**TEACHER’S GUIDE**

**Unit**

**2**

**FURTHER NUMBERS**

# KEY CONCEPTS

**Standard Form**

A number expressed in standard form is written as

A × 10n

where 1 ≤ A < 10 and *n* is an integer.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **10n** | **Name** | **SI prefix** | **Symbol** | **Numeral value** |
| 1012 | Trillion | Tera | T | 1 000 000 000 000 |
| 109 | Billion | Giga | G | 1 000 000 000 |
| 106 | Million | Mega | M | 1 000 000 |
| 103 | Thousand | Kilo | k | 1 000 |
| 10–3 | Thousandth | Milli | m | 0.001 |
| 10–6 | Millionth | Micro |  | 0.000 001 |
| 10–9 | Billionth | Nano | n | 0.000 000 001 |
| 10–12 | Trillionth | Pico | p | 0.000 000 000 001 |

# Common Conversions

**Length** 1 cm = 10 mm

1 m = 100 cm

1 km = 1000 cm

**Area** 1 m2 = 10 000 cm2

1 km2 = 1 000 000 m2

1 km2 = 100 hectares

**Volume** 1 m3 = 1 000 000 cm3

1 km3 = 1 000 000 000 m3

**Mass** 1 g = 1000 mg

1 kg = 1000 g

1 tonne = 1000 kg

**Volume** 1 litre = 1000 ml =1000 cm3

**Time** 1 hour = 60 minutes

1 minute = 60 seconds

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# Rules of Indices

The constants *m*, *n*, *a* and *b* are real numbers.

1. *am* × *an* = *am* + *n*
2. *a*m ÷ *a*n = *a*mn
3. (*am*)*n* = *amn*
4. *am* × *bm* = (*ab*)*m*
5. =

6. *a*0 = 1

7. *a*–*n* =

8. =

9. =

10. ==

# Rules of Surds

The constants *m*, *n*, *a* and *b* are real numbers. 1. *m*+*n*=

2. *m*–*n*= (*m* – *n*)

3. =

4.=

5. *m*×*n*=*mn*

# Maps and Scales

The scale of a map is given in the form 1 : *n*.

It can also be expressed as a representative fraction (R.F.) of .

If a linear scale is 1 : *n*, then the area scale is 12 : *n*2 = 1 : *n*2.

Unit 2 **Further Numbers**

# WORKED EXAMPLES

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## Worked Example 1

Mars is about 228 million km away from the Sun. The speed of light is 3.0 × 108 m/s. Calculate the time taken, in seconds, for light to reach the earth.

Solution

228 million = 228 × 106 = 2.28 × 108 km = 2.28 × 1011 m

|  |  |
| --- | --- |
|  | **ClassWiz steps**  Press |
| is 760 seconds. |
|  | (2.28OG11) |
|  | P(3.0OG8)= |
|  | and 760 will be displayed. |

(2.28 × 1011 m) ÷ (3.0 × 108) = 760 s

The time taken for light to reach earth

## Worked Example 2

Solve the equation 49*x* + 3 ÷ 74 – *x* = 3432*x*.

Solution

49*x* + 3 ÷ 74 – *x* = 3432*x*

72*x* + 6 ÷ 74 – *x* = 76*x*

The key approach here is to make the bases the same. Here, we can tell clearly that the base is 7.

72*x* + 6 – 4 + *x* = 76*x*

2*x* + 6 – 4 + *x* = 6*x*

2 = 3*x*

*x* =

## Worked Example 3

Solve the equation 22*x* × 72*x* = 14*x* – 6.

Solution

22*x* × 72*x* = 14*x* – 6

The key approach here is to make the bases the same.

However, the base is not a prime number here. If you do see more than 2 primes, it is likely that you can multiply the primes to make the common base. Otherwise, one of the prime number will likely be cancelled off as common factor.

142*x* = 14*x* – 6

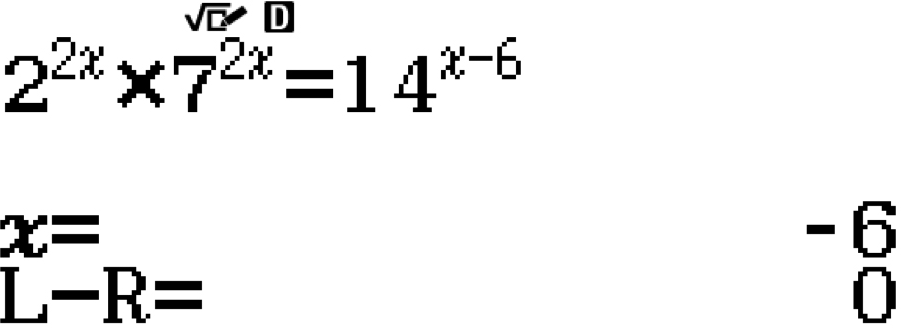
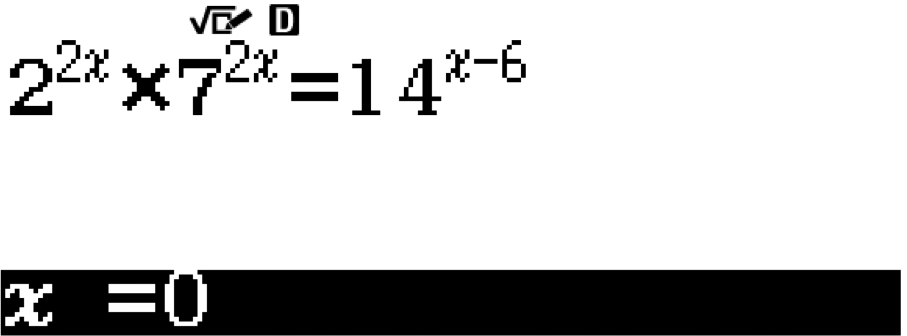
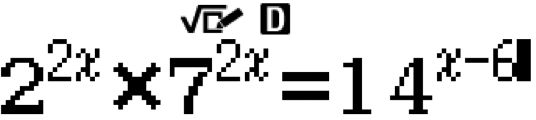
2*x* = *x –* 6

*x* = –6

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### ClassWiz steps



Check your answer using ClassWiz calculator

Tip: Use the Solve function to check your answers. Press Qr for “=” sign to input entire equation.

On the left hand side of the equal sign, press

2^2Q)$O7^2Q)

On the right hand side of the equal sign, press 14^Q)p6

Press qr to solve the equation.

Press 0 to input the value of *x* as “0” and press = to lock in the value.

Press = again and the calculator will solve the equation within the parameters set. The calculator should display *x* = –6 after some time.

Unit 2 **Further Numbers**

## Worked Example 4

**TEACHER’S GUIDE**

Solve the following simultaneous equations 3432*x* ÷ 72*y* = 492 – *x*,

13*x* × = 2197.

Solution

32*x* ÷ 72*y* = 492 – *x* …(1)

13*x* × = 2197 …(2)

From (1) 76*x* ÷ 72*y* = 74 – 2*x*

The key approach here is to use the law of indices to make the bases the same, so that we can compare the index and form an equation.

76*x* – 2*y* = 74 – 2*x*

6*x* – 2*y* = 4 – 2*x* 8*x* = 4 + 2*y*

4*x* = 2 + *y* …(3)

From (2) 13*x* ×= 133

13*x* – 2*y* = 133

The key approach here is to use the law of indices to make the bases the same, so that we can compare the index and form an equation.

*x* – 2*y* = 3

*x* = 3 + 2*y* …(4)

Sub (4) into (3) 4(3 + 2*y*) = 2 + *y*

12 + 8*y* = 2 + *y*

7*y* = 10

*y* =

*x* = 3 + 2 =

and the calculator displays: – .

The CLASSWIZ calculator has the function of rationalizing the denominator for you.

Key in the expression ,

and – is displayed.

a4s2$ps3Rs3$p2s2=,

**ClassWiz steps**

Press

## Worked Example 5

Rationalise and simplify

Solution

=

=

=

*a*, and  *× = .*

Recall that × =

=

=

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## Worked Example 6

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Solve the equation = 2*x*.

Solution

Do you know why we reject – ?By substituting

this value into the equation, you will get

=

7 – 3*x* = 4*x*2

= 2(- ) which simplifies to

4*x*2 + 3*x* – 7 = 0 (4*x* + 7)(*x* – 1) = 0

= –, which is not possible.

*x* = – (reject) or 1

Press Qr for “=” sign to input entire equation of = 2*x*.

Press qr to solve the equation.

Press 0 to input the value of *x* as “0” and press = to lock in the value.

Press = again and the calculator will solve the equation within the parameters set. The calculator should display *x* = 1 promptly.

Tip: Use the Solve function to check your answers.

**ClassWiz steps**

Check your answer using ClassWiz calculator

***Worked Example 7***

Casey travelled to Switzerland and exchanged $50 000 Taiwan dollars (TWD) for Swiss Franc (CHF). The prevailing exchange rate was TWD1 = CHF0.033.

1. Calculate the total amount of Swiss Franc she received.

For the whole trip, Casey spent CHF1063.28. She wanted to exchange the remaining amount for Thai Baht (THB) at an exchange rate of THB1 = CHF0.032.

1. Calculate the total amount of Thai Baht she received.

Solution

As there are 2 maps, it can be quite confusing as each map has its own linear scale and area scale. Hence, labelling out the first and second map can help you to see the big picture better.

(a) 50 000 × 0.033 = CHF1650

She received CHF1650.

(b) CHF1650 – CHF1063.28 = CHF586.72 CHF586.72 ÷ CHF0.032 = THB18335

She received THB18335.

## Worked Example 8

On the first map with a scale of 3 : 81 000, the area of a forest is 380 cm2. Find the area of the forest on a second map with a scale of 1 : 200 000.

Solution First map

|  |  |
| --- | --- |
|  | As there are 2 maps, it can be quite confusing |
|  |
| 3 : 81 000 | as each map has its own linear scale and area |
| 1 : 27 000 | scale. Hence, labelling out the first and second |
| 1 cm : 0.27 km | map can help you to see the big picture better. |

1 cm2 : 0.0729 km2

Actual area of forest = 380 × 0.0729 = 27.702 km2 Second map 1 : 200 000

1 cm : 2 km

1 cm2 : 4 km2

Area of forest on second map = 27.702 km2 ÷ 4 km2 = 6.9255 cm2

Unit 2 **Further Numbers**

# CLASSWIZ WORKSHEETS

**CLASSWIZ WORKSHEETS**

**Level 1 **

1. Convert the following measurements into the given unit.

|  |  |
| --- | --- |
| (a) 0.0372 km to m | (b) 0.84 hours to minutes and seconds |
| (c) 0.000 738 tonne to grams | (d) 7 842 000 cm3 to litres |
| (e) 479 258 200 mg to kg | (f ) 1.8 m2 to cm2 |
| (g) 5 372 538 000 cm2 to km2 | (h) 87 047 200 m3 to km3 |
| (i) 184600 cm3 to m3 | (j) 0.01085 m3 to cm3 |

1. Evaluate the following indices. You may use the ^ function in the ClassWiz calculator.

|  |  |  |
| --- | --- | --- |
| (a) 52 + 34 = | (b) 30 – = | (c) + 5–2 = |
| (d) – 24 = | (e) 102 × 2–3 = | (f ) 73 ÷ = |
| (g) (–8)0 + 53 = | (h) = | (i) = |

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**CLASSWIZ WORKSHEETS**

1. Evaluate the following surds, leaving your answer to 3 significant figures if it is non-exact.

**ClassWiz set-up**

You may use the s, S or F

function in the ClassWiz calculator.

|  |  |  |
| --- | --- | --- |
| (a) = | (b) = | (c) = |
| (d)– = | (e) – + = | (f ) – = |
| (g) × = | (h) ÷ = | (i) × = |

1. Simplify the following algebraic indices, leaving your answer in positive index notation.

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 1. = | 1. = | | | | (c) = | | | |
| (d) = | (e) | 28*a*–1*b*3 =  343*a*–5*b*–2 |  |  | (f ) | 81*p*3*q*0 =  72(*pq*)–4 |  |  |
| (g) = | (h) × | | | | (i) | ÷ | | |

Unit 2 **Further Numbers**

1. Convert the following currencies using the currency table, giving your answers to the nearest 2 decimal places.

**CLASSWIZ WORKSHEETS**

|  |  |
| --- | --- |
| **Currency** | **1 Singapore dollar (SGD) to** |
| American Dollar | 0.747756 USD |
| Euros | 0.623167 € |
| Pounds | 0.553693 £ |
| Indian Rupee | 47.741239 ₹ |
| Australian Dollar | 0.957395 AUD |
| Canadian Dollar | 0.940170 CAD |
| Swiss Franc | 0.728788 CHF |

|  |  |  |
| --- | --- | --- |
| (a) SGD 74.29 to Indian Rupee | (b) SGD 18.04 to Canadian Dollar | (c) SGD 39.11 to Euros |
| (d) SGD 46.20 to Swiss Franc | (e) SGD 104.25 to US Dollar | (f ) SGD 218.99 to Australian Dollar |
| (g) USD 53.85 to SGD | (h) €94.28 to SGD | (i) AUD29.52 to SGD |
| (j) £48.26 to SGD | (k) CHF65.82 to SGD | (l) ₹94720 to SGD |

1. The actual length of the bridge is 105 metres. Calculate the length, in cm, of the bridge on the map with a scale of 1 : 50 000.
2. The actual length of a highway is 158 km. Calculate the length, in cm, of the highway on the map with a scale of 1 : 2 000 000.

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**CLASSWIZ WORKSHEETS**

1. The actual height of the vase is 24.8 cm. Calculate the scale in the form of 1 : *k* given that the height of the miniature version of the vase is 3.2 cm.
2. The length of the field on the map is 5.4 cm. Calculate the actual length, in km, of the field on the map with a scale of 1 cm : 1 500 m.
3. The length of a cornfield on the map is 8.5 cm. Calculate the actual length, in km, of the cornfield on the map with a scale of 1 : 280 000.
4. The distance between 2 towns on the map is 27.8 cm. Calculate the actual distance, in m, between two towns on the map with a scale of 1 cm : 8.8 km.
5. The actual area of a lake is 78 m2. Calculate the area, in cm2, of the lake on the map with a scale of 1 cm2 : 50 m2.
6. The actual area of a field is 0.073 km2. Calculate the area, in cm2, of the field on the map with a scale of 1 cm2 : 0.55 km2.
7. The actual area of a forest is 29 km2. Calculate the area, in cm2, of the forest on the map with a scale of 1 cm2 : 40 km2.
8. The actual area of a plantation is 128 Hectares. Calculate the area, in cm2, of the plantation on the map with a scale of 1 cm2 : 2 580 000 000 cm2.

Unit 2 **Further Numbers**

1. The actual area of a country is 75 km2. Calculate the area, in cm2, of the country on the map with a scale of 1 cm2 : 3 000 000 m2.

**CLASSWIZ WORKSHEETS**

1. The actual area of a nature park is 25 000 Hectares. Calculate the area, in cm2, of the nature park on the map with a scale of 1 cm2 : 450 000 m2.
2. A square field measures 8 cm by 8 cm on the map with a scale of 1 cm : 2 km. Calculate the actual area, in km2, of the square field.
3. A rectangular field measures 10 cm by 25 cm on the map with a scale of 2 cm : 100 m. Calculate the actual area, in km2, of the rectangular field.
4. A circular piece of land has a diameter of 6 cm on the map with a scale of 1 : 250 000. Calculate the actual area, in km2, of the circular piece of land.
5. A square field has an actual area of 16 km2 on the map with an area scale of 2 cm2 : 18 m2. Calculate the length of a side of the square field, in cm, on the map.
6. A circular field has an actual area of 49π km2 on the map with an area scale of 4 cm2 : 9 km2. Calculate the length of the radius of the circle, in cm, on the map.

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**CLASSWIZ WORKSHEETS**

# Level 2

1. Express 592.47 picolitres in litres, expressing your answer in standard form.
2. A helium atom has an atomic radius of 31 picometres. Express the atomic radius in metres in standard form.
3. 6.02 × 1023 oxygen molecules has a mass of 16g. Calculate the mass, in grams, of 1 oxygen molecule, expressing your answer in standard form.
4. The total land surface area of a country is about 10 million square kilometres and the total human population is estimated to be 0.00117 trillion. Calculate the average number of people per square kilometre of land.
5. An MP3 player has a memory capacity of 64 GB, of which only 96% can be utilised for storage. Given that a song has an average capacity of 5.3 MB, find the maximum number of songs that can be stored in the player.
6. Solve the following equations.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| (a) 2*x* = 64 | | (b) 10*x* = 0.0001 | (c) = | |
| (d) | 2.25*x* – 1 = | (e) = 4 | (f ) | 5*x*4 = 80 |
| (g) | = 250 | (h) 8 = 40.5 | (i) | = 1.512 |

Unit 2 **Further Numbers**

1. Solve the following equations.

**CLASSWIZ WORKSHEETS**

|  |  |  |
| --- | --- | --- |
| (a) 32 + *x* × 81 = 32*x* | (b) × 125 = 5*x* – 1 | (c) 16 ÷ = 2–*x* |
| (d) × 3–2*x* = | (e) 493 ÷ = | (f ) 82*x* × = 4*x* |

1. Solve the following equations.

(a) 22*x* × 5*x* = 203*x* + 5 (b) × = 63*x* – 4 (c) × = 15–*x* + 3

1. Given that 5*a* = 4 and 5*b* = 3, find the value of each of the following.

(a) 5*a* + 2*b* (b) 52*a* – 3*b* (c)

1. Given that 3m = 8 and 5m = 11, find the value of 15*m* + 1 + .

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**CLASSWIZ WORKSHEETS**

1. Simplify the following.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| (a) | 7 – 5 + 3 | (b) | 5 – 2+ 3 | (c) | 5– 3+ 8 |
| (d) | (2 – 5)(6 – ) | (e) | (1 + )(5 – 3) | (f ) | (5 – 3)(3 + ) |

1. By rationalising the denominators, simplify the following.

|  |  |  |
| --- | --- | --- |
|  |  |  |
|  |  |  |

1. Solve the following equations.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| (a) | = 5 | (b) – 2 = 1 | (c) | = |
| (d) | = 2 | (e) = *x* | (f ) | *x* = |

Unit 2 **Further Numbers**

1. A money exchanger exchanges 1000 Japanese Yen (¥) for $1.20 Singapore dollar (S$). Given that a customer requires S$580, calculate how much Japanese Yen the money exchanger needs to receive.

**CLASSWIZ WORKSHEETS**

1. A bank exchanges Malaysian Ringgit (RM) and Euros (€) at a rate of RM1 = €0.21. Given that Jonathan exchanged RM 4500 Malaysian Ringgit to Euros and had €370.92 left, calculate the total amount of Euros he has spent.
2. A bank exchanges Thai Baht (THB) and Philippine Peso (PHP) at a rate of THB1 = PHP1.54. Given that Adeline exchanged 48700 Thai Baht to Philippine Peso and had PHP28649 left, calculate the total amount of Philippine Peso she has spent.
3. Candice exchanged British pounds (€) to Hong Kong dollar (HKD) at a rate of €1 = HKD10.45. Given that Candice received HKD8882.50, calculate how much pounds she exchanged.
4. Penelope exchanged Chinese Yuan (CNY) to Korean Won (KRW) at a rate of CNY1 = KRW164. Given that Penelope received 1.7 million Korean Won, calculate how much Chinese Yuan she exchanged.
5. Sydney is ahead of Frankfurt by 10 hours. David took a flight from Frankfurt to Sydney and the flight duration is 22 hours and 40 minutes. Given that David took off from Frankfurt at 9:18 pm on 21st November 2016, calculate the arrival date and time in Sydney.
6. Jakarta is 6 hours ahead of Amsterdam. Vicky took an evening flight from Jakarta at 6:25 pm on 5th April 2017. Calculate the arrival date and time in Amsterdam given that the flight duration is 13 hours and 50 minutes.

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**CLASSWIZ WORKSHEETS**

1. Bali is 7 hours ahead of Switzerland. Given that the flight duration is 17 hours and 5 minutes, find the latest timing in Bali for a flight that Fernandez should take so that he can reach Switzerland on 12 midnight of 24 December 2017.
2. On the first map with a scale of 1 : 250 000, the length of a highway is 3 cm. Find the length of the highway on a second map with a scale of 1 : 80 000.
3. On the first map with a scale of 1 : 150 000, the distance between 2 towns are 8.2 cm. Find the distance on a second map with a scale of 1 : 20 000.
4. On the first map with a scale of 1 : 180 000, the area of a plantation is 3.25 cm2. Find the area of the plantation on a second map with a scale of 1 : 50 000.
5. On the first map with a scale of 1 : 80 000, the area of a field is 28.5 cm2. Find the area of the field on a second map with a scale of 1 : 200 000.
6. On the first map with a scale of 1 : 75 000, the area of a lake is 5.5 cm2. Find the area of the lake on a second map with a scale of 1 : 100 000.

Unit 2 **Further Numbers**

# Level 3

**CLASSWIZ WORKSHEETS**

1. Solve the following simultaneous equations. 32*x* ÷ 81*y* =

25*x* × =

1. Solve the following simultaneous equations

×= ,

73–*x* ÷ 492*y* + 1 = 2401.

1. Solve the following simultaneous equations.

÷=

*y* 2*x* + 1 63*y* 2(4*x*)

6 × 3 =

1. Kelly went to Australia for a holiday and bought a branded bag which cost $1839.45 in Australian dollar (AUD). Given that the exchange rate was USD1 = AUD1.28,
   1. calculate how much the branded bag costs in American dollar (USD).

America sells the same branded bag but it is cheaper in Australia by 12%.

* 1. Calculate the selling price of the branded bag in America, giving your answer in American dollar.

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**CLASSWIZ WORKSHEETS**

1. Mr Ong travelled to America and exchanged $3000 Singapore dollars (SGD) for American dollar (USD). The prevailing exchange rate was SGD1 = USD0.75.
   1. Calculate the total amount of American dollar he received.

For the whole trip, Mr Ong spent USD1292.71. He wanted to exchange the remaining amount for Singapore dollar at an exchange rate of SGD1 = USD0.79.

* 1. Calculate the total amount of Singapore dollar he received.

1. A small town has an area of 84 km2. It is represented by an area of 5.25 cm2 on Map A.
   1. Find the linear scale of Map A.
   2. A road is 11 km 470 m long. Calculate, in cm, the length of the road on Map A.

Given that the road is 1.85 cm on Map B,

* 1. Find the linear scale of Map B.
  2. Calculate the area of the small town on Map B, giving your answer to the nearest 3 significant figures.

1. A monument is built in the centre of a circular floor that has a radius of 2080 m.
   1. Given that the linear scale of Map X is 1 : 250 000, calculate the area of the circular floor on Map X.
   2. Given that the area scale of Map Y is 1 cm2 : 2.56 km2, calculate the diameter of the circular floor on Map Y.

Unit 2 **Further Numbers**

1. A cuboid has a rectangular base and a height of *x* cm. Given that the dimensions of rectangular base are (3 + 2) cm by (8 – ) cm and the volume of the cuboid is given by (141 + 75) cm3, find the 　value of x in the form of 𝑎 + *b*.

**CLASSWIZ WORKSHEETS**

1. A pyramid has a square base and a height of *x* cm. The length of one side of the square base is (1 +

3) cm. The volume of the pyramid is given as ( the value of x in the form of 𝑎

+ *b*.

1. A trapezium has parallel sides measuring (2 + 3) cm and (5 – ) cm. Given that the area of the.

trapezium is c, find the height of the trapezium in the form of 𝑎 + *b* .

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**CLASSWIZ WORKSHEETS**

# Level 4

1. Show that 3n + 1 + 3n + 2 + 3n + 4 is exactly divisible by 31.
2. Show that 42*x* + 1 + 16*x* + 1 – 24*x* + 3 is exactly divisible by 3.

3. Solve 9(32*x*) – 10(3*x*) + 1 = 0.

4. Solve 2 – 2*x* + 3 + 3(22*x* + 1) = 0.

Unit 2 **Further Numbers**

5.By rationalising each denominator, simplify

**CLASSWIZ WORKSHEETS**

* 1. + + .
  2. Hence, using the result observed in part (a), simplify

+ + + + … + .

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