### Numerical patterns

**ALGEBRA**

02

**Finite sums**

Being still a child, the German mathematician Johann Carl Friedrich Gauss (1777 - 1855) was able to solve in a few seconds the challenge posed by his Mathematics teacher:

«What is the value of the sum of the first 100 natural numbers?»

Gauss observed that the first natural number, 1, and the last of the list, 100, add the same as the second number, 2, and the second last, 99, and so on, so he concluded that the sum of the first 100 numbers is:

2

The following figure shows graphically this result for the first 12 natural numbers.

(1 + 100) *·* 100 = 5 050

1

2

3

4

5

6

7

8

9

10

11

12

6

5

4

3

2

1

1

2

3

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9 10 11 12

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In the figure above it is observed that the sum of the first 12 natural numbers is 13 x 12 = 78.



**1**

2

**a)** Calculate 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 11 + 12 + 13.

* 1. Calculate the sum of the first 500 natural numbers.
  2. Use the finite sums function of the calculator to calculate the sums of the previous sections.
  3. Generalize the results obtained.

Find the following sums:



**2**

1. Sum of the first 500 even numbers.
2. Sum of the first 100 multiples of 3.
3. Sum of the first 100 perfect squares.
4. Sum of the first 50 perfect cubes.

**e)** 8 + 9 + 10 + ..... + 100 =.

**f)** 1 – 2 + 3 – 4 + 5 – 6 + ..... + 99 – 100 =.

**g)** – 4 + 5 – 6 + ..... + 99 – 100 + 101 =.

1. Sum of the first 20 terms of the succession 2, 4, 8, 16, 32, 64, 128…

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