# Activity 5: The Need to Sum

####  CONCEPTUAL CATEGORY: ALGEBRA

**DOMAIN: SEEING STRUCTURE IN EXPRESSIONS**

**Write expressions in equivalent forms to solve problems.**

1. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
2. Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. *For example, calculate mortgage payments.*

#### LEARNING OBJECTIVES

Students will be able to write and rewrite expressions. Students should understand *what* these different expressions can tell us about the quantities they represent. Students will understand a geometric sequence.

#### Use properties of operations to generate equivalent expressions.

1. Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.
2. Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. For example, ɑ + 0.05ɑ = 1.05ɑ means that “increase by 5%” is the same as “multiply by 1.05.”

#### Activity 5: Getting Started

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#### GETTING STARTED

In mathematics, a **sequence** is a bunch of numbers listed one after the other. In many cases, it is possible to determine a particular member of a sequence simply from its location. Some

sequences are determined by adding (arithmetic) a real number to each subsequent term while others are determined by multiplying (geometric) a real number by each successive term.

Geometric series appear in many places, but one case which touches just about everyone is in the area of finance and economics. Calculations involving loan payments (for a mortgage, for instance) often make use of geometric series.

Suppose you took out a $200,000 loan for a unicorn. It sounds expensive, but it's worth it if you're a frequent rider. Plus, it's a *unicorn*. The fantastical $200,000 loan has a 30-year fixed interest rate of 3.6% per year paid monthly. By convention, this means that each month, an interest rate of 0.036 ÷ 12 = 0.003, or 0.3% will be applied to the unpaid principal to calculate the interest payment for that month. This will continue for each month until the term of 30 years has been reached, which is (12)(30) = 360 months. If *m* is the monthly amount, *P* is the principal amount on the loan, *r* is the rate each month, and *n* is the number of months, we can use the following formula to find the monthly amount.

$$m=\frac{Pr\left(1+r\right)^{n}}{\left(1+r\right)^{n}-1}$$

There are calculators online which will do this sort of calculation for you, but it is nice to under- stand how the calculation is done so you can verify it for yourself. As far as unicorns go, nobody wants to pay more than they have to. On second thought, that probably applies to just about anything.

1. Determine the monthly payment for your unicorn.

#### UNDERSTAND

1. Carlos reads an advertisement from a bank that offers a 2.9% APR (Annual Percentage Rate (APR)) savings account compounded monthly. Over 10 years, how does this account compare to Destiny’s 3% APR account that is compounded annually, if they both have $600 to invest?
2. Which is better to have, an account at 6% APR compounded annually, or an account at 5.9% APR compounded monthly?
3. A source for large prime numbers are the so-called “Mersenne Primes.” These prime numbers are in the form 2*n* – 1. For example, 23 – 1 = 7, so 7 is a Mersenne prime. Not all numbers of this form are prime. For example, 24 – 1 = 15, but 15 is not prime.

Make a table of output values for 2*n* – 1. Is there any pattern to the values of *n* that produce prime numbers and those that do not? Explain.

#### Activity 5: Getting Started

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#### PRACTICE

1. Write at least four pairs of sequences. A pair will consist of an arithmetic sequence *ɑ* and ageometric sequence *g* with the same initial term and the common difference of *ɑ* equal to the common ratio of *g*.

For example,

*ɑ*1= 2, 5, 8, 11, 14…

*g*1 =2, 6, 18, 54, 162…

Both sequences have the initial term 2. The common difference for the arithmetic sequence *a* is 3. The common ratio for the geometric sequence *g* is 3.

Compare the behavior of a sequence in each pair. Do both sequences increase or decrease? [Be certain to have first term, common differences, and common ratios with various signs and values.]

1. Karen took out a loan for a car that cost $11,000. She paid $1,000 down and got a 4% interest rate.
	1. If the term of the loan is 36 months, what is the monthly payment?
	2. If instead Karen pays $500 per month, how long will it take her to pay of the car?
2. An arithmetic series is the sum of an arithmetic sequence. Think about what series might represent 0.2222.
	1. What is the initial term, *a*?
	2. What is the common ratio, *r*?
	3. What is the number of terms, *n*?
	4. Find the fraction equivalent to 0.2222.

#### EXTEND

1. Draw a square. Connect the midpoints of the sides of this square to form a second, smaller square. Connect the midpoints of the sides of the second square to form the next one. You can continue to make more squares in this manner.
	1. If the side length of the first square you drew is 1, what is the side length of the second square?
	2. Do these lengths form a geometric sequence? If so, what are the first term and common ratio?
	3. If the area of the first square is 1, what is the area of the second square?
	4. Do these areas form a geometric sequence? If so, what are the first term and common ratio? If not, what kind of sequence do they form?

**Activity 5: Getting Started**

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1. Show that each statement is true.

a. $x^{3}+ 1 = \left(x + 1\right)\left(x^{2} – x + 1\right)$

b. $x^{5}+ 1 =(x + 1)(x^{4} – x^{3} + x^{2} – x + 1)$

c. $x^{7}+ 1 =(x + 1)(x^{6} – x^{5} + x^{4} – x^{3}+ x^{2} – x + 1)$

d. $x^{9}+ 1 =(x + 1)(x^{8} – x^{7} + x^{6} – x^{5}+ x^{4} – x^{3}+ x^{2} – x + 1)$

1. Suppose that k is an odd integer greater than 1.

Prove that $2^{k} + 1$ is divisible by 3 and is not a prime number.