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Investigating Mathematics with ClassWiz

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CLASSWIZ

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Notes for Teachers

This set of mathematical investigations is intended to illustrate some possible ways in which students might use *ClassWiz* productively for learning purposes. Each investigation is related to some Activity Book modules. The investigations have a number of common features:

- Each investigation occupies a single page, so that they might be easily copied and used in a class with students.
- The purpose of the investigations is generally to support the development of student understanding of mathematical ideas, so it is important for sufficient time to be allocated for students to engage with the tasks, discuss their understanding with a partner and explore the ideas a little more for themselves. When ideas are new to students, it is important for them to have sufficient time to think about them. If students are used to thinking of a worksheet as an invitation to quickly complete some set tasks and then check 'the answer', they may need help in realising that a different approach is needed here. Depth of thinking is more important than speed of thinking for most of these investigations.
- Many (but not all) of the investigations are intended to help with the introduction of a new idea, or with deepening understanding of a familiar idea. It is expected that different teachers might use them in different ways with different classes.
- They are generally a little more 'open-ended' than typical school tasks, in the sense that students are invited to undertake some investigations of their own choosing, and it is expected that students will sometimes use the tasks as a launching pad for further investigations. Students are given some responsibility to choose tasks by themselves, rather than merely completing tasks set by the teacher or the textbook.
- They are intended to illustrate student-centred learning, at least to some extent. The focus is on the students' own activity, and the investigations explicitly ask students to think about, talk about and often to write about what they have observed with the *ClassWiz*, in order to increase their understanding of some mathematical ideas.
- The investigations are written on the assumption that students are not working by themselves, but have at least one partner undertaking the investigation at the same time. So there are often explicit requests to engage with other people, so that learning mathematics is not a solitary activity and discussion is recognised as educationally valuable. (One way to encourage such joint activity is to provide a copy of the investigation to each pair of students, rather than to each individual student, requiring them to work collaboratively to some extent.)
- In most cases, a whole-class discussion after an investigation has been attempted by students will also be productive. If available, a *ClassWiz* emulator will be useful to support the discussions.
- Each investigation has more than one activity, generally to highlight a different aspect of the mathematics involved. In general, it is assumed that students will undertake these activities in the sequence given.
- Investigations are written to be generally self-sufficient, so that students will not need extensive knowledge of how to use the *ClassWiz* before undertaking the task. Teachers will need to decide for their own class whether advice on how to access particular keyboard or menu features is needed, however.

The brief notes accompanying each particular Investigation highlight key features and offer some advice on classroom use.

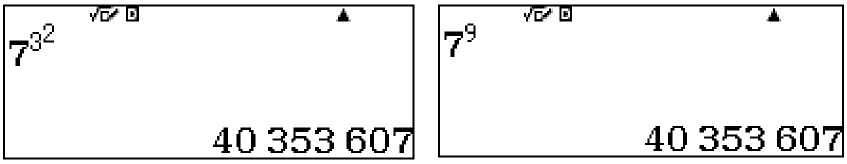
1. Investigating powers

The index laws are the key feature of this investigation, and it is assumed that students are not yet familiar with these. The FACT command of *ClassWiz* is critical to resolve numbers into prime factors and thus to reveal some properties of indices. Make sure that the students know that the command is used only *after* a result is obtained. Students can undertake this investigation before getting detailed advice on using the power key $\boxed{x^{\blacksquare}}$ or the FACT command, but some students might need a little help with these calculator operations in that case.

It is important that students have their *ClassWiz* set to use Math mode, through the use of the SET UP menu, so that powers are represented appropriately. Choose *Input/Output* setting to be *MathI/MathO*.

The examples shown in Activity 1 use prime number bases, but students will quickly encounter 'strange' results if they use a composite number, following the request to try some more examples for themselves. The second screen of Activity 2 deliberately provokes this, in case it hasn't occurred naturally. Examples of that kind give rise to opportunities to learn more about how the index laws work, revealing that $(ab)^n = a^n b^n$ and that $a^m \div a^n = a^{m-n}$.

Students might also be encouraged to investigate the interpretation by the calculator of repeated use of the exponent key, $\boxed{x^{\blacksquare}}$, when parentheses are not used:



As shown above, they can see for themselves that the result is interpreted as 7^9 (since $3^2 = 9$), which will make it clear why parentheses were used on the printed sheet to obtain $(7^3)^2$.

Make sure that students use the writing space to record their observations, both to help them to verbalise what they have concluded, but also to serve as a basis for discussion with a partner or with the whole class.

This investigation might also be used just before students study exponential functions.

2. Investigating fractions and decimals

This investigation is concerned with the relationships between fractions and decimals, and makes extensive use of both the natural display features of *ClassWiz* and the $\boxed{S\Leftrightarrow D}$ key for converting between standard and decimal formats.

It is important that students have their *ClassWiz* set to use Math mode, through the use of the SET UP menu. Choose *Input/Output* setting to be *MathI/MathO*. This will ensure that both fractions and decimals are displayed as simplified fractions in the conventional way.

The intention is for students to appreciate that fractions are single numbers (not pairs of numbers), and that many different fractions have the same numerical value, which allows them to be placed at a distinct point on the real number line. It is a valuable exercise for students to actually plot the fractions on a number line on paper. Students may at first need some help in entering fractional numbers into their calculator, but this will be quickly mastered (in Math mode … it is sometimes a little more difficult in other modes).

ClassWiz automatically simplifies fractions to have a denominator as small as possible, so that 4/10 is represented as 2/5, but are the same number, since each is a fractional representation of 0.4.

Make sure that students use the writing space to record their observations, both to help them to verbalise what they have concluded, but also to serve as a basis for discussion with a partner or with the whole class.

Activity 3 involves both the important ideas of proper and improper fractions as well as the critical idea that integer divisions can be represented as fractions. Neither the entry of mixed fractions (with SHIFT (■⊟)) nor the use of percentages (with SHIFT Ans) are treated in this investigation, but could easily be included if desired, to extend the investigation.

3. Investigating functions

The focus of this investigation is the patterns related to various functions. These patterns are readily discernible from a table (and of course are visually illustrated well in the case of a graph). The investigation assumes that students are already familiar with the use of a table on *ClassWiz*, including the use of x for the independent variable. Make sure that they know how to enter x.

Make sure that students use the writing spaces to record their observations, both to help them to verbalise what they have concluded, but also to serve as a basis for discussion with a partner or with the whole class.

Activities 1 and 2 involve linear functions, for which students should be able to see the pattern of steady increases (or decreases) between values of the function, which illuminate the idea of the slope of the function. Notice that the last two examples in Activity 2 involve a Step of 2 (rather than a Step of 1), requiring students to consider carefully what is happening. The functions in Activity 2 are respectively $f(x) = 3x + 5$, $f(x) = 51 - 4x$, $f(x) = 4x + 11$, $g(x) = 9x + 2$.

You might find it convenient to provide students with some grid paper and to suggest that they draw graphs of functions, especially if they have not done so extensively before. This will highlight the linear nature of the functions still further.

The quadratic functions in Activity 3 have been chosen to contrast with the linear functions in Activities 1 and 2, as they do not have consistent slopes. Students will recognise the first function as $f(x) = x^2 + 1$, but are likely to find it harder to identify the second function, $g(x) = 51 - x^2$ and the third function, $f(x) = x(x + 1)$, so encourage them to persist rather than telling them the answer. Some might even identify the third function as $f(x) = x^2 + x$, providing an opportunity for a class discussion to realise that $x(x + 1)$ is equivalent to $x^2 + x$. Once again, it may be fruitful to provide students with grid paper to sketch the functions concerned.

If desired, similar sorts of investigations could be undertaken at a later stage with other functions, such as exponential functions, logarithmic functions and reciprocal functions.

4. Investigating trigonometry

This investigation requires students to use Table mode and to have their *ClassWiz* set to degrees.

Activity 1 focuses on the relationship for complementary angles that $\sin x = \cos (90^{\circ} - x)$, through identifying the appearance of the same numbers in tables for sine and cosine. It is expected that the investigation will also reinforce for students how the ratios of sine and cosine vary over the first quadrant, with sine increasing from 0 to 1, while cosine decreases from 1 to 0.

Make sure that students use the writing space to record their observations, both to help them to verbalise what they have concluded, but also to serve as a basis for discussion with a partner or with the whole class.

Some students might consider checking their relationship by tabulating at the same time both sin x and cos $(90^{o} - x)$, or cos x and sin $(90^{o} - x)$, which will display the identities in a convincing way.

This idea might be extended to supplementary angles, if desired (although note that a table step of 5^{o} will generate more table values than *ClassWiz* permits).

Activity 2 is again illustrative of a range of possibilities, focusing on the idea of an identity: that two quantities are the same, regardless of the value of the variable. A table of values shows this kind of relationship very well, and can be further used by changing one of the x-values in the table. This mechanism is a supplement to the idea of a formal symbolic proof of the identity.

As a further example, the relationship in Activity 1 can also be studied in this way as an identity, with $f(x) = \sin x$ and $g(x) = \cos (90^{o} - x)$.

If a graph facility is available, graphing both 'sides' of an identity is instructive, too.

More sophisticated students can undertake investigations of this kind using radians rather than degrees.

5. Investigating logarithms

The first two activities in this investigation are concerned with developing a strong understanding of the idea of a logarithm: that it is the power to which a base must be raised to get a particular number. This idea is often emphasised in textbooks; the idea here is for students to experiment personally with it. Once students have a good grasp of this idea, many other aspects of logarithms become more accessible.

It is expected that students will try a variety of bases in each of Activities 1 and 2, so that the fundamental idea of logarithms is not restricted to one particular base (such as base 10, for common logarithms).

Make sure that students use the writing space to record their observations, both to help them to verbalise what they have concluded, but also to serve as a basis for discussion with a partner or with the whole class.

Activity 3 is concerned with the helpful property of logarithms that the logarithm of a product is the sum of their respective logarithms. This is the property that made logarithms so useful for computational purposes for hundreds of years. Notice that the idea of an 'antilogarithm', for which tables have been provided in the past, is replaced on the calculator by a process of raising the base to a power. ClassWiz has special commands for this purpose for the cases of common logarithms (base 10) and natural logarithms (base e).

Activity 3 invites students to explore other logarithmic properties (such as the logarithm of a power); they might also explore the logarithm of a quotient or of a root, too.

More sophisticated students can undertake investigations of this kind using natural logarithms.

6. Investigating data

This investigation is concerned with understanding the use of univariate statistics to summarise a set of data. It assumes that students have already learned how to use the Statistics mode of their *ClassWiz* with univariate data, and thus is appropriate for use after an introduction to the topic.

Activity 1 is concerned with transforming a data set additively, a form of translation, which has the same effect on the mean and the five-number summary data (minimum, Q_1, median, Q_3 and maximum) , but which has *no* effect on measures of spread. (To enter the adjusted data, students should be able to start with the initial data given, and then edit each value in turn, replacing it by the next higher value). Students should be able to readily compare their results with the printed results on the Investigation sheet.

Activity 2 is concerned with a multiplicative transformation, in which each data point is multiplied (by 100 in the first instance). This scaling will affect both the mean and the standard deviation (multiplying each by the same factor), but will affect the variance by the *square* of the multiplier. This is one of the reasons that the standard deviation is often preferred as a measure of spread over the variance – it is scaled in the same way as the original data.

Make sure that students use the writing spaces to record their observations, both to help them to verbalise what they have concluded, but also to serve as a basis for discussion with a partner or with the whole class.

A later investigation might examine the effects of a transformation that involves *both* a translation and a scaling – a good example is a set of temperatures in the Celsius scale that is converted to a set of temperatures in the Fahrenheit scale by the transformation $F = 1.8C + 32$.

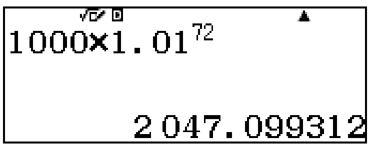
7. Investigating models

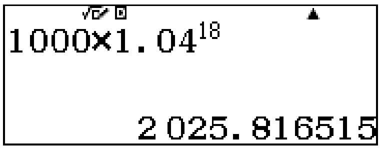
This investigation is concerned with modelling population growth, in several different ways. It is set in the context of Australia, to illustrate the ideas concerned, but it is expected that at some stage students will use the same techniques to study similar questions for their own country. In that case, they may need some help regarding how to locate suitable data for their own country.

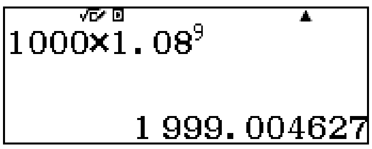
Activities 1 and 2 involve the use of exponential functions, while Activity 3 uses the bivariate statistics capabilities of *ClassWiz*. It is assumed that students already have some familiarity with the bivariate statistics capabilities, so you will need to monitor their calculator work if this is not the case and provide help where appropriate.

Assuming a constant growth rate of 1.4% per annum, the population of Australia is estimated as 24 234 297 in 2016 and 38 879 520 in 2050. With the same assumption, the population is estimated to reach 36 million late in 2044. Encourage students to use multiplication by 1.014 to increase a value by 1.4%.

Students will find that the Rule of 72 gives quite a good approximation, especially if the growth rate of r% per annum is relatively small (as real annual growth rates typically are). The following screens give examples of this, showing $r = 1$, $r = 4$ and $r = 8$. In each case, the original quantity of \$1000 is almost doubled in $72/r$ years:







Activity 3 adopts an approach of imagining someone was trying to predict population growth many years ago (in the 1960's). This allows us to determine the extent to which various methods would have been in fact successful, with the benefit of hindsight. The example shown for prediction uses an exponential model, as we know that population growth is often approximately exponential in character. If students use a linear model, they will see that the predictions made in the 1960's are quite inadequate now. (For example, an Australian population of 15 556 820, much lower than the actual population, would have been predicted for 2016.) Depending on their experience with Statistics mode, students might need help in both changing models and using models to predict.

Make sure that students use the writing spaces to record their observations, both to help them to verbalise what they have concluded, but also to serve as a basis for discussion with a partner or with the whole class.

Make sure that students understand that the models studied are simplistic to an extent: each assumes that population growth rates are static over time, when clearly this is not the case. However, a mathematical model provides a convenient way of estimating the future and exploring the consequences of hypothetical scenarios; the results must always be interpreted in the light of the assumptions involved. More sophisticated models are available at a later stage, but the basic ideas can be explored profitably with these relatively simplistic approaches.

8. Investigating probability

This investigation makes use of the two random number functions on the *ClassWiz*, *Ran#* and *RanInt*, and also requires students to be comfortable with the use of tables. It is assumed that students already have some familiarity with these. The essential point of the investigations is to see in practice how well theoretical ideas related to probability and chance apply in practice, using various kinds of simulated data.

By the very nature of randomness, results are unpredictable to some extent. Accordingly, in all three activities, it is very important for students to compare their answers with others and it is essential for the results of the whole class to be discussed in some suitable way, as this will increase the extent to which the long-term relationships become clear. To help these discussions, make sure that students use the writing spaces to record their observations, both to help them to verbalise what they have concluded, but also to serve as a basis for discussion with a partner or with the whole class.

Activity 1 focuses on the idea of a uniform distribution, and gives some insight into what this means in practice, with the (pseudo-) random numbers generated by *ClassWiz*. Expect that there will be some anomalies in the short term (such as students with substantially more or less than half the numbers below 0.5), but these can be explained as short-term fluctuations if enough data are generated collectively to see a 'big picture'. Encourage students to conduct other experiments, as well as the two suggested (proportions less than 0.5 or 0.2).

Activity 2 highlights the *ClassWiz* practice of producing numbers rounded to three decimal places, and then representing them to two or one decimal place if trailing digits are zero. Once again, short-term variations will be common, but the compilation of results will help students to see that they are merely short-term fluctuations. Make sure that they commit to a guess before generating any data.

Activity 3 offers a way for students to explore the familiar problem of tossing a pair of coins. Many students (naïvely) think there are three possibilities: two heads, two tails or one of each, with probabilities of 1/3 for each. The simulation is intended to help them to understand why this is incorrect, as a head followed by a tail (HT) is a different outcome from a tail followed by a head (TH), and so the probability of getting two heads is only ¼.

As for the previous two activities, stronger and more convincing results will be obtained by pooling the results of individuals into pairs and of pairs into the whole class to stimulate a discussion.

An alternative approach to generating each coin toss separately is to generate a sum of the number of heads as a single function: $f(x) = RanInt(0,1)+RanInt(0,1)$. This will result in 0, 1 or 2 (heads), and make it easier to see the totals more quickly, and probably easier to see that a result of one head occurs around half of the time. It is also offers a method of handling the tossing of more than two coins, should students choose to extend the activity.

9. Investigating sequences

This investigation uses an inherent capability of a *ClassWiz* for recursive operations, as explained in the opening paragraph. It also uses the Table facility.

Recursively defined sequences such as 7, 13, 19, … for which each term is six more than the previous term, can be generated on *ClassWiz* and careful counting allows students to obtain particular terms. Activity 1 uses this idea in the case of two arithmetic (or linear) sequences.

Activity 2 extends this idea to arithmetic sequences that are decreasing as well as increasing and also to a geometric sequence. The three examples given all share the property that the eighth term is 60, presenting students with a challenge to find other examples with this property. The first two examples given arise from arithmetic sequences, with first term 4 and common difference 8, and first term 137 and common difference -11; the third example is a geometric sequence with first term 15/32 and common ratio 2. There are many examples to be found, but students will need to understand thoroughly how the sequences are constructed to become adept at finding other examples.

Make sure that students use the writing spaces to record their observations, both to help them to verbalise what they have concluded, but also to serve as a basis for discussion with a partner or with the whole class.

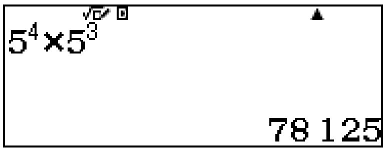
Activity 3 extends the idea of a sequence from those that are recursively defined to those that are explicitly defined, through the use of a table. (It is assumed that students are already familiar with the use of *ClassWiz* tables, but they will need some help if that is not the case.) This is intended to extend understanding of the idea of a sequence to that of a function on the natural numbers.

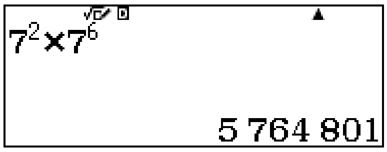
Students might observe that a definition of $f(x) = 6x + 1$ has the same effect as the one shown, although a less obvious connection between the formula and the first term of the sequence.

It is instructive for students to try to represent other sequences, such as geometric sequences, in the form of a function, rather than recursively.

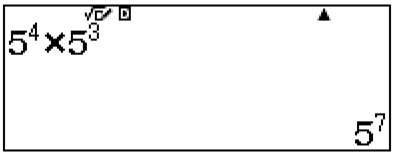
1. Investigating powers

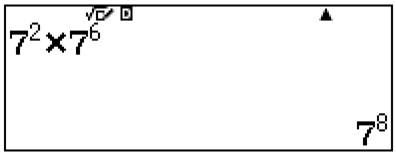
1. When powers of a whole number are multiplied together, the results are often large numbers:





When the FACT command (SHIFT °’’’) is used *after the result is obtained*, factors of the result are displayed, as shown below. Try these two examples on your *ClassWiz*.

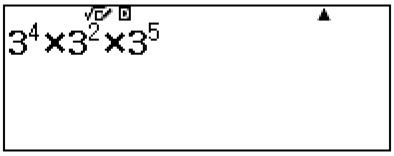


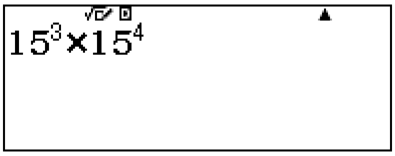


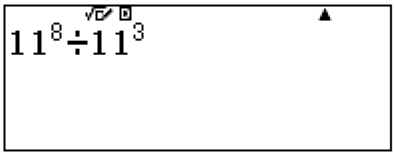
Then try several other examples like these for yourself. Look for a pattern to predict the results before you find the factors. Discuss with your partner what you notice.

Explain what is happening and why it is happening. Write your conclusions here:

2. Predict what will happen with these results *before trying them on your ClassWiz*:



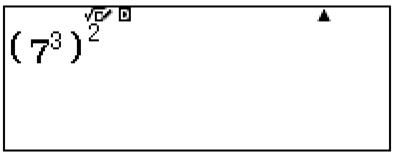


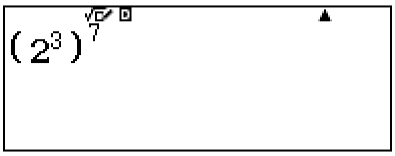


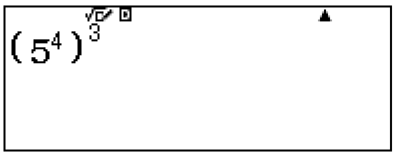
Explain any incorrect predictions.

Try some other examples of these three kinds until you can confidently predict the results.

3. Use factors to see what happens with powers like these:





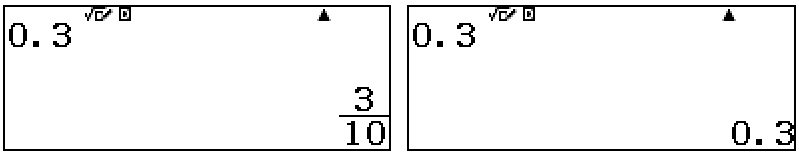


Discuss these with your partner what is happening, and explain the results to each other.

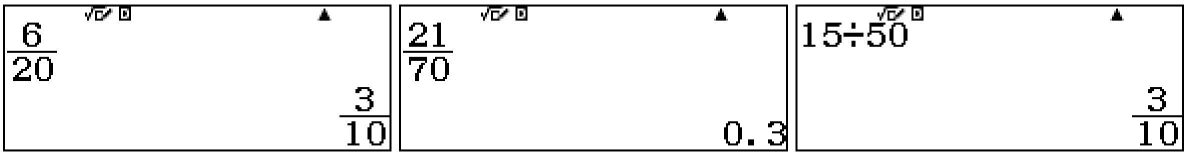
Use your explanation to predict some fresh examples of your own like these. Test your predictions on your *ClassWiz*.

2. Investigating fractions and decimals

1. In Math mode, when a number is entered into your *ClassWiz*, it is usually displayed as a fraction. You can tap the [S⇔D] command to represent a number either as a fraction or a decimal.

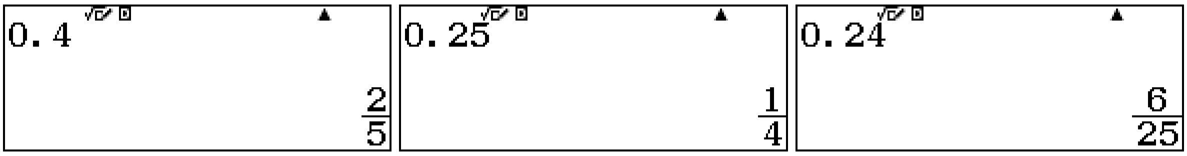


There are many other numbers with the same representation. Here are three more:



Use your *ClassWiz* to find several others. How many can you find? Discuss this with your partner.

2. Kai Fai expected that 0.4 would be represented as four tenths, but the *ClassWiz* represented it as two fifths. He found some other unexpected examples like this below.



Discuss these with your partner to explain what is happening. Make some more examples like these for yourself. Look for a pattern to predict the results before you tap the [=] key.

Explain what is happening. Write your conclusions here:

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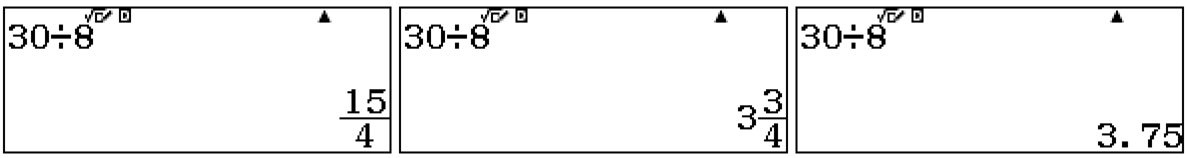
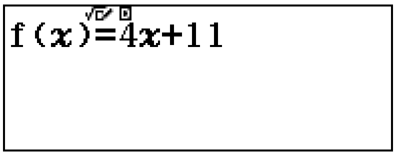
3. Numbers larger than 1 can be represented as fractions in two different ways. Tap [SHIFT] [S⇔D] on your *ClassWiz* to switch between these two representations; you can also tap [S⇔D] to switch between fractions and decimals. Here is an example:

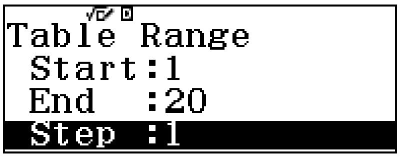


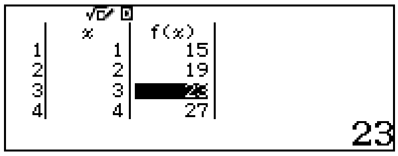
Discuss this example with your partner. Make several more examples of your own like these and predict the results *before using your calculator*. Test your predictions on your *ClassWiz*.

3. Investigating functions

1. The *ClassWiz* is very effective to generate tables of values for functions. The example below shows the linear function $f(x) = 4x + 11$:



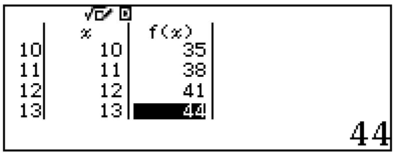


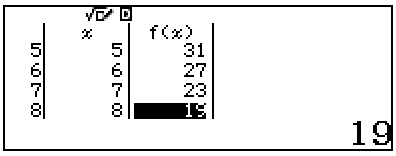


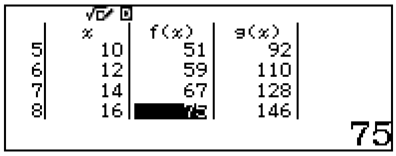
Look carefully at the pattern of values in the table. What will be the next numbers?

Scroll down with ⏷ to check your answer.

2. Once you understand the pattern of values of a function, it is possible to identify the function by analysing the values.



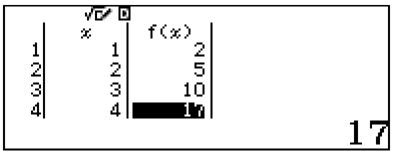


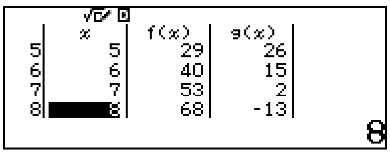


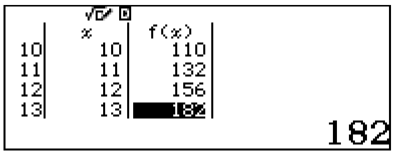
Study the three screens above. Which four functions are involved?

Use the *ClassWiz* to test your answers. Then make up some more examples of this kind to share with a partner.

3. The examples in Activities 1 and 2 all involve linear functions. Other kinds of functions result in different kinds of patterns, for which the difference between successive values is not always the same. The first screen below shows some values for the quadratic function, $f(x) = x^2 + 1$.







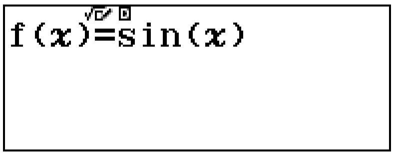
Look carefully at the pattern of values in the tables. What will be the next numbers in each case?

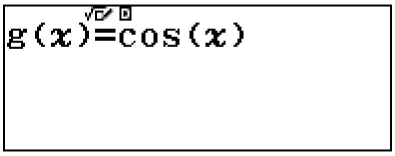
Use the *ClassWiz* to test your answers.

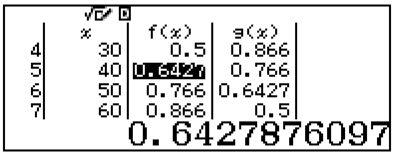
What are the other three functions tabulated?

4. Investigating trigonometry

1. There are some interesting relationships between sines and cosines of angles that can be seen by studying their values in tables. In the example below, the calculator has been set to degrees, and the tables show angles in the first quadrant ($0^o \leq x \leq 90^o$). Use your *ClassWiz* to study these tables carefully.







Notice in the third screen that sin 40^o = cos 50^o and that sin 30^o = cos 60^o. Scroll the table on your *ClassWiz* to find some more examples of pairs of values that are the same.

Record your results on paper and discuss them with your partner.

Check if the same relationship holds for other values by changing the step in the table to 5^o, so that there are more values displayed. Describe the pattern you see:

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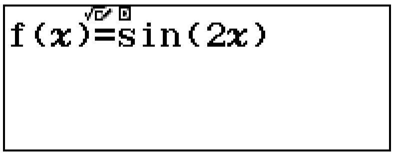
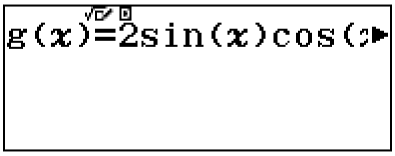
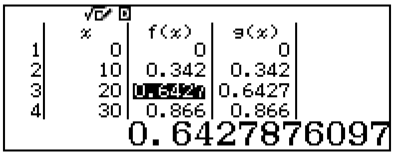
Explain *why* the sine and cosine values are related in this way. A right triangle or a unit circle may be helpful for this.

2. Trigonometric *identities* are important, as they show general relationships. A good example is $\sin 2x = 2\sin x \cos x$. To appreciate what this means, enter each side of the identity into a table, as shown in the screens below.







Check for yourself that this relationship is *always* true by changing the table values: use different limits, use a different step, or change some x-values in the table directly. Make sure that you check positive and negative values, large and small.

Check some other possible identities in this way, like those below.

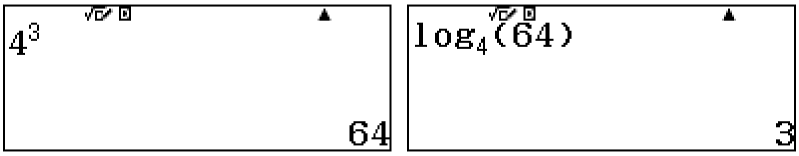
$$\tan x = \frac{\sin x}{\cos x} \qquad \tan 2x = \frac{2\tan x}{1-\tan^2 x}$$

$$\cos 2x = \sin^2 x - \cos^2 x \qquad \cos(90 \; + x) = \sin x$$

Use your *ClassWiz* to check which of these, if any, are *not* identities.

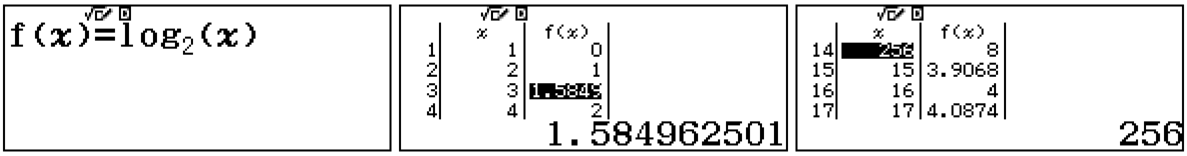
5. Investigating logarithms

1. The $\log_{\square}\square$ command finds the logarithm of a number to a particular base. The logarithm is the power of the base needed to produce the number. Study carefully these two screens showing that the logarithm of 64 to base 4 is 3.



Use your *ClassWiz* to make several other pairs of screens like these. Use a variety of bases. Use positive and negative powers. Record your results on paper and discuss them with your partner.

2. Most logarithms are not whole numbers. A table of logarithms can show many examples, as shown below. (Notice in the third screen that you can replace any number in the *x* column with a number of your own choice. In this case, 14 was replaced by 256.)



Explore this table by scrolling, changing the *x*-numbers or changing the limits of the table.

Larger numbers have larger logarithms. Explain why this is the case.

Which numbers have logarithms that are whole numbers? Explain why.

Make some tables with logarithms to a different base, to check that your explanations still apply; check your observations with your partner. Discuss why, *for all bases*, the logarithm of the base equals 1 and the logarithm of 1 equals zero. Write your explanation here:

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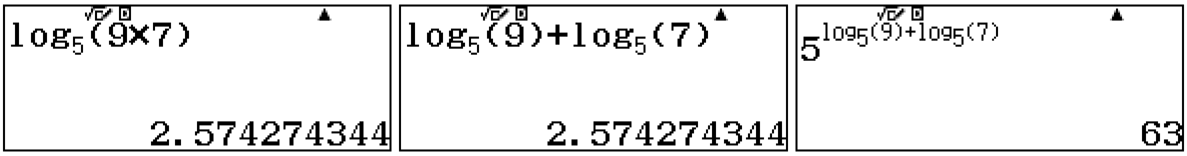
3. Study the screens below to see that the logarithm of a product of two numbers is the sum of their logarithms, thus connecting multiplication and addition.



Make several more examples of your own like this, using different numbers and different bases.

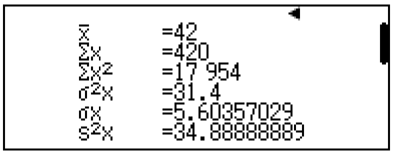
Discuss your examples with your partner. Explain how to use this property of logarithms to find the logarithm of a *power*. (For example, how does the logarithm of 35^7 compare with the logarithm of 35?) Check your answer with the *ClassWiz*, using several different bases.

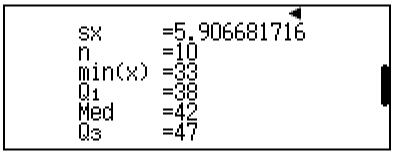
6. Investigating data

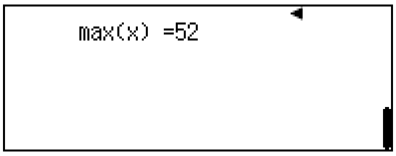
1. To report data to the government, a Principal counted the number of students in each of the ten classes of a school. Here are the results:

35, 38, 42, 47, 45, 39, 33, 47, 42, 52

The Principal used a *ClassWiz* to analyse these data, with the following results:







The Principal then realised that she needed to include all of the class occupants, and so needed to add one person (the teacher) to each class size. What effect will this change have on these statistics?

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Enter the corrected data into your *ClassWiz* and analyse them to check your prediction.

Discuss with your partner the effect of adding 2 (instead of 1) to each number. Check your prediction with the *ClassWiz*.

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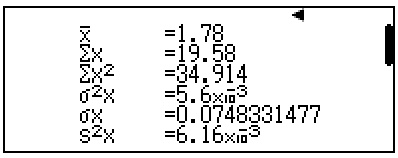
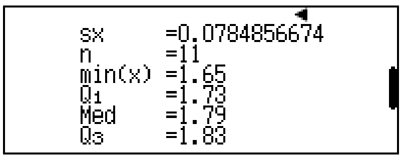
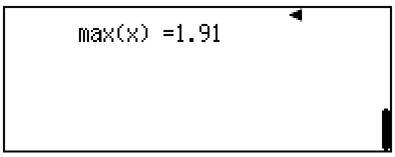
2. The manager measured the heights of players in a football team with the following results:

1.65, 1.78, 1.82, 1.66, 1.85, 1.81, 1.79, 1.75, 1.91, 1.83, 1.73

These data were to be used to include in the team's annual magazine, and a *ClassWiz* was used to summarize them:







Heights of players in other teams were measured in centimetres, however, not metres. So all of the heights above needed to be multiplied by 100. What effect will this change have on the statistics?

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Enter the corrected data into your *ClassWiz* and analyse them to check your prediction.

Discuss with your partner the effect of multiplying each number by 10 (instead of 100). Check your prediction with the *ClassWiz*.

7. Investigating models

1. Mathematics can help with modelling, to understand the world better and to make predictions. For example, the population of Australia was estimated by the Australian Bureau of Statistics (ABS) on 17 September 2015 to be 23 899 701. The annual population growth rate of Australia was reported by ABS to be 1.4% in December 2014.

Use your calculator to estimate the population of Australia at future times, such as in September 2016 or in 2050, near the middle of the 21st century, making reasonable assumptions about the annual growth rate. When would you expect the Australian population to reach 36 million? Try to be efficient in making the estimates. Compare your methods with others.

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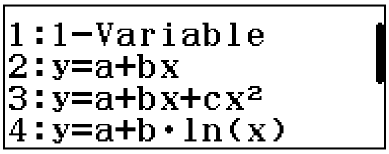
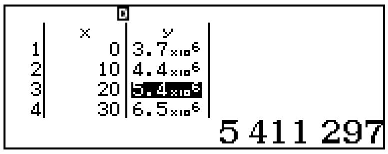
2. A rule of thumb (called *The Rule of 72*) for compounding growth at r% per annum is that a quantity will double after about $72/r$ years. Investigate how well this rule works.

3. One approach to modelling involves predicting from known historical data (assuming they are a reliable guide to the future). Here are the ABS population figures from the early 20th century.

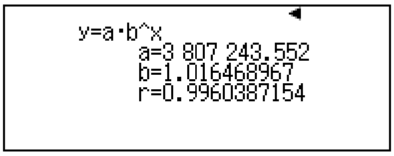
Year	1900	1910	1920	1930	1940	1950	1960
Population	3 765 339	4 425 083	5 411 297	6 500 751	7 077 586	8 307 481	10 391 920

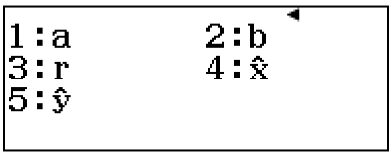
Enter these data into the calculator. Enter Statistics mode and start by choosing a linear model $y = a + bx$ with [2], entering data list-wise, followed each time by [=].

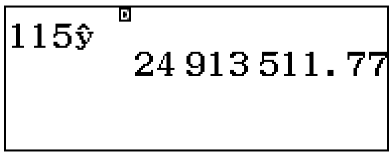




After entering data, tap [AC] and then [OPTN] to extract statistics or change the type of model used. The regression statistics will be most helpful, allowing various models to be fitted and predictions made. To make a prediction for x or y, enter the value first and then enter the relevant estimator via the Regression menu ([OPTN] [▼] and then [4]).







To change models start with [OPTN] [1]. The data above came from the exponential model, $y = ab^x$.

Discuss with others how your conclusions to questions in Activity 1 would differ if you were doing this exercise in the 1960s. E.g., what population would you have predicted for Australia today?

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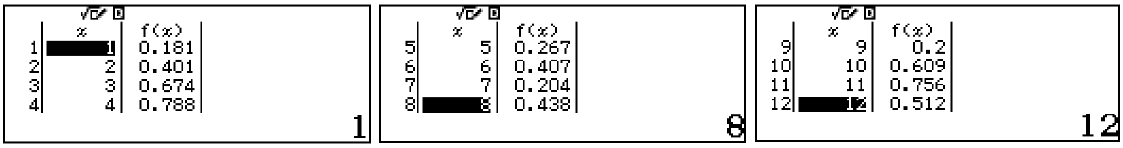
Obtain the data from the Internet and examine some population models for your own country.

8. Investigating probability

1. When a calculator is used to generate random numbers, the numbers will be uniformly spread between 0 and 1. So, about half the numbers should be less than 0.5. The tables below were generated using $f(x)$ = Ran#. In this case, seven of the 12 numbers were less than 0.5.



Use your *ClassWiz* to generate 30 random numbers and check to see how many are less than 0.5. Compare your results with others.

How many of 30 random numbers would you expect to be less than 0.2?

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Generate a fresh set of 30 random numbers and check your prediction. Check with others too.

2. You may have noticed that, when a 3-digit random number ends in 0, it is shown as a 2-digit number. (The ninth random number above is an example, showing 0.2 instead of 0.200.) How many 2-digit numbers would you expect in 20 random numbers? Explain why:

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Use the *ClassWiz* to test your answers. Generate and test several sets of 20 random numbers.

3. The *ClassWiz* can simulate tossing a fair coin with the function *RanInt*(0,1). If the result is 0, we will regard it as a tail, with 1 representing a head. To simulate tossing a pair of coins, use both functions in the table, as shown below

In this case, only one of the four tosses (the fourth one) shows a pair of heads.

How often would you expect to get a pair of heads? Explain why:

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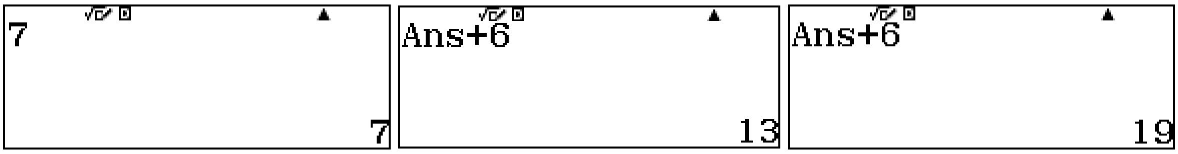
Test your prediction by pairs of coin tosses like this. Compare your results with others.

How likely are the other results? (a pair of tails or a head and a tail)?

9. Investigating sequences

1. A sequence is a set of numbers in order. You can start a sequence on the *ClassWiz* with any number, followed by [=]. Then use the calculator to generate the next term. In the example below, we used [+] [6] [=] to make the second term six more than the first. (Notice the calculator interprets this as adding 6 to the previous answer, called *Ans*.)



Now tap [=] several times to repeat the same operation and get further terms: 7, 13, 19, …

Use your *ClassWiz* to generate the sequence 8, 15, 22, … What is the tenth term of this sequence?

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2. You can make sequences with other operations, including subtraction, multiplication and division. Use your *ClassWiz* to generate each of these sequences for yourself. Check for yourself that the eighth term of each sequence is 60.

4, 12, 20, … 137, 126, 115, … $\frac{15}{32}, \frac{15}{16}, \frac{15}{8}, \ldots$

Find some other sequences for which the eighth term is 60. How many *different* examples can you find?

Write them below. Compare your results with a friend.

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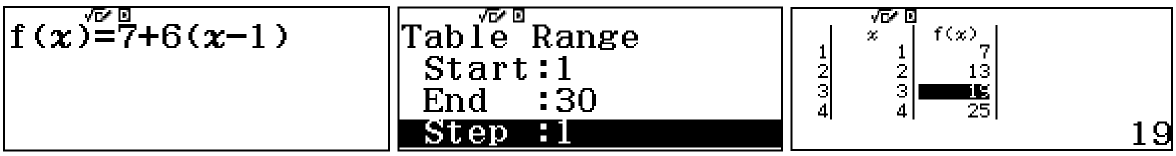
3. You can also make a sequence using a table. The *x* value refers to the term number and the function value gives the terms of the sequence. The third term of the sequence below is 19.



You might notice that this is the same sequence as the first one described in Activity 1. Explain the meaning of the 7 and the 6 in the function definition:

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Make tables for the other sequences you have used, including those involving addition and multiplication.

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