# Activity 3: Dimensional Analysis



### CONCEPTUAL CATEGORY: NUMBER & QUANTITY

### DOMAIN: QUANTITIES

**Reason quantitatively and use units to solve problems.**

1. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
2. Define appropriate quantities for the purpose of descriptive modeling.
3. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

### LEARNING OBJECTIVES

Students will be able to use units as a tool to help solve problems. Students will be able to define appropriate quantities to describe the characteristics of interest.

Students will also understand that the tool used to measure a quantity determines the level of accuracy that can be reported for the measurement.

### Convert like measurement units within a given measurement system.

* 1. Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real world problems.

### Apply and extend previous understandings of operations with fractions.

1. Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.

### Activity 3: Getting Started

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### GETTING STARTED

When we measure something, we always have to specify what units we are using to measuring the quantity. For instance, if I tell you I am 2 tall, you haven't learned much about my height: your response may be "2 what?" If I respond 2 meters, you know that I am a tall person; on the other hand, if I tell you I am 3 meters, you can call the record books.

Some units are made up by a combination of other units: for instance, you might measure the speed of a car in kilometers/hour. So "90 kilometers/hour" (or 90 km/h) means for every hour, the car travels 90 kilometers. This is quite different from a car travelling 90 kilometers/second. No matter what you are measuring, you must specify the units in order to give a measurement.

When solving problems, dimensional analysis can help you determine what to multiply or divide. Suppose that I want to convert 90 kilometers/hour into millimeters/second. I know that 1 kilometer is 1000 meters, that 1 meter is 1000 millimeters, that 1 hour is 60 minutes, and that 1 minute is 60 seconds. Now what? This can be a very tricky question, but using dimensional analysis, it becomes a snap. First of all, I write all the above conversions as fractions, remembering that I might have to turn some of them upside-down.

$$\frac{1 kilometer}{1000 meters}, \frac{1 meter}{1000 millimeter}, \frac{1 hour}{60 minutes}, \frac{1 minute}{60 seconds}$$

Now I write an expression to convert from miles/hour to meters/second, watching the units to make sure they cancel.

$$\frac{90 km}{1 hour}∙\frac{1000 m}{1 km}∙\frac{1000 mm}{1 m}∙\frac{1 hour}{60 minutes}∙\frac{1 minute}{60 seconds}=\frac{90∙1000∙1000 mm}{60∙60 s}=2500 mm/s$$

Use dimensional analysis to assist in solving the following problem: Lewis estimates that the faucet in his bathroom sink drips at the rate of 1 drop for every 3 seconds.

1. Estimate how many times the faucet drips in a week.
2. Lewis determined that there are 592 drops in 29.6 mL. Estimate how many liters of water his leaky bathroom faucet wastes in a year.

### UNDERSTAND

1. Define appropriate quantities for the following situations:
	1. What type of measurement could be used to determine a person’s income or expenses for one month?
	2. How could the number of accidents in the U.S.A be described?
	3. What quantities could be used to describe the best city in Italy?
	4. What quantities could be used to compare two tennis players?

**Activity 3: Getting Started**

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1. Sodium Fluoride:

Fluoride in various forms is the most popular active ingredient in toothpaste to prevent cavities. Fluoride occurs in small amounts in plants, animals, and some natural water sources. The additional fluoride in toothpaste has beneficial effects on the formation of dental enamel and strengthening bones. Sodium fluoride (NaF) is the most common source of fluoride, but stannous fluoride (SnF2), olaflur (an organic salt of fluoride), and sodium monofluorophosphate (Na2 PO3F) are also used.

Unfortunately Sodium Fluoride has a downside. It is poisonous. Merriam-Webster Dictionary defines Sodium Fluoride as “a poisonous crystalline salt that is used in trace amounts in the fluoridation of drinking water and toothpastes, in metallurgy, as a flux, and as a pesticide.” The lethal dose for a 70 kg. human is estimated at 7 g. Sodium fluoride is classed as toxic by both inhalation (of dusts or aerosols) and ingestion. In high enough doses, it has been shown to affect the heart and circulatory system.

Five popular brands of toothpaste from America were examined for the amount of Fluoride contained in a tube. The accompanying table gives information about the concentration of active ingredient in each tube, the size of each tube, and the price of each tube of toothpaste.

|  |  |  |  |
| --- | --- | --- | --- |
| **Name** | **Concentration** | **Tube Size** | **Price** |
| Crest | 0.243% |  190 g. | $2.45 |
| Sensodyne | 0.15% | 118 g. | $5.89 |
| Rembrandt | 0.243% | 89 g. | $8.95 |
| Aquafresh | 0.15% | 190 g. | $2.55 |
| Colgate | 0.24% | 177 g. | $3.49 |

* 1. Make a guess about which brand of toothpaste has the most Fluoride. How did you arrive at your guess? Verify your guess.
	2. Create an ordered list of brands by Fluoride content.
	3. Which brand charges the most based on Fluoride content?
	4. How many tubes of Crest would have to be ingested in order for the amount of Fluoride to be lethal for a 70 kg. human?

**Activity 3: Getting Started**

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1. These nonsense words where taken from the poem Jabberwocky (from Lewis Carroll’s

Through the Looking Glass) to reinforce the concept that some problems can be set up and solved based on units.

There are 20 tumtum trees in the tugley wood.

In each tugley wood is one frumious Bandersnatch.

There are 5 slithy toves in 2 borogroves.

There are 2 mome raths per Jabberwock.

There are 2 Jubjub birds in 200 tumtum trees. There are 200 mome raths in each borogrove. There are 5 Jubjub birds per slithy tove.

Therefore, if there are 5 frumious Bandersnatches, how many Jabberwocks are there?

### PRACTICE

1. Use dimensional analysis to solve the following problems. Show all steps needed to convert from starting units to ending units. Indicate all relationships needed before setting up and solving the problem.
	1. A runner competed in a 5k (kilometer) walk. It takes approximately 131 steps to walk 1 meter. How many steps did she take?
	2. If you go to school for 180 days each year and each day is 7 hours long, how many hours are spent in school in one year?
	3. In the United States, cigarettes are smoked at a rate of 20,000 cigarettes/second. At this rate, how many are smoked in one day? If the population of the United States is 300 million, how many cigarettes are smoked per day per person?

### EXTEND

1. A team of farm-workers was assigned the task of harvesting two fields, one twice the size of the other. They worked for the first half of the day on the larger field. Then the team split into two groups of equal number. The first group continued working in the larger field and finished it by evening. The second group harvested the smaller field, but did not finish by evening. The next day one farm-worker finished the smaller field in a single day's work. Assuming that all farm-workers work at the same rate, how many farm-workers were on the team?