**TEACHER’S GUIDE**

**Unit**

**11**

**PROBABILITY**

# KEY CONCEPTS

**Probability**

Probability of an event *E* happening = P(*E*) = $\frac{Number of ways in which event E can happen}{Total number of possible outcomes}$

For mutually exclusive events, i.e. it must be this or that, the probabilities are added.

P(*A* or *B*) = P(*A*) + P(*B*)

For independent events, i.e. one event has no effect on the probability of the other, the probabilities are multiplied.

P(*A* and *B*) = P(*A*)  P(*B*)

For dependent events, *A* and *B*, the probability of Event *B* depends on the outcome of Event *A*, should Event *A* occur before Event *B*. We multiply the probabilities of Events *A* and *B*.

# Permutations

A permutation of a given set of objects is an **arrangement** of some or all the objects in a defined order.

## Addition Principle

If 2 events are mutually exclusive and the first operation can be done in *n*1 ways and the second operation can be done in n2 ways, then one operation or the other can be done in (*n*1 + *n*2) ways.

## Multiplication Principle

If the first operation can be done in *n*1 ways and the second operation can be done in *n*2 ways, then the two operations can be done in (*n*1  *n*2) ways.

Both the Addition and Multiplication Principles can be extended to include any finite number of operations.

## The factorial

*n*! = *n*  (*n* – 1)  (*n* – 2)  (*n* – 3)  …  3  2  1

Permutation of *n* different objects taken all at a time = *n*! Permutation of *n* different objects taken *r* at a time = $\frac{n!}{\left(n - r\right)!}$

Permutation of *n* different objects where not all objects are distinct = $\frac{n!}{\left(n\_{1}!n\_{2}! …n\_{k}!\right)'}$

where there are *n*1 of the first kind, *n*2 of the second kind, and *nk* for *k* kinds of objects.

# Combinations

A combination is a **selection** of objects, where the order of selection is not of importance.

Combination of *n* objects, taken *r* at a time = $\left(\genfrac{}{}{0pt}{}{n}{r}\right)$ = $\frac{n!}{r!\left(n - r\right)!}$ = $\left(\genfrac{}{}{0pt}{}{n}{n - r}\right)$

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# WORKED EXAMPLES

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**Level 1**

## Worked Example 1

A four-sided fair die with faces 1, 2, 3, 4 and a six-sided fair die with faces 1, 2, 3, 4, 5, 6 were thrown together and the score on the die was recorded.

1. Complete the possibility diagram in the given axes.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | 1 | 2 | 3 | 4 |
| 1 |  |  |  |  |
| 2 |  |  | 2,3 |  |
| 3 |  |  |  |  |
| 4 |  | 4,2 |  |  |
| 5 |  |  |  |  |
| 6 |  |  |  | 6,4 |

1. Find the probability of getting the same score.
2. Find the probability of getting a score of 7.

Solution (a)

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | 1 | 2 | 3 | 4 |
| 1 | 1,1 | 1,2 | 1,3 | 1,4 |
| 2 | 2,1 | 2,2 | 2,3 | 2,4 |
| 3 | 3,1 | 3,2 | 3,3 | 3,4 |
| 4 | 4,1 | 4,2 | 4,3 | 4,4 |
| 5 | 5,1 | 5,2 | 5,3 | 5,4 |
| 6 | 6,1 | 6,2 | 6,3 | 6,4 |

1. P(getting the same score) = $\frac{4}{24}$ = $\frac{ 1 }{6}$

1. P(getting a score of 7) =$ \frac{4}{24} $= $\frac{ 1 }{6}$

## Worked Example 2

Two balls are to be chosen at random from 3 green balls, 2 red balls and 3 white balls, without replacement.

1. Find the probability that both balls chosen are green.
2. Find the probability that the first ball chosen is a red ball, followed by a white ball.

Solution

(a) $\frac{ 3 }{8}×\frac{ 2 }{7}=\frac{3}{28}$

(b) $\frac{ 2 }{8}×\frac{ 3 }{7}=\frac{3}{28}$

Unit 11 **Probability**

# Level 2

**TEACHER’S GUIDE**

## Worked Example 3

Evaluate the following

1. 7P4
2. 10C5
3. 7!

Solution

(a) 7P4 = 840

 **ClassWiz steps**

Press 7qO4= and the calculator will display 840.

(b) 10C5 = 252

 **ClassWiz steps**

Press 10qP5= and the calculator will display 252.

(c) 7! = 5040

 **ClassWiz steps**

Press 7%= and the calculator will display 5040.

## Worked Example 4

Find the number of permutations of the letters of the word ‘MISSISSIPPI’.

Solution

 **ClassWiz steps**

Press

11%a4%O4%O2%=

and the calculator will display 34 650.

$\frac{11!}{4!4!2!}$ = 34 650

## Worked Example 5

Find the number of permutations, taken 4 at a time, of the letters of the word ‘PROTEIN’.

Solution

$\frac{8!}{4!} $= 1680

## Worked Example 6

Find the number of ways to select 6 students from a team of 16 students.

Solution

 **ClassWiz steps**

Press 16qP6= and the calculator will display 8008.

16C6 = 8008

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# CLASSWIZ WORKSHEETS

**CLASSWIZ WORKSHEETS**

**Level 1 **

1. One student is to be chosen at random from 7 boys and 6 girls. Find the probability that the student chosen is a girl.
2. A letter is selected at random from each of the words ‘RICH’ and ‘RICE’.
	1. Represent the sample space using the possibility diagram given.

E C I R

I

R

C

H

* 1. Find the probability of choosing two letters that are
		1. the same.
		2. vowels.
1. James has one five-cent coin, one ten-cent coin and one fifty-cent coin in his pocket. One coin is taken out from his pocket at random and then placed back in the pocket. A second coin is then taken out at random.
	1. Complete the given tree diagram to represent the sample space.

5

5

10 10

50

50

* 1. Find the probability that the sum of the coins taken out is
		1. 60 cents.
		2. less than 20 cents.
		3. one dollar.

Unit 11 **Probability**

1. Box P contains 1 red ball and 2 white balls. Box Q contains 2 red balls and 1 white ball. A ball is taken out at random from each box and its colour is noted.

**CLASSWIZ WORKSHEETS**

* 1. Complete the tree diagram to show the sample space.

Box P Box Q R

R R

W

R

W

* 1. Find the probability of getting
		1. all red balls.
		2. only one white ball.
		3. all white balls.
1. A six-sided fair die with faces 1, 2, 3, 4, 5, 6 and a four-sided fair die with faces 1, 2, 3, 4 were thrown together. The sums of the numbers obtained from the thrown dice were recorded in the possibility diagram below.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | 1 | 2 | 3 | 4 | 5 | 6 |
| 1 |  |  |  |  |  |  |
| 2 |  |  | 5 |  |  |  |
| 3 |  |  |  |  |  | 9 |
| 4 |  |  |  | 8 |  |  |

* 1. Complete the possibility diagram.
	2. By using the diagram, find the probability that the sum of the 2 numbers is
		1. even.
		2. odd.
		3. less than 7.
		4. divisible by 3.

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1. Angel has two red pants, one black pants and two blue pants in her closet. She picks up one pants at random from the closet. Find the probability that pants picked is

**CLASSWIZ WORKSHEETS**

* 1. red.
	2. blue or black.
	3. red or blue.
1. A bag contains 7 pink balls, 8 yellow balls and 4 blue balls. A ball is drawn at random from the bag.

What is the probability that the ball drawn is

* 1. pink?
	2. blue?
	3. pink or yellow?
	4. yellow or blue?
1. Jim, Kelly and Lionel take part in a competition. The probabilities of Jim, Kelly and Lionel winning

the competition are $\frac{1}{9}$ , $\frac{5}{17}$ and $\frac{1}{7}$ respectively. Assuming that there can be only one winner, find the

probability that

* 1. either Jim or Kelly wins.
	2. either Jim, Kelly or Lionel wins.
	3. Jim, Kelly or Lionel all do not win.
1. A box contains some marbles of different colours. A marble is picked at random from the box. The probabilities of picking a green marble, an orange marble and a yellow marble are $\frac{5}{11}$, $\frac{4}{33}$ and $\frac{1}{6}$

respectively. Find the probability that

* 1. the picked marble is either green or orange.
	2. the picked marble is either yellow or orange.
	3. the picked marble is neither yellow nor orange.
	4. the picked marble is neither green nor orange nor yellow.
	5. If there are 30 green marbles in the bowl, find the total number of marbles.

Unit 11 **Probability**

1. Two pens are to be chosen at random from 3 blue pens, 2 red pens and 3 black pens, without replacement. Find the probability that both pens chosen are black.

**CLASSWIZ WORKSHEETS**

|  |  |  |  |
| --- | --- | --- | --- |
|  |  |  | 1 |
| 2 | 2 |  | 1 2 |
| 3 | 1 |  | 4 3 |
|  |  |  | 4 |

The diagram shows two spinners. For each spinner, the pointer is equally likely to point to each number on it after spinning. Let *A* be the event that the sum of the numbers of two spinners is 5 and B be the event that at least one of the numbers shown is 1.

* 1. Find P(A).
	2. Find P(B).
	3. Find P(A and B).
1. A person throws a fair six-sided die and a fair coin together. Find the probability that
	1. an odd number and a tail are obtained.
	2. a prime number and a head are obtained.
	3. a number divisible by 3 and a tail are obtained.
2. The probability that Jonathan passes his Geography test is 0.3 and the probability that he passes his Mathematics test is 0.7. Calculate the probability that Jonathan
	1. passes both subjects.
	2. fails both subjects.
	3. passes at least one subject.

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1. A carton contains 30 apples. It was found that there were 7 rotten apples and 23 good ones in the carton. Two apples are chosen from the carton at random, without replacement.

**CLASSWIZ WORKSHEETS**

* 1. The probability tree diagram shows the possible outcomes and their corresponding probabilities. Complete the tree diagram.

First apple Second apple Good

Good

$$\frac{7}{30}$$

Rotten

$$\frac{6}{29}$$

Rotten Good

Rotten

* 1. Find the probability that
		1. the first apple is rotten and the second apple is good.
		2. at least 1 apple is rotten.
		3. only 1 apple is rotten.
1. In a club, there are 28 students. 13 are girls and 15 are boys. Two students are selected at random, one after another for the positions for President and Vice-President.
	1. Draw a probability tree diagram to show the possible outcomes and the corresponding probabilities.
	2. Find the probability that
		1. the President is a boy and the Vice-president is a girl.
		2. the President is a girl and the Vice-president is a boy.
		3. at least one girl is selected.

Unit 11 **Probability**

1. Bag A contains 5 red balls and 7 black balls. Bag B contains 6 red balls and 4 black balls. A ball is drawn at random from each bag. Find the probability that

**CLASSWIZ WORKSHEETS**

* 1. both balls are red.
	2. at least one ball is black.
	3. the two balls are of different colour.
1. In a bread-making factory, Machine A produces $\frac{1}{2}$ of the bread production, Machine B produces $\frac{ 1 }{5}$ of

the bread production and Machine C produces the rest. Two loaves of bread are selected at random.

Find the probability that

* 1. both loaves of bread are produced by Machine C.
	2. one loaf of bread is produced by Machine A and another loaf is produced by Machine B.
	3. both loaves of bread are not produced by Machine B.
1. Two six-sided fair dice are thrown and the numbers on each die are recorded. Find the probability that
	1. both dice show the same number.
	2. both numbers are even.
	3. the sum of the numbers is 7.
	4. the sum of the numbers is divisible by 5.

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1. A four-sided fair die with faces 1, 2, 3 and 4 and a five-sided fair die with faces 1, 2, 3, 4 and 5 were thrown together and the number on each die was recorded. Find the probability that

**CLASSWIZ WORKSHEETS**

* 1. the same number was obtained in both throws.
	2. the numbers obtained in both throws were even.
	3. the sum of the numbers is 8.
1. A box contains 6 blue balls and 8 red balls. A ball is drawn from the box at random. The ball is replaced if it is a blue ball and removed if it is a red ball. A second ball is then drawn from the box at random and not replaced.
	1. Draw a probability tree diagram to show the possible outcomes and its corresponding probabilities.
	2. Find the probability that
		1. both balls are red.
		2. the two balls are of different colours.
	3. A third ball is drawn at random following the same rules. Find the probability that all three balls are blue.

Unit 11 **Probability**

# Level 2

**CLASSWIZ WORKSHEETS**

1. Evaluate the following permutations.

 **ClassWiz set-up**

Use the ClassWiz calculator function by pressing

qO for the Permutation function.

(a) 7P3 (b) 8P5

(c) 8P3 (d) 11P10

1. Evaluate the following combinations.

(a) 4C3 (b) 9C4

(c) 9C5 (d) 7C0

1. Evaluate the following factorials.

(a) 6! (b) 8!

 **ClassWiz set-up**

Use the ClassWiz calculator function by pressing

q/ for the Factorial, %, function.

(c) 5! (d) 10!

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1. Find the number of permutations of

**CLASSWIZ WORKSHEETS**

* 1. 9 different posters.
	2. the letters of the word ‘KINDLY’.
1. Given that there are 11 airlines operating between Singapore and San Francisco, find the total number of ways that a traveler can fly from Singapore to San Francisco and return on a different airline.
2. Find the number of permutations, taken 5 at a time, of the letters of the word
	1. ‘MATRICES’.
	2. ‘OBJECT’.
	3. ‘MICROBAND’.
3. Find the number of ways to arrange 8 different photographs into 4 empty spaces in a photo album.
4. Find the number of permutations of the letters of the following words.
	1. MATHEMATICS
	2. ARRANGING
	3. INDEPENDENT

Unit 11 **Probability**

1. Find the number of ways to arrange 3 black balls, 2 red balls and 1 green ball in a row.

**CLASSWIZ WORKSHEETS**

As this involves permutation of objects that are not all distinct AND taken 5 at a time, remember to consider the scenarios carefully.

As this involves permutation of objects that are not all distinct AND taken 5 at a time, remember to consider the scenarios carefully.

1. Find the number of ways to arrange 3 identical black balls and 4 identical red balls into 5 empty spaces.
2. Find the number of ways to arrange the 5 identical dresses and 3 identical trousers on to a rack that can hold 4 pieces of clothing.
3. There are 12 clothing items on sale. Find the number of ways to select 3 clothing items.
4. Find the number of ways to select 5 musicians from a team of 13 musicians.
5. Find the number of ways to select 2 green balls, 4 orange balls and 3 pink balls from a bag containing 6 green balls, 7 orange balls and 6 pink balls. Next, find the number of ways to arrange it.

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# Level 3

**CLASSWIZ WORKSHEETS**

1. Find the number of ways to arrange the letters of the word ‘FANTASTIC’, if
	1. the two A’s and the two T’s are not to be separated.
	2. the two A’s and the two T’s are to be separated.
2. Find the number of ways to arrange the letters of the word ‘EXCELLENT’ given that
	1. it must start with L.
	2. it must start with E.
3. Find the total number of ways to form numbers larger than the value of 2800 from the digits 0, 1, 2, 3 and 4, given that no digit can be used more than once.
4. Find the number of ways to select 6 people from 6 married couples given that
	1. the selection must have more women than men.
	2. the selection cannot have a pair of husband and wife.

Unit 11 **Probability**

1. Show that $\left(\genfrac{}{}{0pt}{}{n + 1}{r}\right)$ – $\left(\genfrac{}{}{0pt}{}{n}{r}\right)$ = $\left(\genfrac{}{}{0pt}{}{n}{r - 1}\right)$.

**CLASSWIZ WORKSHEETS**