large |
| Chocolate buns | 45 | 38 |
| Butter buns | 56 | 45 |

Given that **P** = $\left( \genfrac{}{}{0pt}{}{45 38}{56 45} \right)$　and **Q** = $\left( \genfrac{}{}{0pt}{}{3.90}{6.90} \right)$.

Evaluate **PQ**.

Explain what the elements of **PQ** represent. Solution

 $\left( \genfrac{}{}{0pt}{}{45 38}{56 45} \right)$ $\left( \genfrac{}{}{0pt}{}{3.90}{6.90} \right)$ = $\left( \genfrac{}{}{0pt}{}{45\left(3.90\right)+38(6.90)}{56\left(3.90\right)+45(6.90)} \right)$

.

the answer $\left( \genfrac{}{}{0pt}{}{437.70}{528.90} \right)$

Press W

Press T3OT4=. The calculator will display

.

and MatB to be $\left( \genfrac{}{}{0pt}{}{3.90}{6.90} \right)$

 **ClassWiz steps**

Press w4.

Define MatA to be $\left( \genfrac{}{}{0pt}{}{45 38}{56 45} \right)$

 = $\left( \genfrac{}{}{0pt}{}{437.70}{528.90} \right)$

The elements of PQ represent the total revenue from selling chocolate buns and butter buns respectively.

**TEACHER’S GUIDE**

Unit 7 **Matrices**

#### Level 3

**TEACHER’S GUIDE**

###### Worked Example 10

Solve the following linear simultaneous equations using matrices. 2*y* = 3𝑥– 6

$\frac{ 1 }{3}$*y* =$ –\frac{ 2 }{5}$𝑥+ 2

Solution

From 2*y* = 3𝑥– 6, 3𝑥– 2*y* = 6 ---(1)

From $\frac{ 1 }{3}$*y* =$–\frac{ 2 }{5}$𝑥+ 2, $\frac{ 2 }{5}$𝑥+ $\frac{ 1 }{3}$*y* = 2

 6𝑥+ 5*y* = 30 ----(2)

Expressing equations (1) and (2) into matrices: $\left( \genfrac{}{}{0pt}{}{3 -2}{6 5} \right)$ $\left( \genfrac{}{}{0pt}{}{x}{y} \right)$ = $\left( \genfrac{}{}{0pt}{}{6}{30} \right)$

From $\left( \genfrac{}{}{0pt}{}{x}{y} \right)$ = $\left( \genfrac{}{}{0pt}{}{3 -2}{6 5} \right)^{-1}\left( \genfrac{}{}{0pt}{}{6}{30} \right)$

 **ClassWiz steps**

$\left( \genfrac{}{}{0pt}{}{3 -2}{6 5} \right)^{-1}\left( \genfrac{}{}{0pt}{}{3 -2}{6 5} \right)\left( \genfrac{}{}{0pt}{}{x}{y} \right)$ = $\left( \genfrac{}{}{0pt}{}{3 -2}{6 5} \right)^{-1}\left( \genfrac{}{}{0pt}{}{6}{30} \right)$

$\left( \genfrac{}{}{0pt}{}{x}{y} \right)$ = $\left( \genfrac{}{}{0pt}{}{3 -2}{6 5} \right)^{-1}\left( \genfrac{}{}{0pt}{}{6}{30} \right)$

$\left( \genfrac{}{}{0pt}{}{x}{y} \right)$ = $\frac{1}{[3(5) –(–2)(6)]}\left( \genfrac{}{}{0pt}{}{ 5 2}{-6 3} \right)\left( \genfrac{}{}{0pt}{}{6}{30} \right)$

$\left( \genfrac{}{}{0pt}{}{x}{y} \right)$ = $\frac{1}{27}\left( \genfrac{}{}{0pt}{}{5\left(6\right)+2(30)}{-6\left(6\right)+3(30)} \right)$

$\left( \genfrac{}{}{0pt}{}{x}{y} \right)$ = $\frac{1}{27}\left( \genfrac{}{}{0pt}{}{90}{54} \right)$

$\left( \genfrac{}{}{0pt}{}{x}{y} \right)$ = $\left( \genfrac{}{}{0pt}{}{3\frac{ 1 }{3}}{2} \right)$

Hence, 𝑥= 3$\frac{ 1 }{3}$ and *y* = 2.

and MatB to be $\left( \genfrac{}{}{0pt}{}{6}{30} \right)$.

Press w4.

Define MatA to be $\left( \genfrac{}{}{0pt}{}{3 -2}{6 5} \right)$

display the answer $\left( \genfrac{}{}{0pt}{}{3\frac{ 1 }{3}}{2} \right)$.

Press W

Press T3/OT4=. The calculator will

**Casio ClassWiz Mathematics Workbook**

**CLASSWIZ WORKSHEETS**

#### CLASSWIZ WORKSHEET

**Level 1**

1. State the order of the following matrices.

|  |  |
| --- | --- |
| (a) $\left( \genfrac{}{}{0pt}{}{3 -2}{4 7} \right)$ | (b) $\left( \genfrac{}{}{0pt}{}{0 0}{\begin{array}{c}0 0\\0 0\end{array}} \right)$  |
| (c) $\left( 5 -9 \right)$ | (d) $\left( 12 \right)$ |
| (e) $\left( \genfrac{}{}{0pt}{}{ 4 8 11 2}{\begin{array}{c}-2 0 -7 5\\ 1 5 0 -12\end{array}} \right)$  | (f) $\left( \genfrac{}{}{0pt}{}{4}{\begin{array}{c}3\\2\\-1\end{array}} \right)$  |
| (g) $\left( \genfrac{}{}{0pt}{}{1 0 0}{\begin{array}{c}0 1 0\\ 0 0 1 \end{array}} \right)$  | (h) $\left( \genfrac{}{}{0pt}{}{a b c d e}{v w x y z} \right)$ |
| (i) $\left( \genfrac{}{}{0pt}{}{3 3 3}{3 3 3} \right)$  |  |

Unit 7 **Matrices**

 2. Evaluate the following addition and subtraction of row and column matrices without the use of the matrix function of a calculator. (Note: Not all expressions can be evaluated.)

**CLASSWIZ WORKSHEETS**

|  |  |
| --- | --- |
| (a) $\left( 11 18 \right)$ + $\left( -15 3 \right)$ | (b) $\left( 6 2 -9 \right)$ – $\left( 14 -5 17 \right)$ |
| (c) $\left( 1 2 3 4 \right)$ – $\left( 5 6 7 \right)$ | (d) $\left( \genfrac{}{}{0pt}{}{-1}{5} \right)$ + $\left( \genfrac{}{}{0pt}{}{3}{-2} \right)$  |
| (e) $\left( \genfrac{}{}{0pt}{}{3}{\begin{array}{c}2\\ 1 \end{array}} \right)$ – $\left( \genfrac{}{}{0pt}{}{1}{\begin{array}{c}2\\ 3 \end{array}} \right)$ | (f) $\left( \genfrac{}{}{0pt}{}{8}{\begin{array}{c}1\\ 3 \end{array}} \right)$ + $\left( \genfrac{}{}{0pt}{}{11}{12} \right)$  |
| (g) $\left( 3 4 \right)$ – $\left( \genfrac{}{}{0pt}{}{11}{12} \right)$ | (h) $\left( 3 7-1 5 \right) $– $\left( 2-2 6 4\right)$ + $\left( 5 0 7-6\right)$ |
| (i)$ \left( \genfrac{}{}{0pt}{}{4}{5} \right)$ +$ \left( \genfrac{}{}{0pt}{}{2}{4} \right)$ – $\left( \genfrac{}{}{0pt}{}{5}{0} \right)$ | (j) $\left( \genfrac{}{}{0pt}{}{-3}{\begin{array}{c}0\\ 4 \end{array}} \right)$ – $\left( \genfrac{}{}{0pt}{}{7}{\begin{array}{c}6\\-9 \end{array}} \right)$ – $\left( \genfrac{}{}{0pt}{}{5}{\begin{array}{c}-8\\0 \end{array}} \right)$ |
| (k) $\left( 1 9 -8 \right)$ – $\left[\left( 8 4 0 \right)+\left( -5 6-9 \right)\right]$ – $\left( 7 0 -5\right)$ |
| (l) $\left( \genfrac{}{}{0pt}{}{-8}{\begin{array}{c}2\\-3\\0\end{array}} \right)$ – $\left[\left( \genfrac{}{}{0pt}{}{6}{\begin{array}{c}1\\-8\\9\end{array}} \right)–\left( \genfrac{}{}{0pt}{}{9}{\begin{array}{c}-1\\9\\-6\end{array}} \right)\right]$ |

**Casio ClassWiz Mathematics Workbook**

**CLASSWIZ WORKSHEETS**

3. Evaluate the following addition and subtraction of matrices. Note: Not all expressions can be evaluated.

 **ClassWiz set-up**

You may press w4 to access the matrix function. Press T1 to define the matrices accordingly.

|  |  |
| --- | --- |
| (a) $\left( 5 \right)$ + $\left( 6 \right)$ |  |
| (b) $\left( \genfrac{}{}{0pt}{}{-8 4}{3 0} \right)$ + $\left( \genfrac{}{}{0pt}{}{1 -6}{-4 11} \right)$  | (c) $\left( \genfrac{}{}{0pt}{}{3 -2}{4 7} \right)$ + $\left( \genfrac{}{}{0pt}{}{10}{9} \right)$ |
| (d) $\left( \genfrac{}{}{0pt}{}{4 3}{1 5} \right)$ + $\left( \genfrac{}{}{0pt}{}{2 -7}{0 1} \right)$ – $\left( \genfrac{}{}{0pt}{}{1 2}{3 -3} \right)$ | (e) $\left( \genfrac{}{}{0pt}{}{1 0 0}{\begin{array}{c}0 1 0\\ 0 0 1 \end{array}} \right)$ + $\left( \genfrac{}{}{0pt}{}{0 0 3}{\begin{array}{c}0 5 0\\10 0 0 \end{array}} \right)$ |
|  (f ) $\left( \genfrac{}{}{0pt}{}{0 -5}{\begin{array}{c}1 3\\5 9\end{array}} \right)$ – $\left( \genfrac{}{}{0pt}{}{6 8}{\begin{array}{c} 7 -2\\-3 0\end{array}} \right)$  |  (g) $\left( \genfrac{}{}{0pt}{}{4 3 7}{-1 -9 8} \right)$ + $\left( \genfrac{}{}{0pt}{}{5 11}{\begin{array}{c}6 -3\\2 0\end{array}} \right)$  |
| (h) $\left( \genfrac{}{}{0pt}{}{4 -3 9 2 1}{-1 2 1 -3 0} \right)$ + $\left( \genfrac{}{}{0pt}{}{8 -6 4 2 0}{9 -7 8 -5 3} \right)$ |
| (i)$ $ $\left(\begin{matrix}\begin{matrix}5&8\end{matrix}&\begin{matrix}9&-7\end{matrix}\\\begin{matrix}-2&9\end{matrix}&\begin{matrix}-1& 8\end{matrix}\\\begin{matrix}3&4\end{matrix}&\begin{matrix}2&10\end{matrix}\end{matrix}\right)$ +$ \left(\begin{matrix}\begin{matrix}3&3\end{matrix}&\begin{matrix}-5&13\end{matrix}\\\begin{matrix}5&0\end{matrix}&\begin{matrix}7& 6\end{matrix}\\\begin{matrix}8&1\end{matrix}&\begin{matrix}-4&-13\end{matrix}\end{matrix}\right)$ |

Unit 7 **Matrices**

 4. Evaluate the following multiplication of a matrix with a scalar quantity.

 **ClassWiz set-up**

You may press w4 to access the matrix function. Press T1 to define the matrices accordingly.

**CLASSWIZ WORKSHEETS**

|  |  |
| --- | --- |
| 1. 5$ \left( \genfrac{}{}{0pt}{}{2 -2}{3 -3} \right)$
 |  |
| (b) –3 $\left( \genfrac{}{}{0pt}{}{1 6}{\begin{array}{c}-3 8 \\7 -9\end{array}} \right)$ |  (c) 3 $\left( 15 -8 \right)$ |
| (d) –$ \frac{ 1 }{4}(32)$ | (e) 0 $\left( \genfrac{}{}{0pt}{}{6 -9 2 0}{\begin{array}{c}-2 4 1 1\\ 1 0 -6 -2\end{array}} \right)$ |
| (f) $\frac{ 1 }{3}\left( \genfrac{}{}{0pt}{}{81}{\begin{array}{c}18\\36\\6\end{array}} \right)$ | (g) 5 $\left( \genfrac{}{}{0pt}{}{13 5 -7}{\begin{array}{c}0 1 10\\4 -3 1 \end{array}} \right)$ |
| (h) *k* $\left( \genfrac{}{}{0pt}{}{3 b -8 d 0}{v 7 x -1 z} \right)$ | (i) 30$\left( \genfrac{}{}{0pt}{}{\frac{1}{10} \frac{4}{15} \frac{-5}{6}}{\frac{ 2 }{3} \frac{-3}{5} \frac{ 1 }{2}} \right)$ |

**Casio ClassWiz Mathematics Workbook**

**CLASSWIZ WORKSHEETS**

1. Evaluate the following multiplication of row and column matrices.

 **ClassWiz set-up**

You may press w4 to access the matrix function. Press T1 to define the matrices accordingly.

|  |  |
| --- | --- |
| (a) $\left( \genfrac{}{}{0pt}{}{5}{2} \right)$ $\left( 3 \right)$  |  |
| (b) $\left( 6 9 \right)\left( \genfrac{}{}{0pt}{}{-3}{4} \right)$ | (c) $\left( \genfrac{}{}{0pt}{}{-3}{4} \right) \left( 6 9 \right)$ |
| Teachers to note: Multiplication of matrices is not commutative. i.e. **AB** ≠ **BA** |
| (d) $\left( 1 2 3 \right)\left( \genfrac{}{}{0pt}{}{4}{\begin{array}{c}5\\6\end{array}} \right)$ | (e) $\left( \genfrac{}{}{0pt}{}{4}{\begin{array}{c}5\\6\end{array}} \right)\left( 1 2 3 \right)$ {{Highlight d and e}}[[]] |
| Teachers to note: Multiplication of matrices is not commutative. i.e. **AB** ≠ **BA** |
| (f ) $\left( 5 -8 -9 5\right)\left(\genfrac{}{}{0pt}{}{0}{\begin{array}{c}-2\\7\\3\end{array}}\right)$  | (g)$ \left(\genfrac{}{}{0pt}{}{0}{\begin{array}{c}-2\\7\\3\end{array}}\right)$ $\left(5 -8 -9 5\right)$ {{Highlight f and g}}[[]] |
| Teachers to note: Multiplication of matrices is not commutative. i.e. **AB** ≠ **BA** |

Unit 7 **Matrices**

**CLASSWIZ WORKSHEETS**

|  |  |
| --- | --- |
| (h)$ \left( -2 -1 \right)\left( \genfrac{}{}{0pt}{}{3}{-5} \right)\left( 4 \right)$ | (i) $\left( 4 –1 5 \right)$ $\left( \genfrac{}{}{0pt}{}{8}{\begin{array}{c}-9\\2\end{array}} \right)\left( 2 \right)$  |

1. Calculate the value of determinant for the following matrices.

 **ClassWiz set-up**

You may press w4 to access the matrix function. Press TR2 to access the determinant function.

|  |  |
| --- | --- |
| 1. $\left( \genfrac{}{}{0pt}{}{9 -9}{8 1} \right)$
 |  |
| (b) $\left( \genfrac{}{}{0pt}{}{7 4}{8 3} \right)$ | (c) $\left( \genfrac{}{}{0pt}{}{1 0}{-3 5 }\right)$ |
| (d) $\left( \genfrac{}{}{0pt}{}{2 7}{3 3} \right)$ | (e) $\left( \genfrac{}{}{0pt}{}{3 4}{6 8} \right)$  |
| (f ) $\left( \genfrac{}{}{0pt}{}{1 2}{3 4} \right)$ | (g) $\left( \genfrac{}{}{0pt}{}{5 3}{-8 -2} \right)$  |
| (h) $\left( \genfrac{}{}{0pt}{}{3 0}{0 -8} \right)$  | (i) $\left( \genfrac{}{}{0pt}{}{3 2}{9 6} \right)$  |

**Casio ClassWiz Mathematics Workbook**

**CLASSWIZ WORKSHEETS**

1. Following question 6, evaluate the inverse function of the following matrices.

 **ClassWiz set-up**

You may press w4 to access the matrix function. Use the / button to find the inverse of a matrix.

|  |  |
| --- | --- |
| 1. $\left( \genfrac{}{}{0pt}{}{9 -9}{8 1} \right)$
 |  |
| (b) $\left( \genfrac{}{}{0pt}{}{7 4}{8 3} \right)$ | (c) $\left( \genfrac{}{}{0pt}{}{1 0}{-3 5 }\right)$ |
| (d) $\left( \genfrac{}{}{0pt}{}{2 7}{3 3} \right)$ | (e) $\left( \genfrac{}{}{0pt}{}{3 4}{6 8} \right)$ |
| (f ) $\left( \genfrac{}{}{0pt}{}{1 2}{3 4} \right)$  | (g) $\left( \genfrac{}{}{0pt}{}{5 3}{-8 -2} \right)$ |
| (h) $\left( \genfrac{}{}{0pt}{}{3 0}{0 -8} \right)$ | (i) $\left( \genfrac{}{}{0pt}{}{3 2}{9 6} \right)$  |

**CLASSWIZ WORKSHEETS**

#### Level 2

Unit 7 **Matrices**

1. Find the values of the unknown constants in the following equations. (a) $\left( 3 -2 \right)$ +$\left( 8 b \right)$ = $\left( a 3 \right)$

(b) $\left( 4 a -3 \right)$ – $\left( -1 2 b \right)$ = $\left( c 4 5 \right)$

(c) $\left( 3 a 4 \right)$ – $\left( 5 1 b \right)$ =$c\left( 2 6 5 \right)$

(d) $\left( 8 3 7 -9 \right)$ – 3$\left( a 4 9 -2\right)$ +$\left( 5 3 b 0 \right) $=$\left( 5-6 -9 c \right)$

(e) $\left( \genfrac{}{}{0pt}{}{-1}{3} \right)$ +5$\left( \genfrac{}{}{0pt}{}{1}{b} \right)$ =$\left( \genfrac{}{}{0pt}{}{a}{4} \right)$

(f ) $\left( \genfrac{}{}{0pt}{}{-7}{0} \right)$ +$\left( \genfrac{}{}{0pt}{}{a}{3} \right)$ – 2 $\left( \genfrac{}{}{0pt}{}{4}{c} \right)$ =3$\left( \genfrac{}{}{0pt}{}{2}{6} \right)$

 **Casio ClassWiz Mathematics Workbook**

(g) $\left( \genfrac{}{}{0pt}{}{a}{\begin{array}{c}5\\9\end{array}} \right)$ – 3 $\left( \genfrac{}{}{0pt}{}{3}{\begin{array}{c}b\\6\end{array}} \right) $=$\left( \genfrac{}{}{0pt}{}{4}{\begin{array}{c}-1\\c\end{array}} \right)$

(h) $\left( \genfrac{}{}{0pt}{}{-5}{\begin{array}{c}-2\\13\end{array}} \right)$ +$\left[\left( \genfrac{}{}{0pt}{}{15}{\begin{array}{c}a\\-4\end{array}} \right)–2\left( \genfrac{}{}{0pt}{}{0}{\begin{array}{c}-3\\c\end{array}} \right)\right]$ =$\left( \genfrac{}{}{0pt}{}{b}{\begin{array}{c}11\\6\end{array}} \right)$

**CLASSWIZ WORKSHEETS**

(i) $\left(\genfrac{}{}{0pt}{}{4}{\begin{array}{c}-3\\7\\1\end{array}}\right)$ –$ \left[\frac{ 1 }{2}\left(\genfrac{}{}{0pt}{}{8}{\begin{array}{c}a\\-1\\6\end{array}}\right)–\left( \genfrac{}{}{0pt}{}{5}{\begin{array}{c}1\\0\\b\end{array}} \right)\right]$ =2$\left(\genfrac{}{}{0pt}{}{c}{\begin{array}{c}4\\-8\\9\end{array}}\right) $– $\left(\genfrac{}{}{0pt}{}{12}{\begin{array}{c}-6\\d\\3\end{array}}\right)$

1. Evaluate the following matrices.

|  |  |
| --- | --- |
| 1. $\left( \genfrac{}{}{0pt}{}{ 1 9}{-6 1} \right)$ $\left( \genfrac{}{}{0pt}{}{6}{-1} \right)$
 | 1. $\left( \genfrac{}{}{0pt}{}{6}{-1} \right)\left( \genfrac{}{}{0pt}{}{ 1 9}{-6 1} \right)$
 |
| Teachers to note: Multiplication of matrices is not commutative. i.e. **AB** ≠ **BA** |
| (c) $\left( 4 3 0 \right)\left( \genfrac{}{}{0pt}{}{6 2}{\begin{array}{c}-2 -4 \\0 1\end{array}} \right)$ | (d) $\left( \genfrac{}{}{0pt}{}{6 -3 4}{8 -2 1} \right)\left( \genfrac{}{}{0pt}{}{5 0}{\begin{array}{c}-4 -2\\-3 0\end{array}} \right)$ |
| Teachers to note: Multiplication of matrices is not commutative. i.e. **AB** ≠ **BA** |

Unit 7 **Matrices**

**CLASSWIZ WORKSHEETS**

|  |  |
| --- | --- |
| (e) $\left( \genfrac{}{}{0pt}{}{ 5 0}{\begin{array}{c}-4 -2 \\-3 0\end{array}} \right)$ $\left( \genfrac{}{}{0pt}{}{6 -3 4}{8 -2 1} \right)$  | (f )$ \left( \genfrac{}{}{0pt}{}{5 -3}{6 -5} \right)^{2}$  |
| Teachers to note: Multiplication of matrices is not commutative. i.e. **AB** ≠ **BA** |
| (g) $ \left( \genfrac{}{}{0pt}{}{5 -3}{6 -5} \right)^{3}$  | (h) $\left( \genfrac{}{}{0pt}{}{ 2 0 -1}{\begin{array}{c}-8 3 0 \\ 1 -3 4 \end{array}} \right)^{2}$ |
| (i) $\left( \genfrac{}{}{0pt}{}{ 2 0 -1}{\begin{array}{c}-8 3 0 \\ 1 -3 4 \end{array}} \right)\left( \genfrac{}{}{0pt}{}{ 2 0 -1}{\begin{array}{c}-8 3 0 \\ 1 -3 4 \end{array}} \right)^{-1}$ | (j)$ \left( \genfrac{}{}{0pt}{}{ 2 0 -1}{\begin{array}{c}-8 3 0 \\1 -3 4 \end{array}} \right)^{-1}\left( \genfrac{}{}{0pt}{}{ 2 0 -1}{\begin{array}{c}-8 3 0 \\ 1 -3 4 \end{array}} \right)$  |
|  | You may use the ClassWiz calculator to find these. |  |
|  |  |
| Teachers to note: For any matrix **A**, **AA**-1 = **A**-1**A** = **I**. |

**Casio ClassWiz Mathematics Workbook**

1. Find the value(s) of the unknown constants in the following equations. (a) $\left( 3 -1 \right)\left( \genfrac{}{}{0pt}{}{5}{a} \right)$ +$\left( 3 \right)$ =$\left(-7 \right)$

(b) $\left( \genfrac{}{}{0pt}{}{4 7}{-9 11} \right)$ –$ \left( \genfrac{}{}{0pt}{}{a}{b} \right)\left( 3 -2 \right)$ = $\left( \genfrac{}{}{0pt}{}{-5 13}{3 3} \right)$

 (c) $\left( \genfrac{}{}{0pt}{}{8}{\begin{array}{c}-9\\3\end{array}} \right)\left( 7 0 1 \right)$ – $\left(\genfrac{}{}{0pt}{}{ 25 4 -1}{\begin{array}{c}-48 11 0\\ 2 -6 9 \end{array}}\right)$ = $\left( \genfrac{}{}{0pt}{}{ a -4 9}{\begin{array}{c}-15 b -9\\ 19 6 c \end{array}} \right)$

**CLASSWIZ WORKSHEETS**

(d) $\left( \genfrac{}{}{0pt}{}{4 1}{3 -1} \right)\left( \genfrac{}{}{0pt}{}{a 8}{6 0} \right)$ = $\left( \genfrac{}{}{0pt}{}{10 b}{-3 c} \right)$

(e) $\left( \genfrac{}{}{0pt}{}{3 -2}{4 7} \right)\left( \genfrac{}{}{0pt}{}{a}{b} \right)$ –3$\left( \genfrac{}{}{0pt}{}{a}{2} \right)$ = $\left( \genfrac{}{}{0pt}{}{10}{19} \right)$

Unit 7 **Matrices**

(f ) $\left( \genfrac{}{}{0pt}{}{7 3}{10a -9} \right)$ +$\left( \genfrac{}{}{0pt}{}{b 7}{6 -5} \right)\left( \genfrac{}{}{0pt}{}{-7 0}{ 5 -3} \right)$ = $\left( \genfrac{}{}{0pt}{}{0 c}{3 6} \right)$

**CLASSWIZ WORKSHEETS**

(g) $\left(-5 3 \right)\left( \genfrac{}{}{0pt}{}{4 -3 0}{a -1 2} \right)$ +$\left( 1 b 3 \right)$ = 2$\left( 1 2 c \right)$

(h) $\left( \genfrac{}{}{0pt}{}{-1 0 2}{\begin{array}{c}3 -4 1\\ 9 5 a \end{array}} \right)\left( \genfrac{}{}{0pt}{}{ 4 3}{\begin{array}{c} 2 -6 \\-3 0\end{array}} \right)\left( \genfrac{}{}{0pt}{}{3}{5} \right)$ – $\left( \genfrac{}{}{0pt}{}{-4}{\begin{array}{c}b\\-3\end{array}} \right)$ = 3$\left( \genfrac{}{}{0pt}{}{c}{\begin{array}{c}5\\4\end{array}} \right)$

(i) $\left(\genfrac{}{}{0pt}{}{a}{\begin{array}{c}8\\-3\\6\end{array}}\right)\left(-2 3 \right)\left( \genfrac{}{}{0pt}{}{0 1}{2 b} \right) $=$ \left(\genfrac{}{}{0pt}{}{ 12 -10}{\begin{array}{c} 48 -40\\ c 15\\36 d\end{array}}\right)$

**Casio ClassWiz Mathematics Workbook**

**CLASSWIZ WORKSHEETS**

1. Evaluate the product of matrices with identity matrix.
2. $\left( \genfrac{}{}{0pt}{}{4 1}{7 9} \right)\left( \genfrac{}{}{0pt}{}{1 0}{0 1} \right)$

(b) $\left( \genfrac{}{}{0pt}{}{1 0}{0 1} \right)\left( \genfrac{}{}{0pt}{}{4 1}{7 9} \right)$

Teachers to note: For any matrix **A**, **AI** = **IA** = **A**.

 (c) $\left( \genfrac{}{}{0pt}{}{4 4 2}{\begin{array}{c}1 -6 3\\ 8 11 9 \end{array}} \right)\left( \genfrac{}{}{0pt}{}{1 0 0}{\begin{array}{c}0 1 0\\ 0 0 1 \end{array}} \right)$

 (d) $\left( \genfrac{}{}{0pt}{}{1 0 0}{\begin{array}{c}0 1 0\\ 0 0 1 \end{array}} \right)$ $\left( \genfrac{}{}{0pt}{}{4 4 2}{\begin{array}{c}1 -6 3\\ 8 11 9 \end{array}} \right)$

Teachers to note: For any matrix **A**, **AI** = **IA** = **A**.

Unit 7 **Matrices**

1. Given that **A** = $\left( \genfrac{}{}{0pt}{}{ 2 8}{-1 5} \right)$ and **B** = $\left( \genfrac{}{}{0pt}{}{ 5 0}{-2 3} \right)$, find the matrix C such that 2**A** + **C** = **BI**.

**CLASSWIZ WORKSHEETS**

1. Given that **A** =$ \left( \genfrac{}{}{0pt}{}{1 3}{5 6} \right)$ and **B** = $\left( \genfrac{}{}{0pt}{}{-2 -5}{-4 1} \right)$, find the matrix C such that 4**C** + **B** = **AI**.
2. Given that **A** = $\left( \genfrac{}{}{0pt}{}{ 4 7}{-3 5} \right)$ and **B** = $\left( \genfrac{}{}{0pt}{}{3 3}{ 5 -1} \right)$, find the matrix C such that – **B** + 3**A** = **IA** + **C**.
3. Given that **M** = $\left( \genfrac{}{}{0pt}{}{2 1}{ 1 -2} \right)$ and **M**2 = *k***I**, find the value of *k*.
4. Given that **Q** = $\left( \genfrac{}{}{0pt}{}{ 5 -3}{-3 -5} \right)$ and **Q**2 = *k***I**, find the value of *k*.
5. Given that **M** = $\left( \genfrac{}{}{0pt}{}{ 4 -2}{-2 -4} \right)$ and **M**2 = *k***I**, find the value of *k*.

**Casio ClassWiz Mathematics Workbook**

1. Given that **A** = $\left( \genfrac{}{}{0pt}{}{ 4 -7}{3 5} \right)$, **B** = $\left( \genfrac{}{}{0pt}{}{2 0}{-5 6} \right)$ and **C** = $\left( \genfrac{}{}{0pt}{}{ 7 9}{-1 1} \right)$, find unknown matrix **D** in the

following equations. You may use the ClassWiz calculator to evaluate the following expressions.

1. **AD** = **B**
2. **DC** = **A**
3. **A** + **B** = **CD**
4. **DB** = **AC**
5. **A**2**D** = 2**C**
6. 3**C** – **A** = **DB**2
7. Find the transpose of the following matrices.

 **ClassWiz set-up**

You may press w4 to access the matrix function. Press TR3 to find the transpose of a matrix.

1. $\left( \genfrac{}{}{0pt}{}{ 5 3}{-8 -2} \right)$ (b)$ \left( \genfrac{}{}{0pt}{}{ 7 1}{\begin{array}{c}-4 0 \\ 9 -5\end{array}} \right)$

1. $\left( \genfrac{}{}{0pt}{}{4 4 -2}{7 -3 1} \right)$ (d)$ \left( \genfrac{}{}{0pt}{}{ 4 -8 3}{\begin{array}{c}6 7 -1\\0 2 5\end{array}} \right)$

## $\left( \genfrac{}{}{0pt}{}{-3 8 }{\begin{array}{c} 3 5\\ 2 -6\\ 9 11\end{array}} \right)$ (f)$ \left( \genfrac{}{}{0pt}{}{5 7 -2 9}{3 8 -1 0} \right)$

**CLASSWIZ WORKSHEETS**

Unit 7 **Matrices**

1. Following question 12, find the matrix multiplication of the given matrix with its transpose. Use the ClassWiz calculator. Press w4 to access the matrix function. Press TR3 to find the transpose of a matrix.

**CLASSWIZ WORKSHEETS**

1. $\left( \genfrac{}{}{0pt}{}{ 5 3}{-8 -2} \right)$ (b) $\left( \genfrac{}{}{0pt}{}{7 1}{\begin{array}{c}-4 0 \\9 -5\end{array}} \right)$

1. $\left( \genfrac{}{}{0pt}{}{4 4 -2}{7 -3 1} \right)$ (d) $ \left( \genfrac{}{}{0pt}{}{4 -8 3}{\begin{array}{c} 6 7 -1\\0 2 5\end{array}} \right)$

## $\left( \genfrac{}{}{0pt}{}{-3 8 }{ \begin{array}{c}3 5\\ 2 -6\\ 9 11\end{array}} \right)$ (f)$ \left( \genfrac{}{}{0pt}{}{5 7 -2 9}{3 8 -1 0} \right)$

1. A cake shop sells chocolate cakes and strawberry cakes in two sizes, small and large. The selling price of a small cake is $6.80 and the selling price of a large cake is $19.80. The quantities sold in a particular week are as listed.

|  |  |  |
| --- | --- | --- |
| Type of cake/Size | Small | Large |
| Chocolate cake | 20 | 67 |
| Strawberry cake | 34 | 58 |

Given that **P** = $\left( \genfrac{}{}{0pt}{}{20 67}{34 58} \right)$ and **Q** =$ \left( \genfrac{}{}{0pt}{}{6.80}{19.80} \right)$.

1. Evaluate **PQ**.
2. Explain what the elements of **PQ** represent.

**Casio ClassWiz Mathematics Workbook**

1. Two cafés sell coffee and tea. The price of a cup of coffee is $3.40 and the price of a cup of tea is $2.80. The quantities sold in a particular week are as listed.

|  |  |  |
| --- | --- | --- |
| Type of drink/Café | Café A | Café B |
| Coffee | 250 | 219 |
| Tea | 173 | 198 |

Given that **C** = $\left(\genfrac{}{}{0pt}{}{250 173}{219 198}\right)$ and T = $\left( \genfrac{}{}{0pt}{}{3.40}{2.80} \right)$.

1. Evaluate **CT**.
2. Explain what the elements of **CT** represent.

**CLASSWIZ WORKSHEETS**

1. A florist store offers two types of hampers, Floral Hamper and Congratulations Hamper. The price of Floral Hampers and Congratulations Hampers are $109 and $139 respectively. The quantities sold in two consecutive weeks are as listed.

|  |  |  |
| --- | --- | --- |
| Type of Hamper/Week | Week 1 | Week 2 |
| Floral Hamper | 98 | 72 |
| Congratulations Hamper | 45 | 138 |

Given that **H** = $\left( \genfrac{}{}{0pt}{}{98 72}{45 138} \right)$ and **P** = $\left( 109 139 \right)$.

1. Evaluate **PH**.
2. Explain what the elements of **PH** represent.

Unit 7 **Matrices**

1. Pharmacies A and B offer two types of medication for pain relief, Potarl Relief and Ketger Med. The cost price to the pharmacies of Potarl Relief and Ketger Med are $3.90 and $5.90 respectively. The quantities of the medication sold by the pharmacies in one particular week are as listed.

**CLASSWIZ WORKSHEETS**

|  |  |  |
| --- | --- | --- |
| Pharmacy/Medication | Potarl Relief | Ketger Med |
| Pharmacy A | 144 | 83 |
| Pharmacy B | 120 | 102 |

Given that **M** = $\left( \genfrac{}{}{0pt}{}{144 120}{83 102} \right)$ and **C** = $\left( 3.90 5.90 \right)$.

1. Evaluate **CM**.
2. Explain what the elements of **CM** represent.
3. A farmer sells chicken eggs and duck eggs for 20 cents and 35 cents each respectively. The quantities of the eggs sold by the farmer in two particular weeks are as listed.

|  |  |  |
| --- | --- | --- |
| Egg/Week | Week 1 | Week 2 |
| Chicken eggs | 325 | 368 |
| Duck eggs | 228 | 194 |

Given that **E** = $\left( \genfrac{}{}{0pt}{}{325 228}{368 194} \right) $and **M** = $\left( \genfrac{}{}{0pt}{}{0.20}{0.35} \right)$.

1. Evaluate **EM**.
2. Explain what the elements of **EM** represent.

**Casio ClassWiz Mathematics Workbook**

1. A phone distributor sells three different types of Android phones, A, B and C. Phones A, B and C are priced at $439, $688 and $1098 respectively. The quantities of the phones sold in January and February are as listed.

|  |  |  |
| --- | --- | --- |
| Phone/ Month | January | February |
| A | 14 | 28 |
| B | 23 | 11 |
| C | 13 | 20 |

Given that **C** = $\left( \genfrac{}{}{0pt}{}{14 28}{\begin{array}{c}23 11\\13 20\end{array}} \right)$ and **P** = $\left(439 688 1098\right)$.

1. Evaluate **PC**.
2. Explain what the elements of **PC** represent.
3. A company sells formula milk powder for 3 different age groups, 0 to 12 months, 1 year old to 3 years old and 4 years old to 8 years old. They are respectively priced at $43, $48 and $58 respectively. The quantities of the formula milk powder sold in Store A and Store B for the year 2017 are as listed.

|  |  |  |
| --- | --- | --- |
| Formula milk powder/Country | Store A | Store B |
| 0 to 12 months | 7800 | 4600 |
| 1 to 3 years old | 4900 | 6300 |
| 4 to 8 years old | 10400 | 12800 |

 Given that **F** = $\left(\begin{matrix}7800&4600\\4900&6300\\10400&12800\end{matrix}\right)$ and **S** = $\left(43 48 58\right)$.

1. Evaluate **SF**.
2. Explain what the elements of **SF** represent.

**CLASSWIZ WORKSHEETS**

Unit 7 **Matrices**

1. A fashion store launched three new types of dresses, Ortega, Galvi and Peplum. They are respectively priced at $89, $79 and $69 respectively. The quantities sold in the first and second week after the launch are as listed.

**CLASSWIZ WORKSHEETS**

|  |  |  |
| --- | --- | --- |
| Dress/Week | First week | Second week |
| Ortega | 120 | 189 |
| Galvi | 204 | 145 |
| Peplum | 163 | 179 |

Given that **D** = $\left(\begin{matrix} 120&204&163 \\189&145&179\end{matrix}\right)$ and **S** = $\left(\begin{matrix} 89 \\79\\69\end{matrix}\right)$.

1. Evaluate **DS**.
2. Explain what the elements of **DS** represent.

##### Level 3

**Casio ClassWiz Mathematics Workbook**

**CLASSWIZ WORKSHEETS**

1. Solve the following linear simultaneous equations using matrices.

*y* = 3𝑥+ 2

*y* = –2𝑥– 1

1. Solve the following linear simultaneous equations using matrices. 3*y* = 𝑥+ 3

*y* = –0.5𝑥+ 1

1. Solve the following linear simultaneous equations using matrices.

*y* = –𝑥+ 2 4*y* = –3𝑥+ 1

1. Solve the following linear simultaneous equations using matrices. 5*y* = 5𝑥– 4

2*y* = –3𝑥+ 5

Unit 7 **Matrices**

1. Solve the following linear simultaneous equations using matrices. 3*y* = 2𝑥– 4

**CLASSWIZ WORKSHEETS**

0.5*y* = –0.2𝑥– 1

1. Jimmy sold a total of 28 tickets for his school concert. He sold *x* tickets at $8 each and *y* tickets at $12 each. He collected $276 in total.
	1. Write down two equations connecting *x* and *y*.
	2. Solve the simultaneous equations using matrices.
2. The diagram shows a parallelogram *ABCD* where *AB* = (7𝑥– 5) cm, BC = 2(𝑥+ 3) + 1 cm, *CD* = 2(*y* + 3) cm and *AD* = (4*y* – 7) cm.

*A* 7𝑥 – 5 *B*

4*y* – 7 2(𝑥+ 3) + 1

*D* 2(*y* + 3) *C*

* 1. Write down two equations connecting *x* and *y*.
	2. Solve the simultaneous equations using matrices.
	3. Find the perimeter of *ABCD*.

**Casio ClassWiz Mathematics Workbook**

1. Henry bought 20 apples and 30 oranges for $29.50. Hillary bought 33 apples and 20 oranges for the same amount. Given that the apples cost $*x* each and the oranges cost $*y* each,
	1. Write down two equations connecting *x* and *y*.
	2. Solve the simultaneous equations using matrices.
2. Given that $\left( \genfrac{}{}{0pt}{}{ a 2}{-3 b} \right)$ $\left( \genfrac{}{}{0pt}{}{3}{a^{2}} \right)$ = $\left( \genfrac{}{}{0pt}{}{5}{-4} \right)$, solve for the unknown positive integers *a* and *b.*
3. Given that $\left(-4 2a a^{2}\right)$ $\left(\genfrac{}{}{0pt}{}{ -3 b^{2}}{\begin{array}{c}-a 3\\ 5 2b\end{array}} \right) $= $\left(13a -16b\right)$, solve for the unknown positive integers *a* and *b*.

**CLASSWIZ WORKSHEETS**

**CLASSWIZ WORKSHEETS**

##### Level 4

Unit 7 **Matrices**

1. A factory produces and delivers three types of bakes, cupcakes, chiffon cakes and bread to four different bakeries. The table shows the number of bakes supplied per delivery.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Store/Bake | Cupcakes | Chiffon cakes | Bread | Number of deliveries per month |
| AC Bakery | 100 | 40 | 80 | 5 |
| J solo Bakery | 150 | 35 | 60 | 4 |
| Baketalk | 120 | 35 | 70 | 6 |
| Cakery | 80 | 15 | 30 | 2 |
| Selling price per bake | $1.50 | $5 | $1 |  |

* 1. (i) Write down a 4 × 3 matrix **A** to represent the total quantities of bakes supplied to all 4 bakeries per delivery.

(ii) Write down a 1 × 4 matrix **B** to represent the number of deliveries in a month.

(iii) Calculate the product **BA**. Explain what the elements in the resultant matrix represent.

* 1. (i) Write down a 3 × 1 matrix **C** to represent the selling prices of the bakes to the bakeries.

(ii) Calculate the product of (**BA**)**C**. Explain what the element(s) in the resultant matrix represent.

**Casio ClassWiz Mathematics Workbook**

1. A toy factory produces four types of toys, The Cleaning Kit, Toy Car, Ride-a-bike, Piano set and distributes them to 3 outlets. The quantities sold by each outlet in a week is shown in the table, together with the selling price of each toy.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | The Cleaning Kit | Toy Car | Ride-a-bike | Piano set |
| Outlet 1 | 18 | 15 | 20 | 25 |
| Outlet 2 | 23 | 8 | 13 | 19 |
| Outlet 3 | 20 | 11 | 16 | 28 |
| Selling price of each toy | $17.90 | $39.90 | $29.90 | $19.90 |

The information for the sales volume of the toys can be represented by the matrix

 **S** = $\left( \genfrac{}{}{0pt}{}{18 15 20 25}{\begin{array}{c}23 8 13 19\\20 11 16 28\end{array}} \right)$.

* 1. (i) Write down a matrix **M** representing the selling price of each toy such that the elements of the matrix product will give the total amount of money earned by each outlet respectively.
1. Find this matrix product.
	1. Given that **H** = $\left( \genfrac{}{}{0pt}{}{1}{\begin{array}{c}1\\1\\1\end{array}} \right)$, calculate **P** = **SH**. Explain what the elements in matrix **P** represent.
	2. Given that **K** =$\left(1 1 1\right)$, calculate **Q** = 1/3 **KP**. Explain what the element(s) in matrix **Q** represent.

**CLASSWIZ WORKSHEETS**

Unit 7 **Matrices**

1. A shop sells three brands of microSD cards, Kingsler, Canten and Sonda. Each brand produces their microSD card in three capacities, 16 GB, 32 GB and 64 GB. The sales of the microSD cards in August and September are shown in the table.

**CLASSWIZ WORKSHEETS**

|  |  |  |
| --- | --- | --- |
|  | August | September |
|  | 16 GB | 32 GB | 64 GB | 16 GB | 32 GB | 64 GB |
| Kingsler | 16 | 19 | 21 | 12 | 22 | 21 |
| Canten | 11 | 15 | 19 | 8 | 13 | 17 |
| Sonda | 20 | 28 | 18 | 15 | 25 | 20 |

 The information for August’s sales can be represented by the matrix **X** = $\left(\begin{matrix}16&19&21\\11&15&19\\20&28&18\end{matrix}\right) $.

* 1. The information for September’s sales can be represented by the matrix **Y**. Write down matrix **Y**.
	2. Calculate $\frac{ 1 }{2}$ (**X** + **Y**). Explain what the elements in the resultant matrix represent.
	3. Given that **Z** = $\left(\begin{matrix} 1 \\1\\1\end{matrix}\right)$, calculate the matrix product of **XZ**. Explain what the elements in the

resultant matrix represent.

* 1. Given that **W** =$ \left( 1 1 1 \right)$, calculate the matrix product of **WY**. Explain what the elements in the resultant matrix represent.

**Casio ClassWiz Mathematics Workbook**

**CLASSWIZ WORKSHEETS**

1. A bakery shop sells 2 designs of birthday cakes, Unicorn design and Galaxy design. Each design is available in three sizes, small, medium and large. The sales of these birthday cakes in April and May are shown in the table.

|  |  |  |
| --- | --- | --- |
|  | April | May |
|  | Small | Medium | Large | Small | Medium | Large |
| Unicorn Design | 18 | 30 | 21 | 14 | 28 | 17 |
| Galaxy Design | 20 | 15 | 24 | 23 | 29 | 13 |

The information for April’s sales can be represented by the matrix **A** = $\left( \genfrac{}{}{0pt}{}{18 30 21}{20 15 24} \right)$.

* 1. Write down a matrix **B** to represent the information for May’s sales.
	2. Calculate $\frac{1}{2}$ (**A** + **B**). Explain what the elements in the resultant matrix represent.

The prices of the cakes are shown in the table below.

|  |  |  |  |
| --- | --- | --- | --- |
|  | Small | Medium | Large |
| Unicorn Design | 18.90 | 27.90 | 39.90 |
| Galaxy Design | 15.90 | 23.90 | 34.90 |

* 1. (i) Write down a 3 × 2 matrix **C** to represent the prices of the cakes.
	2. Calculate **D** = (**A** + **B**)**C.** Explain which two elements are not useful in the resultant matrix.
	3. Calculate (1 0)**D**T and explain what the elements represent in this resultant matrix.

Unit 7 **Matrices**

1. A factory produces and sells two types of biscuits, Oatmeal biscuits and Banana biscuits. The table below shows the unit quantities of the required ingredients to make a tin of each biscuit and the unit cost of each ingredient in 2015 and 2016.

**CLASSWIZ WORKSHEETS**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Ingredients | Flour | Sugar | Oatmeal | Banana |
| Oatmeal Biscuit | 4 | 5 | 3 | 0 |
| Banana Biscuit | 3 | 3 | 0 | 4 |

|  |  |  |
| --- | --- | --- |
| Unit Cost for | 2015 | 2016 |
| Flour | 1.50 | 1.70 |
| Sugar | 2.30 | 2.20 |
| Oatmeal | 2.80 | 3.00 |
| Banana | 3.30 | 3.50 |

Given that **X** = $\left(\begin{matrix}4&5&\begin{matrix}3&0\end{matrix}\\3&3&\begin{matrix}0&4\end{matrix}\end{matrix}\right)$ and **Y** = $\left( \begin{matrix}1.5&1.7\\2.3&2.2 \\\begin{matrix}2.8\\3.3\end{matrix}&\begin{matrix}3\\3.5\end{matrix}\end{matrix}\right)$ .

* 1. Evaluate **XY**. Explain what the elements in the resultant matrix represent.
	2. A customer wants to order 26 tins of Oatmeal biscuit and 38 tins of Banana biscuits. By writing down a matrix **A**, evaluate **B** = **AX** where the elements in matrix **B** represent the total amount of flour, sugar, oatmeal and banana respectively required for supplying the customer’s order for oatmeal biscuits and banana biscuits.
	3. Given that **D** = $\left( \begin{matrix}1.7 \\\begin{matrix}2.2\\3\end{matrix}\\3.5\end{matrix}\right)$, evaluate the matrix product **BD** and explain what the elements in the

resultant matrix represent.