**Unit**

**3**

**FUNCTIONS**

# KEY CONCEPTS

**TEACHER’S GUIDE**

**Sketching of linear graphs**

A general formula of a linear graph is *y = mx + c*, where *m* is the gradient of the curve and *c* is the *y*-intercept.

The gradient measures the steepness of the line. A positive gradient yields an upwards sloping line (from the left to the right) and a negative gradient yields a downwards sloping line (from left to right). Also, a line with a gradient of 4 will be steeper than a line with a gradient of 1.

*y*

*y =* 2𝑥– 3

1.5

–3

𝑥

*y*

Example

A horizontal line has gradient = 0 and hence has the formula of *y* = *c*, where *c* is a constant as well as the

*y*-intercept.

*y*

*y =* 2

2

*y*

𝑥

Example

A vertical line does not have a defined gradient. This is because the formula of the gradient is , and a

vertical has a value of 0 for the run, which causes the fraction to be undefined.

The formula for a vertical line is 𝑥= *a*, where *a* is a constant as well as the *x*-intercept.

Example

*y*

–1

𝑥 *=* –1

𝑥

*x*

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# Sketching of quadratic graphs

1. We can sketch quadratic graphs from 2 algebraic forms: **factorised form** or **completed square form**. The 3 main details we must have before sketching a quadratic graph are: (a) the turning point, (b) the *x*-intercepts and (c) the *y*-intercepts.
2. A parabola curve is symmetrical and the line of symmetry cuts through the turning point.
3. A curve that opens upwards has a positive coefficient of 𝑥2 and has a minimum point.
4. A curve that opens downwards has a negative coefficient of 𝑥2 and has a maximum point.

|  |  |
| --- | --- |
| **Factorised form: *y* = ± (𝑥– *a*)(*𝑥* – *b*)** | **Completed square form: *y* = ± (𝑥– *h*)2 + *k*** |
| Shape of graph  Note: the ± sign only indicates if quadratic graph opens upwards or downwards. Do not take + to mean that the curve opens upwards and – to mean that the curve opens downwards.  Always expand out the *x*2 term to double check the coefficient. | |
| This is the first detail you can derive from this form: ***x*-intercepts**.  𝑥= *a* or 𝑥= *b* | This is the first detail you can derive from this form: **coordinates of turning point**.  Coordinates = (*h*, *k*) |
| Turning point:  Note that turning point can be found by the line of symmetry.  The *x*-coordinate of the line of symmetry will be the average of *x*-intercepts: 𝑥= .  The *y-*coordinate can be easily found by substituting the 𝑥value into the equation. | *x*-intercepts:  Let *y* = 0 and find the values of 𝑥.  Note: Do not expand the square. Simply square root both sides and rearrange the terms. |
| *y*-intercepts:  Let 𝑥= 0 and find the value of *y*. | *y*-intercepts:  Let 𝑥= 0 and find the value of *y*. |

Unit 3 **Functions**

***Example***

**CLASSWIZ WORKSHEETS**

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Factorised form: *y*** | | **= –(3 –** | **𝑥)(𝑥** | **+ 7)** | | **Completed square form:** | ***y*** | **= –(𝑥** | **+ 3)2 + 4** | |
| Shape of graph:  Do not be fooled by the negative sign. If you expand the equation, you will realise that the coefficient is actually positive 1. Hence the curve opens upwards. | | | | | | Shape of graph:  If you expand the equation, the coefficient will be negative 1. Hence the curve opens downwards. | | | | |
| This is the first detail you can derive from this form: ***x*-intercepts**.  𝑥= 3 or 𝑥= –7 | | | | | | This is the first detail you can derive from this form: **coordinates of turning point**.  Coordinates = (3, 4)  Note that (𝑥+ 3)2 is not in the same form as (𝑥– *h*)2. Rewrite as [𝑥– (–3)]2. Hence the *x*-coordinate is now −3. | | | | |
| Turning point:  *x*-coordinate: 𝑥= = –2  When 𝑥= –2, *y* = –(3 – 2)((–2) + 7)  = –25  Coordinates = (2, 25) | | | | | | *x*-intercepts: | | | | |
| When *y* = 0, –(𝑥+ 3)2 + 4 = 0 | | | | |
| (𝑥+ 3)2 = 4 | | | | |
| 𝑥+ 3 = ±2 | | | | |
| 𝑥= –5 or –1 | | | | |
| *y*-intercepts:  When 𝑥= 0, *y* = –21 | | | | | | *y*-intercepts:  When 𝑥= 0, *y* = –5 | | | | |
| (–2, –25)  –21  –7  3 |  | *y* |  | |  | (–3, 4)  –5  –1  𝑥  –5  *y* |  |  |  |  |
|  |  |  |  | |  |  |  |  |  |  |
|  |  |  |  | | 𝑥 |  |  |  |  |  |
|  |  |  |  | |  |  |  |  |  |  |
|  |  |  |  | |  |  |  |  |  |  |
|  |  |  |  | |  |  |  |  |  |  |

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**CLASSWIZ WORKSHEETS**

# Sketching of cubic graphs

The general formula for a cubic graph is *y* = *a*𝑥3 + *b*𝑥2 + *c*𝑥+ *d*, where *a*, *b*, *c* and *d* are constants. Under the O-Level syllabus, you need to sketch *y* = *a*𝑥3 + *b* only, where *a* and *b* are constants.

The cubic function *y* = *a*𝑥3 + *b* has a stationary point, i.e. when the gradient = 0. It has been marked out on the diagram as a cross.

Example

*y*

1.5

*y* = 12𝑥3 + 1.5

𝑥

# Sketching of power functions

*y* = , where *a* is a constant. Since the denominator ≠ 0, 𝑥= 0 will be the asymptote. And since there is no

value of 𝑥such that can be 0, hence *y* ≠ 0 and *y* = 0 will be the asymptote.

*y*

y=

*y*

𝑥

Example

*y* = , where *a* is a constant. Since the denominator ≠ 0, 𝑥= 0 will be the asymptote. And since there is no

value of *x* such that can be 0, hence *y* ≠ 0 and *y* = 0 will be the asymptote.

Example 　　　　　*y*

*y*

y =

𝑥

Unit 3 **Functions**

# Long division

**CLASSWIZ WORKSHEETS**

The division algorithm is given as

Dividend = Divisor × Quotient + Remainder

OR

Quotient Divisor )Dividend

•

•

•

Remainder

Example

Find the remainder when 2𝑥3 –4𝑥2 + 𝑥–3 is divided by 𝑥– 1.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  |  |  | 2𝑥2 | – 2𝑥 | – 1 |
| 𝑥– 1 | ) | 2𝑥3 | – 4𝑥2 | + 𝑥 | – 3 |
|  |  | – (2𝑥3 | – 2𝑥2) |  |  |
|  |  |  | – 2𝑥2 | + 𝑥 |  |
|  |  | – | (– 2𝑥2 | +2𝑥) |  |
|  |  |  |  | – 𝑥 | – 3 |
|  |  |  | – | (– 𝑥 | + 1) |
|  |  |  |  |  | – 4 |

# Remainder Theorem

If a polynomial f(𝑥) is divided by a linear divisor (𝑥– *a*), the remainder is f(*a*). Using the same example:

Find the remainder when 2𝑥3 – 4𝑥2 + 𝑥– 3 is divided by 𝑥– 1. Let f(𝑥) = 2𝑥3 – 4𝑥2 + 𝑥– 3.

f(1) = 2(1)3 – 4(1)2 + (1) – 3 = – 4

# Factor Theorem

If (𝑥– *a*) is a factor of a polynomial f(𝑥), then f(*a*) = 0 as there are no remainders.

# Partial Fractions

In order to express a proper rational expression into partial fractions, we need to factorise g(𝑥) completely and write down the forms of the partial fractions according to the following rules.

The very first step to note is that the fraction must be proper and rational. If the degree of the numerator is equal or larger than the degree of the denominator, then a long division is necessary.

|  |  |
| --- | --- |
| **g(**𝑥**) has** | **Corresponding partial fraction(s)** |
| Linear factor *a*𝑥+ *b* |  |
| Repeated linear factor (*a*𝑥+ *b*)2 | + |
| Quadratic factor 𝑥2 + *c*2  (cannot be factorised) |  |

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# WORKED EXAMPLES

**Worked Example 1**

By using the CLASSWIZ calculator, find the roots and the turning point of

Solution

.

5𝑥2 + 17𝑥– 12 = 10𝑥– 4

**ClassWiz steps**

Press wQz for Equation/Function. You will see the 1. Simul Equation and 2. Polynomial displayed on the

calculator. Choose 2 and select 2 again for the degree of the polynomial. As shown, the format is given as *a*𝑥2 + *b*𝑥+ *c*. Key in the values of *a*, *b*, *c* as 5, 7, –8 respectively.

5𝑥2 + 7𝑥– 8 = 0

Roots: 𝑥 = or

Turning point: (– , – )

**CLASSWIZ WORKSHEETS**

# Worked Example 2

By using the CLASSWIZ calculator, find only the real roots of 3𝑥3 + 11𝑥2 –19𝑥+ 5 = 0.

Solution

**ClassWiz steps**

Press wQz for Equation/Function. You will see the 1. Simul Equation and 2. Polynomial displayed on the

calculator. Choose 2 and select 3 for the degree of the polynomial. As shown, the format is given as *a*𝑥3 + *b*𝑥2 + *c*𝑥+

*d*. Key in the values of *a*, *b*, *c*, *d* as 3, 11, –19, 5 respectively.

Roots: *x* = or 1 or –5

# Worked Example 3

By using the CLASSWIZ calculator, find the asymptote/point of discontinuity of *y* = . For the table range, start with –5 and end with 5. Set the step at 1.

Solution using ClassWiz steps

We are going to use the Table functions of the ClassWiz calculator to deduce the asymptote and

*y*-intercept.



**ClassWiz steps**

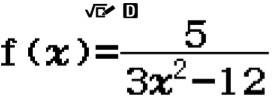
Press w9.

Unit 3 **Functions**

## ClassWiz steps

**CLASSWIZ WORKSHEETS**

Key in 5a3Q)dp12 for f(𝑥).

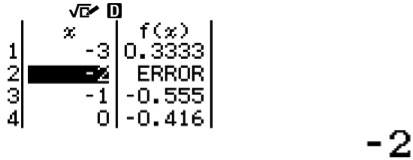


Press == to skip g(𝑥) since there is only 1 equation.

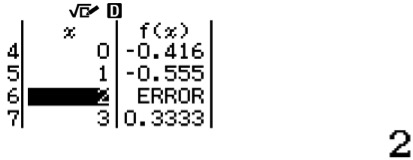


Start the range with –3 and end the range at 25. Leave the step as 1.

Press = and the table of values will be displayed. We will now try to find the **asymptote**.

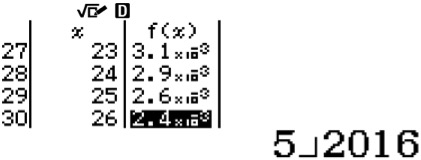


Press R to explore the table of values.



Hence, deduce that asymptotes are 𝑥= 2 or 𝑥= – 2.

Press = to view more values not stated in the range previously.



Hence, deduce that the function converges towards *y* = f(𝑥) = 0. The asymptote is *y* = 0.

*y* cannot be 0 because there are

no values of 𝑥that will cause *y* =

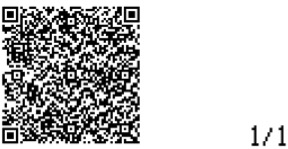
0. The only way for the function

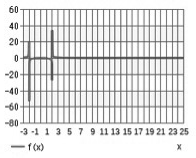
to be 0 is when the numerator

= 0, but clearly in this case the

numerator is 5 and not 0.

Press qT to generate a QR code.



Scan the QR code with CASIO EDU + mobile application to view graph.

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**CLASSWIZ WORKSHEETS**

# Worked Example 4

Sketch 3*y* + 7 = 5𝑥, showing the 𝑥and *y*-intercepts clearly.

Solution

Rearrange the equation to be in the format of *y* = *m*𝑥+ *c* first.

3*y* + 7 = 5𝑥

3*y* = 5𝑥– 7

*y* = *x* –

As we can see, the gradient is positive. There is no need

to show the steepness of the line exactly at gradient

= . Just a general upward sloping line would do. The

*y*-intercept is clearly seen from here as *y* = – . For the

*x*-intercept, we will need to calculate by letting *y* = 0.

When *y* = 0, 𝑥=

𝑥=

*y*

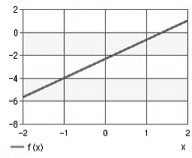
3*y* + 7 = 5𝑥

–

*y*

𝑥

Solution using ClassWiz steps



**ClassWiz steps**

Press w9.

Key in 5a3$Q)p7a3 for f(𝑥).

Press = = to skip g(𝑥) since there is only 1 equation.

Start the range with –2 and end the range at 2. Leave the step as 1.

Press = qT to generate a QR code.

Scan the QR code with CASIO EDU + mobile application to view graph.

Unit 3 **Functions**

# Worked Example 5

**CLASSWIZ WORKSHEETS**

Sketch *y* = – (𝑥 + 1)2 + 4, showing the 𝑥and y-intercepts and turning points clearly.

Solution

Since the coefficient of 𝑥2 is negative, the curve opens downwards. Turning point: (– 1,4)

When 𝑥= 0, *y* = 3.

When y = 0, –(𝑥+ 1)2 + 4 = 0

(𝑥+ 1)2 = 4

𝑥+ 1 = ±2

𝑥= – 3 or 1

*y*

*y =* – (𝑥+ 1)2 + 4

–1

1

–3

𝑥

*y*

Solution using ClassWiz steps

**ClassWiz steps**

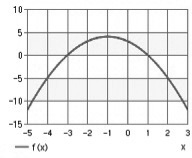
Press w9.

Key in z(Q) + 1)d + 4 for f(𝑥). Press = = to skip g(𝑥) since there is only 1 equation.

Start the range with –5 and end the range at 3. Leave the step as 1.

Press = qT to generate a QR code.

Scan the QR code with CASIO EDU + mobile application to view graph.



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**CLASSWIZ WORKSHEETS**

# Worked Example 6

Sketch *y* = 5𝑥3 –2, showing the 𝑥and *y*-intercepts clearly. Mark out the point of infle*x*ion with a cross.

Solution

When y = 0, 5𝑥3 – 2 = 0

Note that a sketch of y = 5𝑥3 will look the same as y = 𝑥3. Again, the steepness of the curve is not required to be shown in a sketch.

=

𝑥=

*y*

-2

y

𝑥

–2

*y*

-2

Solution using ClassWiz steps

**ClassWiz steps**

Press w9.

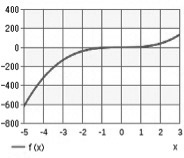
Key in 5Q)Dp2 for f(𝑥).

Press = = to skip g(𝑥) since there is only 1 equation.

Start the range with –5 and end the range at 3. Leave the step as 1.

Press = qT to generate a QR code.

Scan the QR code with CASIO EDU + mobile application to view graph.



Unit 3 **Functions**

# Worked Example 7

**CLASSWIZ WORKSHEETS**

Solve the pair of simultaneous equations.

=

2𝑥+ 3*y* = 9

Solution

= …(1)

2𝑥+ 3*y* = 9 …(2)

From (1), 32 – 12*y* = 15𝑥– 25

57 = 15𝑥+ 12*y* …(3)

From (2), 9 = 2𝑥+ 3*y*

36 = 8𝑥+ 12*y* …(4)

Take (3) – (4): 21 = 7𝑥

𝑥= 3

2(3) + 3*y* = 9

*–y* = 1

Solution using ClassWiz steps

From equation (2), 2𝑥+ 3*y* = 9.

From equation (3), 15𝑥+ 12*y* = 57.

**ClassWiz steps**

Press wQz.

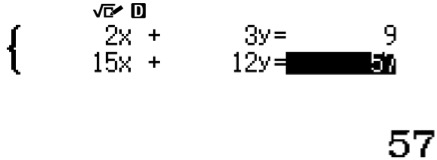
Press 1 for 1: Simul Equation.

Press 2 for 2 unknowns.

Key in equation (2) and (3) accordingly. It should look like this:

Press = to solve the simultaneous equations. It should show 𝑥= 3. Press = again to

show *y* = 1.



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# Worked Example 8

Solve 2𝑥3 + 5𝑥2 – 21𝑥– 36 = 0.

Solution

Let 𝑥= 3.

2(3)3 + 5(3)2 – 21(3) – 36 = 54 + 45 – 63 – 36 = 0

Hence 𝑥– 3 is a factor of 2𝑥3 + 5𝑥2 – 21𝑥– 36.

By long division,

2 𝑥2 + 11𝑥 + 3

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |  |
| 𝑥– 3 |  | ) | 2𝑥3 | + 5𝑥2 | – 21𝑥 | – 36 |
|  |  | – | (2𝑥3 | – 6𝑥2) |  |  |
|  |  |  |  | 11𝑥2 | – 21𝑥 |  |
|  |  |  | – | (11𝑥2 | – 33𝑥) |  |
|  |  |  |  |  | 12𝑥 | – 36 |
|  |  |  |  | – | (12𝑥 | – 36) |
|  |  |  |  |  |  | 0 |

**ClassWiz steps**

Press wQz2 and 3 to select a degree of 3 to

get a cubic equation. Press 2 = for the coefficient of

𝑥3, 5 = for the coefficient of 𝑥2, z21 = for

coefficient of 𝑥, and z36 = as the constant term.

Press = and you will be able to see the answer displayed

as = 3, = – and = –

**CLASSWIZ WORKSHEETS**

2𝑥3 + 5𝑥2 – 21*x* – 36 = 0 (𝑥– 3)(2𝑥2 + 11𝑥+ 3) = 0 (𝑥– 3)(2𝑥+ 3)(𝑥+ 4) = 0

𝑥= 3 or –1.5 or –4

# Worked Example 9

Find the value of *k* if f(𝑥) = 5𝑥3 – 7𝑥2 + 2*k*𝑥+ 1 leaves a remainder of –17 when divided by 𝑥+ 1.

Solution

Given: f(–1) = – 17

f(–1) = 5(– 1)3 – 7(– 1)2 + 2*k*(– 1) + 1 = – 17

– 5 – 7 – 2*k* + 1 = – 17

– 2*k* = – 6

*k* = 3

Unit 3 **Functions**

# Worked Example 10

**CLASSWIZ WORKSHEETS**

Express in partial fractions.

Solution

The fraction is improper. By long division,

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  | 2𝑥 | + 1 |
| 𝑥3 – 3𝑥2 + 𝑥– 3 |  | ) | 2𝑥4 | – 5𝑥3 | + 4𝑥2 | – 3𝑥 | – 4 |
|  |  | – | (2𝑥4 | – 6𝑥3 | + 2𝑥2 | – 6𝑥) |  |
|  |  |  |  | 𝑥3 | + 2𝑥2 | + 3𝑥 | – 4 |
|  |  |  | – | (𝑥3 | – 3𝑥2 | + 𝑥 | – 3) |
|  |  |  |  |  | *5*𝑥2 | + 2𝑥 | – 1 |

= 2𝑥+ 1+

= 2𝑥+ 1+ +

5𝑥2 + 2𝑥– 1 = A(𝑥2 + 1) + (B𝑥+ C)(𝑥– 3)

5𝑥2 + 2𝑥– 1 = (A + B) 𝑥2 + (C – 3B)𝑥+ (A – 3C)

A + B = 5 …(1)

*C* – 3*B* = 2 …(2)

*A* – 3*C* = – 1 …(3)

From (3), *A* = 3*C* – 1 …(4)

Sub (4) into (1): 3*C* – 1 + *B* = 5

*B* = 6 – 3*C* …(5)

Sub (5) into (2): *C* – 3(6 – 3*C*) = 2

*C* – 18 + 9*C* = 2

10*C* = 20

*C* = 2

From (4), *A* = 3(2) – 1 = 5

From (1), 5 + *B* = 5

*B* = 0

= 2𝑥+ 1 + +

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**CLASSWIZ WORKSHEETS**

# CLASSWIZ WORKSHEETS

**Level 1 **

1. Calculate the y-coordinate of each algebraic expression with the given *x*-coordinate.

|  |  |
| --- | --- |
| (a) *y* = 3𝑥+ , 𝑥 = | (b) *y* = 3𝑥2 – 𝑥 + 1, 𝑥= |
| (c) *y* = – 𝑥2 + 5𝑥– , 𝑥= | (d) 5*y* + 2𝑥 = 𝑥2 – 9, 𝑥= 7 |
| (e) 𝑥2 – *y* + 5 = 7𝑥2 + , 𝑥= | (f) *y* – 3𝑥+ = – 5𝑥2, 𝑥 = |
| (g) 7𝑥2 – 8*y* = *y* + 4𝑥, 𝑥= | (h) –2𝑥3 + 4𝑥= *y* – , 𝑥= 2 |
| (i) *y*2 – 𝑥3 + 3 = 9𝑥 + ,𝑥 = |  |

Unit 3 **Functions**

1. Find the *x* and *y* intercept of the following linear functions.

**CLASSWIZ WORKSHEETS**

|  |  |
| --- | --- |
| (a) *y* = 2𝑥– 3 | (b) 8 – *y* = 𝑥 |
| (c) –3+ =𝑥 | (d) 𝑥+ *y* = 1 |
| (e) 5𝑥– *y* = 3*y* – 𝑥+ 1 | (f) 3*y* – 2𝑥+ 2 = 3𝑥– 4*y* + 5 |

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**CLASSWIZ WORKSHEETS**

1. By using the CLASSWIZ calculator, state whether the linear functions are increasing or decreasing.

Since the calculator can only handle ‘f(𝑥) =

this means that you need to manipulate your

equations until you achieve a singular ‘*y*’ on the

left-hand side of the equation, without anymore

‘*y*’ terms on the right-hand side.

**ClassWiz set-up**

Press w9 for the Table. You will see the ‘f(𝑥) =’ displayed on the calculator. ‘f(𝑥)’ is

‘*y*’, and they are both functions of 𝑥. Proceed to key in the equation, note that keying

in the variable 𝑥can be done via Q). Since there isn’t a second equation, you

can press = when you see ‘g(𝑥) =’ displayed on the calculator to skip this function.

Set the table range to start from –5 and end at 5. Set the step as 1. Press qT

to generate a QR code. Scan the QR code with CASIO EDU + mobile application to

view graph. Alternatively, you may simply observe the table of values and see if values of f(𝑥) is increasing as 𝑥is increasing, instead of having to see the graph.

|  |  |
| --- | --- |
| (a) *y* = 3𝑥– 1 | (b) 2𝑥+ *y* + 1 = 7 |
| (c) 5*y* = 2𝑥– 3 | (d) 2*y* – 7 = – 5𝑥 |
| (e) 8𝑥– 2*y* – 3 = 9 | (f) 5*y* + 1 = |

Do you know of another way to know

whether linear functions are increasing

or decreasing? Hint: Observe the

coefficient of 𝑥, in the form of *y* = *m*𝑥+ c.

Unit 3 **Functions**

1. By using the CLASSWIZ calculator, find the roots and the turning point of the following quadratic equations.

**CLASSWIZ WORKSHEETS**

**ClassWiz set-up**

Press wQz for Equation/Function. You will see

the 1. Simul Equation and 2. Polynomial displayed on

the calculator. Choose 2 and select 2 again for

the degree of the polynomial. As shown, the format

is given as *a*𝑥2 + *b*𝑥+ *c*. Key in the values of *a*, *b*, *c*

accordingly.

|  |  |
| --- | --- |
| (a) 8𝑥2 + 6𝑥+ 1 = 0 | (b) 2𝑥2 + 8𝑥+ 5 = 0 |
| (c) 4𝑥2 + 7𝑥+ 3 = 0 | (d) 5𝑥2 – 10𝑥+ 4 = 0 |
| (e) – 7𝑥2 – 9𝑥+ 3 = 0 | (f) – 3𝑥2 – 4𝑥+ 1 = 0 |

**Casio ClassWiz Mathematics Workbook**

**CLASSWIZ WORKSHEETS**

1. By using the CLASSWIZ calculator, find only the real roots of the following cubic equations.

**ClassWiz set-up**

Press wQz for Equation/Function. You will see the 1.

Simul Equation and 2. Polynomial displayed on the calculator.

Choose 2 and select 3 for the degree of the polynomial.

As shown, the format is given as *a*𝑥3 + *b*𝑥2 + *c*𝑥+ *d*. Key in the

values of *a*, *b*, *c*, *d* accordingly. Write down only the answers for

the real roots. Non real roots will have the ‘*I*’ symbol in it.

(a) 2𝑥3 = 0 (b) – 3𝑥3 + 5 = 0

(c) 𝑥3 – 3𝑥+ 2 = 0 (d) 3𝑥3 + 𝑥2 – 4𝑥= 0

(e) 2𝑥3 + 3𝑥2 + 5𝑥+ 1 = 0 (f) – 4𝑥3 + 5𝑥2 – 2𝑥– 3 = 0

1. By using the CLASSWIZ calculator, find the asymptote/point of discontinuity of the following reciprocal functions. For the table range, start with –5 and end with 5. Set the step at 0.5.

**ClassWiz set-up**

Press w9 for the Table. You will see the “f(𝑥) = ” displayed

on the calculator. ‘f(𝑥)’ is ‘*y*’, and they are both functions of 𝑥.

Proceed to key in the equation, note that keying in the variable

𝑥can be done via Q). When you see “g(𝑥)” displayed on the

calculator, do not key in anything. Go to “next” by pressing =.

|  |  |
| --- | --- |
| (a) *y* = | (b) *y* = – |
| (c) *y* = + 1 | (d) *y* = |
| (e) *y* = – 1 | (f) *y* = + 2 |

Do you know of another way to know the asymptotes of

these reciprocal functions? Hint: Observe the denominator of

the fraction. Will the function be valid if the denominator is 0?

Unit 3 **Functions**

1. By using the CLASSWIZ calculator, state which function is increasing faster for the following pair of functions, utilizing Table mode. Set the step at 1.

**ClassWiz set-up**

Press w9 for the Table. You will see the ‘f(𝑥) =’ displayed on

the calculator. ‘f(𝑥)’ is ‘*y*’, they are both functions of *x*. Proceed to

key in the equation, note that keying in the variable 𝑥can be

done via Q). For the second equation, you can key it in

when you see ‘g(𝑥) =’ displayed on the calculator. Set the table

range to start from 0 and end at 10. Set the step as 1. Press

qT to generate a QR code. Scan the QR code with CASIO

EDU + mobile application to view graph. Alternatively, you

may simply observe the table of values and see whether values

of f(𝑥) or values of g(𝑥) is increasing faster as 𝑥is increasing,

instead of having to see the graph.

**CLASSWIZ WORKSHEETS**

|  |  |
| --- | --- |
| (a) *y* = and *y* = | (b) *y* = 5𝑥+ 2 and *y* = 3𝑥+ 2 |
| (c) *y* = and *y* = | (d) *y* = 2𝑥2 and *y* = 2𝑥3 |
| (e) *y* = – and *y* = – | (f) *y* = – and *y* = – |

**Casio ClassWiz Mathematics Workbook**

**CLASSWIZ WORKSHEETS**

1. Solve the following linear algebraic equations.

|  |  |
| --- | --- |
| (a) 𝑥+ 3 = 5 | (b) 2.5𝑥– 3 = 1 |
| (c) 3𝑥– 4 = 5𝑥+ 6 | (d) 2𝑥– 7 = 4𝑥+ 6 |
| (e) 𝑥– 3 = 5*x* + 8 | (f) 𝑥– 11 = – 9𝑥– |
| (g) 6 + 𝑥= 𝑥+ 10 | (h) = – 2𝑥+ 7 |
| (i) = | (j) = |
| (k) = + 2 |  |

Unit 3 **Functions**

# Level 2

For Questions 1 to 4, you may use the CLASSWIZ

calculator to help you check your answers.

**CLASSWIZ WORKSHEETS**

**ClassWiz set-up**

Press w9 for the Table. You will see the ‘f(𝑥) =’ displayed on the calculator. ‘f(𝑥)’ is ‘*y*’, they

are both functions of 𝑥. Proceed to key in the equation, note that keying in the variable 𝑥

can be done via Q). Since there is no second equation, you can press = when you

see ‘g(𝑥) =’ displayed on the calculator to skip this function. Set the table range to start from

–5 and end at 5. Set the step as 1. Press qT to generate a QR code. Scan the QR code

with CASIO EDU + mobile application to view graph.

1. Sketch the following linear functions, showing the *x* and *y*-intercepts clearly.

|  |  |
| --- | --- |
| (a) *y* = 3𝑥– 4 | (b) 2*y* = 𝑥– 5 |
| (c) *y* = –2𝑥+ 1 | (d) *y* = –2.5𝑥– 3 |
| (e) *y* = 𝑥– 1 | (f) 3*y* = –𝑥+ 3 |

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**CLASSWIZ WORKSHEETS**

1. Sketch the following quadratic functions, showing the *x* and *y*-intercepts and the turning point

clearly.

|  |  |
| --- | --- |
| (a) *y* = 𝑥2 | (b) *y* = 2𝑥2 + 3 |
| (c) *y* = (𝑥– 2)(𝑥+ 3) | (d) *y* = (𝑥– 3)2 – 4 |
| (e) *y* = – (𝑥+ 1)2 + 9 | (f) *y* = 2(2𝑥– 1)2 – 8 |

Unit 3 **Functions**

1. Sketch the following cubic functions, showing the 𝑥and *y*-intercepts clearly. Mark out the point of

**CLASSWIZ WORKSHEETS**

inflexion with a cross.

|  |  |
| --- | --- |
| (a) *y* = 𝑥3 | (b) *y* = 2𝑥3 |
| (c) *y* = – 0.5𝑥3 | (d) *y* = 0.5𝑥3 + 1 |
| (e) 3*y* = 2𝑥3 – 6 | (f) 2*y* = – 3𝑥3 – 6 |

**Casio ClassWiz Mathematics Workbook**

**CLASSWIZ WORKSHEETS**

1. Sketch the following reciprocal functions, showing the asymptotes clearly.

|  |  |
| --- | --- |
| (a) *y* = | (b) *y* = – |
| (c) *y* = + 1 | (d) *y* = |
| (e) *y* = – | (f) *y* = + 2 |

Unit 3 **Functions**

For Questions 5 to 12, you may use the CLASSWIZ calculator to help you check your answers.

**CLASSWIZ WORKSHEETS**

**ClassWiz set-up**

Press wQz, choose 1 for ‘Simul Equations’ and 2 for 2

unknowns. The format is shown in the calculator, so you need to rearrange

the equation into the format of ‘*a*𝑥+ *by* = *c*’ where *a*, *b* and *c* are constants.

1. Solve the pair of simultaneous equations.

*y* = 3𝑥– 5

3𝑥– 2*y* = 9

1. Solve the pair of simultaneous equations.

2𝑥– 5*y* = 7

*y* + 1.5𝑥= 4

1. Solve the pair of simultaneous equations.

10𝑥+ 9 = 6*y* – 4

5*y* – 4 = 8𝑥+ 1

1. Solve the pair of simultaneous equations.

𝑥– 8 = *y* + 2

11 = 5𝑥– 6*y*

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**CLASSWIZ WORKSHEETS**

1. Solve the pair of simultaneous equations.

8 – 9𝑥= 7*y* – 8

2𝑥+ 1 = *y*

1. Solve the pair of simultaneous equations.

*y* – 3 = 7𝑥+ 2

𝑥= – *y*

1. Solve the pair of simultaneous equations.

=

2𝑥– 3*y* = 5

1. Solve the pair of simultaneous equations.

= 3

4𝑥– 3 = 2*y*

Unit 3 **Functions**

1. By using the CLASSWIZ calculator, find only the real roots of the following quadratic equations.

**ClassWiz set-up**

Press wQz for Equation/Function. You will see the 1. Simul Equation

and 2. Polynomial displayed on the calculator. Choose 2 and select 2

again for the degree of the polynomial. As shown, the format is given as *a*𝑥2

+ *b*𝑥+ *c*. Key in the values of *a*, *b*, *c* accordingly.

**CLASSWIZ WORKSHEETS**

|  |  |
| --- | --- |
| 1. = 2𝑥2 + 1 | 1. + 𝑥– 1 = 0 |
| (c) – = | (d) = |
| (e) 5𝑥 – 3 = | (f) 5 – = 3𝑥 |
| (g) 2𝑥 – = 1 | (h) = |

**Casio ClassWiz Mathematics Workbook**

**CLASSWIZ WORKSHEETS**

1. Divide the following polynomials by the given divisor using long division.

|  |  |
| --- | --- |
| (a) 5𝑥3 – 2𝑥2 + 𝑥– 4 = 0, 𝑥– 1 | (b) 3𝑥3 + 9𝑥2 – 3𝑥– 7 = 0, 𝑥+ 3 |
| (c) 2𝑥3 – 2𝑥2 + 8𝑥+ 5 = 0, 𝑥+ 2 | (d) 4𝑥3 – 6𝑥2 + 8𝑥+ 11 = 0, 2𝑥– 1 |

1. Solve the following cubic equations.

**ClassWiz set-up**

Press wQz for Equation/Function. You will see the 1. Simul Equation

and 2. Polynomial displayed on the calculator. Choose 2 and select 3

for the degree of the polynomial. As shown, the format is given as a𝑥3 +

b𝑥2 + c𝑥 + d. Key in the values of a, b, c, d accordingly. Write down only the

answers for the real roots. Non real roots will have the “*i*” symbol in it.

You may

check your

answer using

the ClassWiz

calculator.

|  |  |  |  |
| --- | --- | --- | --- |
|  | 𝑥3 + 3𝑥2 – 6𝑥– 8 = 0 |  | 𝑥3 – 2𝑥2 – 9𝑥+ 18 = 0 |
|  | 2𝑥3 – 9𝑥2 + 𝑥+ 12 = 0 |  | 4𝑥3 – 19𝑥2 + 11𝑥+ 4 = 0 |
| (e) | 2𝑥3 – 𝑥2 – 10𝑥– 7 = 0 |  | |

Unit 3 **Functions**

1. Find the remainder when f(𝑥) = 3𝑥3 – 8𝑥2 – 5𝑥+ 4 is divided by 2𝑥– 3.

**CLASSWIZ WORKSHEETS**

1. Find the value of *k* if h(𝑥) = 5𝑥3 – 𝑥2 + *k*𝑥+ 3 leaves a remainder of –11 when divided by 𝑥+ 1.

1. Show that 𝑥+ 2 is a factor of g(𝑥) = 2𝑥3 – 𝑥2 – 8𝑥+ 4.

1. Show that 2𝑥– 1 is a factor of h(𝑥) = 2𝑥3 – 11𝑥2 – 𝑥+ 3.

**Casio ClassWiz Mathematics Workbook**

**CLASSWIZ WORKSHEETS**

1. Find the value of *k* if f(𝑥) = 4𝑥3 + 3𝑥2 – 2*k*𝑥+ 5 leaves a remainder of 3.25 when divided by 2𝑥– 1.

1. Find the value of *k* if h(𝑥) = 3𝑥3 + 5𝑥2 – *k*𝑥+ 8 leaves a remainder of –10 when divided by 𝑥+ 3.

1. Given that (2𝑥– 1) is a factor of g(𝑥) = *k*𝑥3 + 3𝑥2 – 2*k*2𝑥+ 3, calculate the possible values of *k*.

1. Given that (𝑥– *p*) is a factor of g(𝑥) = 2𝑥2 – (*p* + 1)𝑥– *p* – 8, calculate the possible values of *p*.

Unit 3 **Functions**

# Level 3

**CLASSWIZ WORKSHEETS**

1. Jonathan sold *x* premium tickets at $12 each and *y* non-premium tickets at $8 each for the school

concert. The total number of seats that Jonathan sold was 127. The total revenue collected was $1212.

Form two equations in *x* and *y* and solve it to find the value of xand *y*.

1. Casper bought *h* pens for $2.50 each and *k* pencils for 75 cents each. He bought it for a class of 40

students where each student gets one item and it cost him a total of $61.50. Form two equations in *h*

and *k* and solve it to find the value of *h* and *k*.

1. (a) The polynomial p(*x*) = *h*𝑥3 – *k*𝑥2 + *h*𝑥– *k* has a remainder of *R* when divided by 𝑥– 2 and a

remainder of –2*R* when divided by 𝑥+ 2. Show that *h* = 1.5*k*.

(b) Given that *h* = *k* + 1, find the values of *h* and *k*.

**Casio ClassWiz Mathematics Workbook**

1. It is given that f(𝑥) = 2𝑥3 – 𝑥2 – 5*p*𝑥+ 3*q* has a factor of 𝑥+ 3 and has a remainder of 4 when divided by

𝑥– 1. Find the value of *p* and of *q*.

Hint: You need to use simultaneous

equations to help you solve.

1. Express in partial fractions.
2. Express in partial fractions.

**CLASSWIZ WORKSHEETS**

Unit 3 **Functions**

**CLASSWIZ WORKSHEETS**

1. Express in partial fractions.
2. Express in partial fractions.
3. A line *y* = 3𝑥+ 7 intersects a second line y = – 2𝑥– 3 at point *M*.
   1. Find the coordinates of point *M*.
   2. Given that a new line is parallel to the second line and passes through a point (5, 13), find the

equation of this new line.

1. A line passes through points (2, –3) and (8, 10). Find the equation of the line.

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# Level 4

# 

1. Express in partial fractions.

**CLASSWIZ WORKSHEETS**

1. Express in partial fractions.

Unit 3 **Functions**

1. (a) Solve the cubic equation 3𝑥3 – 26𝑥2 + 52𝑥– 24 = 0.

**CLASSWIZ WORKSHEETS**

* 1. Hence, find the values of *y* for which – + 26*y* – 24 = 0.

1. (a) Solve the cubic equation 2𝑥3 – 7𝑥2 – 17𝑥+ 10 = 0.
   1. Hence, find the values of *y* for which 16*y*3 – 28*y*2 – 34*y* + 10 = 0.

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**CLASSWIZ WORKSHEETS**

1. Answer the whole of this question on a piece of graph paper.

The table below shows some values of *x* and the corresponding values of *y* where 2*y* = .

1. Complete the table.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 𝑥 | – 1 | 0 | 1 | 2 |
| *y* |  | 0.75 |  | 6.75 |

1. Use a scale of 4 cm to represent 1 unit on the *x*-axis, for –1 ≤ 𝑥≤ 3 and 2 cm to represent 1 unit on

the *y*-axis and draw the line of 2*y* = .

1. From the graph of 2*y* = , find
   1. the gradient of the line,
   2. the value of *y* when 𝑥= 1.2.
2. By drawing a suitable straight line graph, find the solution for
   1. 2*y* = ,
   2. *y* = –2𝑥+ .

Unit 3 **Functions**

# SOLUTIONS

**TEACHER’S GUIDE**

**Level 1**

1. (a) *y* = 3() + =

* 1. *y* = 3 – () + 1 =
  2. *y* = – + 5 – = –
  3. 5*y* + 2(7) = – 9

5*y* = 5

 *y* = 1

* 1. – *y* + 5 = + 1

*y* =

* 1. *y* – 3() + = –5

*y* = –

* 1. 7 – 8*y* = + 4()

8 *y* =

*y* =

* 1. –2 + 4(2) = –

– 7 = *y*

*y* = –

* 1. *y*2 – + 3 = 9() +

*y*2 =

*y* = 1.35 (to 3 sig. fig.)

2. (a) *y* = 2𝑥– 3

*x*-intercept: When *y* = 0, 2𝑥– 3 = 0

𝑥= 1.5

*y*-intercept: When 𝑥= 0, *y* = –3

(b) 8 – *y* = 𝑥

*x*-intercept: When *y* = 0, 𝑥= 9.6

*y*-intercept: When 𝑥= 0, *y* = 8

1. –3 + = 𝑥

*x*-intercept: When *y* = 0, 𝑥= –3

*y*-intercept: When 𝑥= 0, *y* = 7.5

1. 𝑥+ *y* = 1

*x*-intercept: When *y* = 0, 𝑥= 1

*y*-intercept: When *x* = 0, *y* = 1

1. 5𝑥– *y* = 3*y* – 𝑥+ 1

*x*-intercept: When *y* = 0, 6𝑥= 1

𝑥=

*y*-intercept: When 𝑥= 0, 4*y* = –1

*y* = –

**Casio ClassWiz Mathematics Workbook**

**TEACHER’S GUIDE**

1. 3*y* – 2𝑥+ 2 = 3𝑥– 4*y* + 5

*x*-intercept: When *y* = 0, 5𝑥= –3

𝑥= –

*y*-intercept: When 𝑥= 0, 7*y* = 3

*y* =

Note the value of *y* as *x* is increasing. If *y* is increasing, then the function is increasing.

3. (a) For *y* = 3𝑥– 1, *y* is increasing.

**ClassWiz steps**

Solution using ClassWiz steps Press w9.

Ke*y* in 3Q)p1 for f(𝑥).

Press = = to skip g(𝑥) since there is only1 equation.

Start the range with –5 and end the range at 5. Leave the step as 1.

Press = to see the table of values.

As *x* is increasing down the table, f(𝑥) is increasing as well, from –16 to –7. Hence, *y* = 3𝑥– 1 is an increasing function.

Alternativel*y*, we maysee the graph of this equation to determine if it is increasing or decreasing.

Press qT to generate a QR code.

Scan the QR code with CASIO EDU + mobile application to view graph.

From the graph, we can also tell that the function is increasing.

1. For 2𝑥+ *y* + 1 = 7, *y* is decreasing.



1. For 5*y* = 2𝑥– 3, *y* is increasing.
2. For 2*y* – 7 = –5𝑥, *y* is decreasing.
3. For 8𝑥– 2*y* – 3 = 9, *y* is increasing. (f) For 5*y* + 1 = 2𝑥– , *y* is increasing.



3

Another wayto know whether linear

functions are increasing or decreasing

is to observe the coefficient of *x*, in the

form of *y* = *m*𝑥+ *c*.

For e*x*ample, 8𝑥– 2*y* – 3 = 9

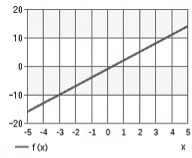
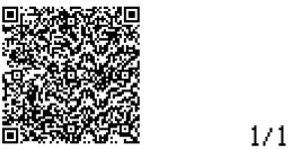
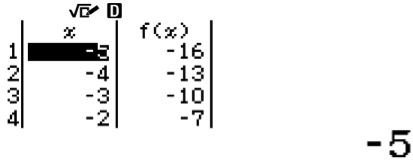
2*y* = 8𝑥– 3 – 9

2*y* = 8𝑥– 12

*y* = 4𝑥– 3

Since the coefficient of *x* is 4 which is

positive, hence the line is increasing.



Unit 3 **Functions**

4. (a) 8𝑥2 + 6𝑥+ 1 = 0

8

8

**ClassWiz steps**

Press = = to find the coordinates of the minimum

point. The calculator displays 𝑥= – and *y* = – .

4

1

**ClassWiz steps**

Press wQz for Equation/Function. You will see

the 1. Simul Equation and 2. Polynomial displayed on the

calculator. Choose 2 and select 2 again for the degree

of the polynomial. As shown, the format is given as a𝑥2 +

b𝑥+ c. Key in the values of 8, 6, 1 for a, b, c respectively.

The calculator will display 𝑥1 = – and 𝑥2= – .

**TEACHER’S GUIDE**

Roots: 𝑥= –0.25 or –0.5

Turning point: (– ,– )

(b) 2𝑥2 + 8𝑥+ 5 = 0

Roots: 𝑥= or

Turning point: (–2, –3)

(c) 4𝑥2 + 7𝑥+ 3 = 0

Roots: 𝑥= –0.75 or –1

Turning point: (– , – )

(d) 5𝑥2 – 10𝑥+ 4 = 0

Roots: 𝑥= or

Turning point: (1, –1) (e) –7𝑥2 – 9𝑥+ 3 = 0

Roots: 𝑥= or

Turning point: (– , )

(f) –3𝑥2 – 4𝑥+ 1 = 0

Roots: 𝑥= or

Turning point: (– , )

5. (a) For 2𝑥3 = 0, 𝑥= 0

**ClassWiz steps**

Press wQz for Equation/Function. You

will see the 1. Simul Equation and 2. Polynomial

displayed on the calculator. Choose 2 and

select 3 for the degree of the polynomial. As

shown, the format is given as *a*𝑥3 + *b*𝑥2 + *c*𝑥+ *d*.

Key in the values of 3, 1, –4, 0 for *a*, *b*, *c* and *d*,

respectively. The calculator will display 𝑥1 = 1 or

𝑥2= – or 𝑥3= 0.

2

(b) For –3𝑥3 + 5 = 0, 𝑥= 1.19 (to 3 sig. fig.)

(c) For 𝑥3 – 3𝑥+ 2 = 0, 𝑥= –2 or 1

(d) For 3𝑥3 + 𝑥2 – 4𝑥= 0, 𝑥= 1 or 𝑥= – or 0 (to 3 sig. fig.)

(e) For 2𝑥3 + 3𝑥2 + 5𝑥+ 1 = 0, 𝑥= –0.226 (to 3 sig. fig.)

(f) For –4𝑥3 + 5𝑥2 – 2𝑥– 3 = 0, 𝑥= –0.524 (to 3 sig. fig.)

**Casio ClassWiz Mathematics Workbook**

**TEACHER’S GUIDE**

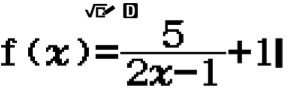
1. (a) For *y* = , asymptote is 𝑥= 0 and *y* = 0
2. For *y* = – , asymptote is 𝑥= –1 and *y* = 0
3. For *y* = + 1, asymptote is 𝑥= 0.5 and *y* = 1

Solution using ClassWiz steps

Press w9.

Key in 5a2Q)p1$ + 1 for f(𝑥).

**ClassWiz steps**



1. For *y* = , asymptote is 𝑥= 0 and *y* = 0.
2. For *y* = – 1, asymptote is 𝑥= 2 and 𝑥=

–2 and *y* = –1.

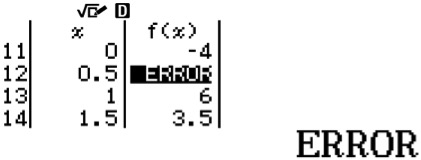
Press = = to skip g(𝑥) since there is only 1 equation.

Start the range with –5 and end the range at 5. Set the

step as 0.5.

1. For *y* = + 2, asymptote is 𝑥= 3 and 𝑥=

–3 and *y* = 2.



At 𝑥= 0.5, *y* is an error. Hence, 𝑥= 0.5 is an asymptote.

When 𝑥= 0.5, the denominator in *y* = + 1 becomes

0. Hence, there is no way for *y* to be equal to 1 as well

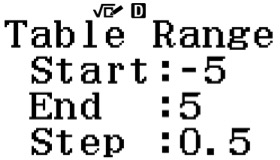
since 𝑥≠ 0.5. Hence, *y* = 1 is also an asymptote.

Alternatively, we may see the graph of this equation to

look for the asymptotes.

Press qT to generate a QR code.

Press = to see the table of values.



Another way to know the asymptotes

of these reciprocal functions is to let

the denominator of the fraction be zero

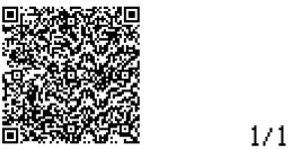
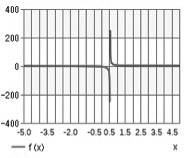
to find the vertical asymptotes. Since a

fraction is present and the numerator

is not 0, hence y cannot be 0. Let the

whole fraction be zero to find the

horizontal asymptote.



Scan the QR code with CASIO EDU + mobile application

to view graph.

From the graph, we can also tell that 𝑥= 0.5 and *y* = 1

are asymptotes.

Unit 3 **Functions**

**TEACHER’S GUIDE**

1. (a) *y* = increases faster
2. *y* = 5𝑥+ 2 increases faster

**ClassWiz steps**

Solution using ClassWiz steps

Press w9.

Key in 5Q) + 2 for f(𝑥).



1. *y* = increases faster
2. *y* = 2𝑥3 increases faster

Press 3Q) + 2 for g(𝑥).

1. *y* = – increases faster

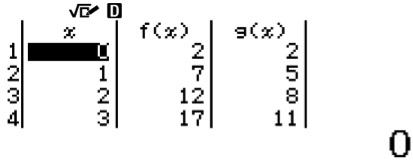


1. *y* = – increases faster

Start the range with 0 and end the range at

10. Leave the step as 1.

Press = to see the table of values.



As *x* is increasing down the table, f(𝑥) is increasing from

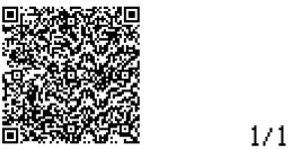
2 to 17 and g(𝑥) is increasing from 2 to 11. Hence, *y* = 5𝑥

+ 2 increases faster than *y* = 3𝑥+ 2.

Alternatively, we may see the graph of this equation to

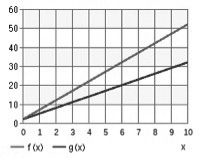
determine if it is increasing or decreasing.

Press qT to generate a QR code.



Scan the QR code with CASIO EDU + mobile application

to view graph.



From the graph, we can also tell that the function *y* = 5𝑥

+ 2 is increasing faster.

**Casio ClassWiz Mathematics Workbook**

**TEACHER’S GUIDE**

1. Solve the following linear algebraic equations.
2. 𝑥+ 3 = 5

𝑥 = 2

1. 2.5𝑥– 3 = 1

2.5𝑥= 4

𝑥= 1.6

1. 3𝑥– 4 = 5𝑥+ 6 2𝑥= –10

𝑥= –5

1. 2𝑥– 7 = 4𝑥+ 6 2𝑥= –13

𝑥= –6.5

1. 𝑥– 3 = 5𝑥+ 8

𝑥= –11

𝑥= –

1. 𝑥– 11 = –9𝑥–

𝑥=

𝑥=

1. 6 + 𝑥= 𝑥+ 10

𝑥= –4

𝑥= –

1. = –2𝑥+ 7

9𝑥– 18 = –6𝑥+ 21

15𝑥= 39

𝑥= 2.6

1. =

21𝑥– 51 = –8𝑥– 18

29𝑥= 33

𝑥=

1. =

–15𝑥– 55 = –56 + 12𝑥

27𝑥= 1

𝑥=

1. = + 2

=

–56𝑥+ 21 = 36𝑥+ 42

92𝑥= –21

𝑥= –

Unit 3 **Functions**

# Level 2

**TEACHER’S GUIDE**

1. (a) *y* = 3𝑥– 4

*y*

*y =* 3𝑥– 4

–4

-4

*y*

𝑥

**ClassWiz steps**

Solution using ClassWiz steps Press w9.

Key in 3Q)p4 for f(𝑥).

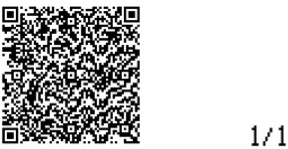


Press = = to skip g(𝑥) since there is only 1 equation.

Start the range with –5 and end the range at 5. Leave

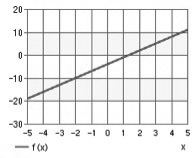
the step as 1.

Press = qT to generate a QR code.



Scan the QR code with CASIO EDU + mobile application

to view graph.



(b) 2*y* = 𝑥– 5

–

5

*2y =* 𝑥– 5

*y*

𝑥

**Casio ClassWiz Mathematics Workbook**

(c) *y* = –2𝑥+ 1

**TEACHER’S GUIDE**

*y*

1

*y*

*y* = *–*2𝑥 + 1

𝑥

(d) *y* = –2.5𝑥– 3

*y*

𝑥

–1.2

–3

*y* = –2.5𝑥– 3

(e) *y* = 𝑥– 1

*y*

*y =* 𝑥– 1

1

𝑥

– 1

(f) 3*y* = –𝑥+ 3

*y*

1

3*y = –*𝑥+ 3

–3

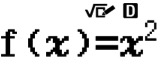
3

𝑥

Unit 3 **Functions**

2. (a) *y* = 𝑥2

**TEACHER’S GUIDE**



**ClassWiz steps**

Solution using ClassWiz steps

Press w9.

Key in Q)d for f(𝑥).

*y*

*y =* 𝑥2

0

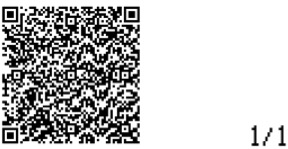
𝑥

Press = = to skip g(𝑥) since there is only 1 equation.

Start the range with –5 and end the range at 5.

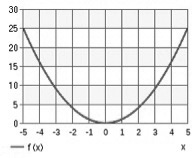
Leave the step as 1.

Press = qT to generate a QR code.



Scan the QR code with CASIO EDU + mobile application

to view graph



(b) *y* = 2𝑥2 + 3

*y*

*y =* 2𝑥2 + 3

3

𝑥

**Casio ClassWiz Mathematics Workbook**

**TEACHER’S GUIDE**

1. *y* = (𝑥– 2)(𝑥+ 3)

*y*

*y =* (𝑥– 2)(𝑥+ 3)

–3

2

𝑥

–6

–6

1. *y* = (𝑥– 3)2 – 4

*y* =(𝑥– 3)2 – 4

5

–4

*y*

𝑥

3

1

5

1. *y* = –(𝑥+ 1)2 + 9

*y*

2

𝑥

–1

–4

9

8

*y* =–(𝑥+ 1)2 + 9

*x*

1. *y* = 2(2𝑥– 1)2 – 8

*y =* 2(2𝑥– 1)2 – 8

*x*

1.5

*y*

–6

𝑥

–

–8

Unit 3 **Functions**

3. (a) *y* = 𝑥3

**TEACHER’S GUIDE**

*y*

*y* =𝑥*3*

0

𝑥

(b) *y* = 2𝑥3

*y*

*y* = 2𝑥*3*

*y*

0

𝑥

(c) *y* = –0.5𝑥3

𝑥

0

*y*

*y*

*y*

*y* =–0.5𝑥*3*

**Casio ClassWiz Mathematics Workbook**

**TEACHER’S GUIDE**

(d) *y* = 0.5𝑥3 + 1

*y*

*y* =0.5𝑥*3* + 1

1

*y*

𝑥

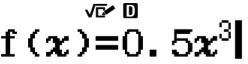
1

**ClassWiz steps**

Solution using ClassWiz steps

Press w9.

Key in 0.5Q)^3 for f(𝑥).

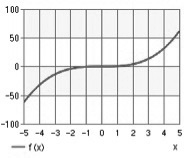


Press = = to skip g(𝑥) since there is only 1 equation.

Start the range with –5 and end the range at 5.

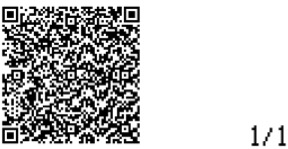
Leave the step as 1.

Press = qT to generate a QR code.



Scan the QR code with CASIO EDU + mobile application

to view graph.



(e) 3*y* = 2𝑥3 – 6

3*y* =2𝑥*3* – 6

-2

–2

*y*

𝑥

(f) 2*y* = –3𝑥3 – 6

–3

2*y* = –3𝑥*3* – 6

*y*

𝑥

Unit 3 **Functions**

*y*

*y* =

*x*

*y*

*y*

*x*

*y* =

*y*

*y*

1

-2

*x*

*y*

2

*y* =+ 1

1

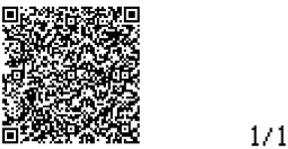
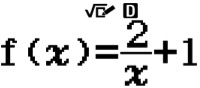
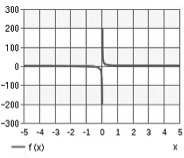
𝑥

4. (a) y =

**TEACHER’S GUIDE**

𝑥

(b) y =



**ClassWiz steps**

Solution using ClassWiz steps Press w9.

Key in 2aQ)$ + 1 for f(𝑥).

Press = = to skip g(𝑥) since there is only 1 equation.

Start the range with –5 and end the range at 5. Leave the step as 1.

Press = qT to generate a QR code.

Scan the QR code with CASIO EDU + mobile application to view graph.

𝑥

(c) y = + 1

**Casio ClassWiz Mathematics Workbook**

**TEACHER’S GUIDE**

1. *y* =

*x*2

*y*

*y*

*y* =

𝑥

1. *y* = –

*y*

*y* = –

*y*

𝑥

*x*2

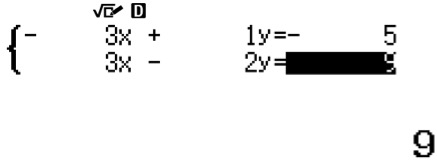
(f ) *y* =+ 2

*y*

*y* =+ 2

*y*

2



3

show 𝑥= . Press = again to show *y* = –4.

Press = to solve the simultaneous equations. It should

**ClassWiz steps**

Solution using ClassWiz steps

From equation (1), –3𝑥+ *y* = –5. From equation (2), 3𝑥– 2*y* = 9. Press wQz.

Press 1 for 1: Simul Equation.

Press 2 for 2 unknowns.

Key in equation (1) and (2) accordingly. It should look

like this:

2

𝑥

5. *y* = 3𝑥– 5 …(1)

3𝑥– 2*y* = 9 …(2)

Sub (1) into (2) 3𝑥– 2(3𝑥– 5) = 9

3𝑥– 6𝑥+ 10 = 9

–3𝑥= –1

𝑥=

3

*y* = 3() – 5 = –4

Unit 3 **Functions**

**TEACHER’S GUIDE**

6. 2𝑥– 5*y* = 7 …(1)

**TEACHER’S GUIDE**

*y* + 1.5𝑥= 4 …(2)

From (2), *y* = 4 – 1.5𝑥…(3)

Sub (3) into (1) 2𝑥– 5(4 – 1.5𝑥) = 7

2𝑥– 20 + 7.5𝑥= 7

9.5𝑥= 27

𝑥=

*y* = 4 – 1.5 = –

7. 10𝑥+ 9 = 6*y* – 4 …(1)

5*y* – 4 = 8𝑥+ 1 …(2)

From (1), 10𝑥= 6*y* – 13

𝑥= …(3)

Sub (3) into (2) 5*y* – 4 = 8() + 1

5*y* – 5 = 4()

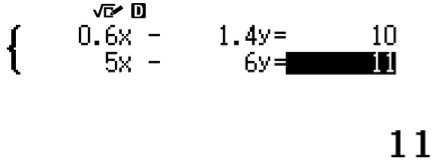
5

25*y* – 25 = 24*y* – 52

*y* = –27

𝑥= = –17.5

10



show 𝑥= –. Press = again to show *y* = – .

Press = to solve the simultaneous equations. It should

From equation (1)𝑥 – *y* = 10.

From equation (2), 5𝑥– 6*y* = 11.

Press wQz.

Press 1 for 1: Simul Equation.

Press 2 for 2 unknowns.

Key in equation (1) and (2) accordingly. It should look like this:

**ClassWiz steps**

Solution using ClassWiz steps

8. 𝑥– 8 = *y* + 2 …(1)

11 = 5𝑥– 6*y* …(2)

From (2), 5𝑥= 11+ 6*y*

5

*x* =　　　 …(3)

Sub (3) into (1) – 8 = *y* + 2

(

= + 10

33 + 18*y* = 35*y* + 250

17*y* = –217

*y* = –

𝑥= = –

**Casio ClassWiz Mathematics Workbook**

**TEACHER’S GUIDE**

9. 8 – 9𝑥= 7*y* – 8 …(1)

2𝑥+ 1 = *y* …(2)

Sub (2) into (1) 8 – 9𝑥= 7(2*x* + 1) – 8

16 – 9𝑥= 14*x* + 7

9 = 23𝑥

𝑥 =

*y* = 2() + 1 =

10. *y* – 3 = 7𝑥+ 2 …(1)

2

𝑥= –*y* …(2)

Sub (2) into (1) *y* – 3 = 7(–*y*) + 2

2

*y* + 7*y* = 5 12.5*y* = 5

2

*y* =

𝑥= –

5

11. = …(1)

2𝑥 – 3*y* = 5 …(2)

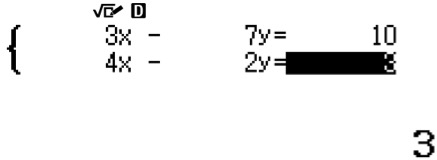
From (2), 3*y* = 2𝑥– 5 …(3) Sub (3) into (1)

=

4(3𝑥 – 5) = 5(3𝑥+ 2)

12𝑥– 20 = 15𝑥+ 10

3𝑥= –30



show 𝑥=. Press = again to show *y* = – .

Press = to solve the simultaneous equations. It should

**ClassWiz steps**

Solution using ClassWiz steps From equation (1),3𝑥– 7*y* = 10. From equation (2), 4𝑥– 2*y* = 3. Press wQz.

Press 1 for 1: Simul Equation.

Press 2 for 2 unknowns.

Key in equation (1) and (2) accordingly. It should look like this:

𝑥= –10

*y* = = –

12. = 3 …(1)

4𝑥– 3 = 2*y* …(2)

From (2), *y* =　　　　…(3)

2

Sub (3) into (1) = 3

2

+ 4 = 3𝑥– 6

2

28𝑥– 21 + 8 = 6𝑥– 12

22𝑥= 1

𝑥=

*y* = = –

2

Unit 3 **Functions**

**TEACHER’S GUIDE**

2

13. By using the CLASSWIZ calculator, find only the real roots and the turning point of the following quadratic equations.

**TEACHER’S GUIDE**

(a)= 2𝑥2 + 1 8𝑥+ 5 = 6𝑥2 + 3

**ClassWiz steps**

Press wQz for Equation/Function. You will see the 1. Simul Equation and 2.

Polynomial displayed on the calculator. Choose 2 and select 2 again for the degree of the polynomial. As shown, the format is given as *a*𝑥2 + *b*𝑥+ *c*, key in the values of 3, –4, –1 for *a*, *b* and *c* respectively.

3

6𝑥2 – 8𝑥– 2 = 0

3𝑥2 – 4𝑥– 1 = 0

Roots: 𝑥= or

3

2

3

1

The calculator displays = and =.

**ClassWiz steps**

(b) + 𝑥– 1 = 0

= 1 – 𝑥

3𝑥2 + 2 = 4 – 4𝑥

3𝑥2 + 4𝑥– 2 = 0

Roots: 𝑥= or

(c) =

3 = –2 – 5𝑥

3 + 5𝑥+ 2 = 0

Roots: 𝑥= or –1

3

(d) (3𝑥– 2)/4 = 4𝑥/(𝑥– 1)

(3𝑥– 2) (𝑥– 1) = 16𝑥

3𝑥2 – 3𝑥– 2𝑥 + 2 = 16𝑥

3𝑥2 – 21𝑥+ 2 = 0

Roots: 𝑥= or

**Casio ClassWiz Mathematics Workbook**

**TEACHER’S GUIDE**

(e) 5𝑥– 3 =

(5𝑥– 3)(2 –𝑥) = 2𝑥

*b*, *c* respectively. The calculator displays 𝑥=

1

**ClassWiz steps**

Press wQz for Equation/Function. You will see the 1. Simul Equation and 2.

Polynomial displayed on the calculator. Choose 2 and select 2 again for the degree of the polynomial. As shown, the format is given as *a*𝑥2 + *b*𝑥+ *c*. Key in the values of 5, –11, 6 for *a*,

and 𝑥2 = 1.

10𝑥– 5𝑥2 – 6 + 3𝑥= 2𝑥

5𝑥2 – 11𝑥+ 6 = 0

Roots: 𝑥= or 1

5

(f ) 5 – = 3𝑥

(5𝑥 + 5 – 3)/(𝑥+ 1) = 3𝑥

5𝑥+ 2 = 3𝑥2 + 3𝑥

3𝑥2 – 2𝑥– 2 = 0

Roots: 𝑥= or

(g)2𝑥– = 1

= 1

2𝑥2 –5𝑥+ 1 = 𝑥

2𝑥2 – 6𝑥+ 1 = 0

Roots: 𝑥= or

(h) =

2(2𝑥2 – 5 + 𝑥) = (3𝑥– 4)(5 – 𝑥)

b, c respectively. The calculator displays =

and 𝑥2 = 1 .

**ClassWiz steps**

Press wQz for Equation/Function. You will see the 1. Simul Equation and 2.

Polynomial displayed on the calculator. Choose 2 and select 2 again for the degree of the polynomial. As shown, the format is given as a𝑥2 + b𝑥+ c. Key in the values of 7, –17, 10 for a,

1

4𝑥2 + 2𝑥– 10 = 15𝑥– 3𝑥2 – 20 + 4𝑥

7𝑥2 – 17𝑥+ 10 = 0

Roots: 𝑥=or 1

7

Unit 3 **Functions**

14. (a) 5𝑥2 + 3𝑥+ 4

**TEACHER’S GUIDE**

𝑥– 1 ) 5𝑥3 – 2𝑥2 + *x* – 4

– (5𝑥3 – 5𝑥2)

3𝑥2 + 𝑥

– (3𝑥2 – 3𝑥)

4𝑥– 4

– (4𝑥– 4)

0

(b) 3𝑥2 – 3

𝑥+ 3 ) 3𝑥3 + 9𝑥2 – 3𝑥– 7

– (3𝑥3 + 9𝑥2)

0 – 3𝑥– 7

– (–3𝑥– 9)

2

(c) 2𝑥2 – 6𝑥 + 20

𝑥+ 2 ) 2𝑥3 – 2𝑥2 + 8𝑥+ 5

– (2𝑥3 + 4𝑥2)

–6𝑥2+ 8𝑥

–(–6𝑥2 – 12𝑥)

20𝑥+ 5

– (20𝑥+ 40)

–35

(d) 2𝑥2 – 2𝑥+ 3

2𝑥– 1 ) 4𝑥3 – 6𝑥2 + 8𝑥+ 11

– (4𝑥3 – 2𝑥2)

–4𝑥2 + 8𝑥

–(–4𝑥2 + 2𝑥)

6𝑥+ 11

* (6𝑥 –3)

14

15. (a) 𝑥3 + 3𝑥2 – 6𝑥– 8 = 0

Let 𝑥= –1.

(–1)3 + 3(–1)2 – 6(–1) – 8 = –1 + 3 + 6 – 8 = 0

Hence, 𝑥 + 1 is a factor of 𝑥3 + 3𝑥2 – 6𝑥– 8. By long division,

𝑥2 + 2𝑥– 8

𝑥+ 1 ) 𝑥3 + 3𝑥2 – 6𝑥– 8

– (𝑥3 + 𝑥2)

2𝑥2 – 6𝑥

– (2𝑥2 + 2𝑥)

–8𝑥– 8

– (–8𝑥– 8)

0

You may check your answers using the ClassWiz calculator. Press wQz for Equation/Function. You will see the 1. Simul Equation and 2. Polynomial

displayed on the calculator. Choose 2 and select 3 for the degree of the polynomial. As shown, the format is given as *ax*3 + + *cx* + *d*. Key in the values of 1, 3,

–6 and –8 for a, *b*, *c*, d respectively. The calculator will display x1 = –4 or x2 = 2 or x3 = –1.

𝑥3 + 3𝑥2 – 6𝑥– 8 = 0

(𝑥+ 1)(𝑥2 + 2𝑥– 8) = 0

(𝑥+ 1)(𝑥+ 4)(𝑥– 2) = 0

𝑥= –1 or –4 or 2

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**TEACHER’S GUIDE**

(b) 𝑥3 – 2𝑥2 – 9𝑥+ 18 = 0

Let 𝑥= 2

(2)3 – 2(2)2 – 9(2) + 18 = 8 – 8 – 18 + 18 = 0

Hence, 𝑥– 2 is a factor of 𝑥3 – 2𝑥2 – 9𝑥+ 18. By long division,

– 9

𝑥– 2 ) – 2𝑥2 – 9𝑥+ 18

– (𝑥3 – 2𝑥2)

0 – 9𝑥+ 18

– (–9𝑥+ 18)

0

𝑥3 – 2𝑥2 – 9𝑥+ 18 = 0 (𝑥– 2)(𝑥2 – 9) = 0

(𝑥– 2)(𝑥– 3)(𝑥+ 3) = 0

𝑥= 2 or 3 or –3

(c) 2𝑥3 – 9𝑥2 + 𝑥+ 12 = 0

Let 𝑥= –1

2(– 1)3 – 9(– 1)2 + (– 1) + 12 = –2 – 9 – 1 + 12 = 0

Hence, 𝑥+ 1 is a factor of 2𝑥3 – 9𝑥2 + 𝑥+ 12. By long division,

2𝑥2 – 11𝑥+ 12

𝑥+ 1 ) 2𝑥3 – 9𝑥2 + 𝑥+ 12

– (2𝑥3 + 2𝑥2)

–11𝑥2 + 𝑥

– (–11𝑥2 – 11𝑥)

12𝑥+ 12

– (12𝑥+ 12)

0

2𝑥3 – 9𝑥2 + 𝑥+ 12 = 0

(𝑥+ 1)(2𝑥2 – 11𝑥+ 12) = 0 (𝑥+ 1)(2𝑥– 3)(𝑥– 4) = 0

𝑥= –1 or 1.5 or 4

(d) 4𝑥3 – 19𝑥2 + 11𝑥+ 4 = 0

Let 𝑥= 1

4(1)3 – 19(1)2 + 11(1) + 4 = 4 – 19 + 11 + 4 = 0

Hence, 𝑥– 1 is a factor of 4𝑥3 – 19𝑥2 + 11𝑥+ 4. By long division,

4𝑥2 – 15𝑥– 4

𝑥– 1 ) 4𝑥3 – 19𝑥2 + 11𝑥+ 4

– (4𝑥3 – 4𝑥2)

–15𝑥2 + 11𝑥

– (–15*x*2 + 15*x*)

–4𝑥+ 4

– (–4𝑥+ 4)

calculator will display = 4 or = 1 or = – .

3

2

1

You may check your answers using the ClassWiz calculator. Press wQz for Equation/Function. You will see the 1. Simul Equation and 2. Polynomial displayed on the calculator. Choose 2 and select 3 for the degree of the polynomial. As shown,

the format is given as a+ b + c𝑥+ d. Key in the values of 4, –19, 11, 4 for a, b, c, d respectively. The

0

4𝑥3 – 19𝑥2 + 11𝑥+ 4 = 0 (𝑥– 1)(4𝑥2 – 15𝑥– 4) = 0

(𝑥– 1)(4𝑥+ 1)(𝑥– 4) = 0

𝑥= 1 or –0.25 or 4

(e) 2*x*3 – *x*2 – 10*x* – 7 = 0

**TEACHER’S GUIDE**

L=

Unit 3 **Functions**

(e) 2𝑥3 – 𝑥2 – 10𝑥– 7 = 0

**TEACHER’S GUIDE**

Let 𝑥= –1

2(– 1)3 – (–1)2 – 10(–1) – 7 = –2 – 1 + 10 – 7 = 0

Hence, 𝑥+ 1 is a factor of 2𝑥3 – 𝑥2 – 10𝑥– 7.

By long division,

2𝑥2 – 3𝑥– 7

𝑥+ 1 ) 2𝑥3 – 𝑥2 – 10𝑥– 7

– (2𝑥3 + 2𝑥2)

–3𝑥2 – 10𝑥

– (–3𝑥2 – 3𝑥)

–7𝑥– 7

– (–7𝑥– 7)

0

You may check your answers using the ClassWiz calculator. Press wQz for Equation/Function. You will see the 1. Simul Equation and 2. Polynomial displayed on the calculator. Choose 2 and select 3 for the degree of the polynomial. As shown, the format is given as *a* + *b* + *c*𝑥+ *d*. Key in the values of 2, –1, –10, –7 for *a*, *b*, *c* and d respectively. The　calculator will display 𝑥1 = 2.765564437 or 𝑥2 = –1 or 𝑥3 = –1.265564437.

2𝑥3 – 𝑥2 – 10𝑥– 7 = 0

(𝑥+ 1)(2𝑥2 – 3𝑥– 7) = 0

(𝑥+ 1) = 0 or (2𝑥2 – 3𝑥– 7) = 0

𝑥= –1 or 𝑥=

𝑥= –1 or 𝑥= or

1. Let 𝑥=

f = 3 – 8 – 5+ 4 = –

1. Find the value of *k* if h(𝑥) = 5𝑥3 – 𝑥2 + *k*𝑥+ 3 leaves a remainder of –11 when divided by 𝑥+ 1. Given: h(–1) = –11.

h(–1) = 5(–1)3 – (–1)2 + *k*(–1) + 3 = –11

–5 – 1 – *k* + 3 = –11

*k* = 8

1. g(–2) = 2(–2)3 – (–2)2 – 8(–2) + 4 = –16 – 4 + 16 + 4 = 0.

To show that *x* + 2 is a factor of g(𝑥) = 2𝑥3 – 𝑥2 – 8𝑥+ 4, we need to show that g(–2) = 0.

Hence, 𝑥+ 2 is a factor of 2𝑥3 – 𝑥2 – 8𝑥+ 4.

1. h = 2– 11+ 3 = + 3 = 0.

Hence, 2𝑥– 1 is a factor of h(𝑥) = 2𝑥3 – 11𝑥2 – 𝑥 + 3.

1. Given: f = 3.25

f = 4+ 3– 2*k* + 5 = 3.25

+ – *k* + 5 = 3.25

*k* = 3

1. Given: h(–3) = –10

h(–3) = 3(–3)3 + 5(–3)2 – *k*(–3) + 8 = –10

–81 + 45 + 3*k* + 8 = –10

3*k* = 18

*k* = 6

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22. Given: g = 0,

g = *k* + 3 – + 3 = 0

+ + 3 = 0

*k* + 6 – 8*k*2 + 24 = 0

8*k*2 – *k* – 30 = 0

(8*k* + 15)(*k* – 2) = 0

*x* = – – or 2

8

23. Given: g(p) = 0

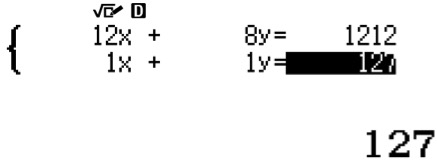
**TEACHER’S GUIDE**

g(*p*) = 2*p*2 – (*p* + 1)*p* – *p* – 8 = 0 2*p*2 – *p*2 – *p* – *p* – 8 = 0

*p*2 – 2*p* – 8 = 0 (*p* – 4)(*p* + 2) = 0

*p* = 4 or –2

# Level 3



**ClassWiz steps**

Solution using ClassWiz steps

From equation (1), 12𝑥+ 8*y* = 1212. From equation (2), 𝑥+ *y* = 127.

Press wQz.

Press 1 for 1: Simul Equation.

Press 2 for 2 unknowns.

Key in equation (1) and (2) accordingly. It should look like this:

Press = to solve the simultaneous equations. It should show 𝑥= 49. Press = again to show *y* = 78.

1. 12𝑥+ 8*y* = 1212 …(1)

𝑥+ *y* = 127 …(2)

From (2), 𝑥= 127 – *y* …(3) Sub (3) into (1)

12(127 – *y*) + 8*y* = 1212

1524 – 12*y* + 8*y* = 1212

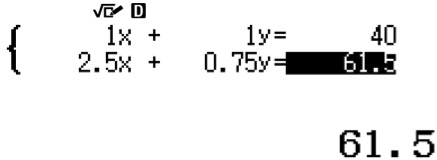
312 = 4*y*

*y* = 78

𝑥= 127 – 78 = 49

Unit 3 **Functions**

2. *h* + *k* = 40 …(1)



**ClassWiz steps**

Solution using ClassWiz steps

From equation (1), 𝑥+ *y* = 40. (let h be 𝑥and k be *y*.) From equation (2), 2.5𝑥+ 0.75*y* = 61.5.

Press wQz.

Press 1 for 1: Simul Equation.

Press 2 for 2 unknowns.

Key in equation (1) and (2) accordingly. It should look like this:

Press = to solve the simultaneous equations. It should show 𝑥= 18. Press = again to show *y* = 22.

Hence *h* = 18 and *k* = 22.

**TEACHER’S GUIDE**

2.5*h* + 0.75*k* = 61.50 …(2)

From (1), *h* = 40 – *k* …(3) Sub (3) into (2) 2.5(40 – *k*) + 0.75*k* = 61.50

100 – 2.5*k* + 0.75*k* = 61.50

1.75*k* = 38.5

*k* = 22

*h* = 40 – 22 = 18

3. (a) p(2) = *R* …(1)

p(– 2) = –2*R* …(2)

From (1), p(2) = *h*(2)3 – *k*(2)2 + *h*(2) – *k* = *R*

8*h* – 4*k* + 2*h* – *k* = *R*

10*h* – 5*k* = *R* …(3)

From (2), p(– 2) = *h*(– 2)3 – *k*(– 2)2 + *h*(– 2) – *k* = –2*R*

– 8*h* – 4*k* – 2*h* – *k* = –2*R*

– 10*h* – 5*k* = –2*R* …(4)

Sub (3) into (4) – 10*h* – 5*k* = –2(10*h* – 5*k*)

– 10*h* – 5*k* = –20*h* + 10*k* 10*h* = 15*k*

*h* = 1.5*k* (shown)

(b) Given that *h* = *k* + 1, find the values of *h* and *k*. *h* = 1.5*k* …(1)

*h* = *k* + 1 …(2)

Sub (1) into (2) 1.5*k* = *k* + 1

0.5*k* = 1

*k* = 2

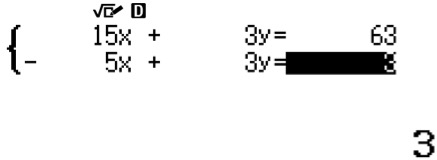
*h* = 1.5(2) = 3

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**TEACHER’S GUIDE**

4.

|  |  |  |  |
| --- | --- | --- | --- |
| f(–3) = 0 …(1)  f(1) = 4 …(2) | |  | **ClassWiz steps** |
| From (1), | f(–3) = 2(–3)3 – (–3)2 – 5*p*(–3) + 3*q* = 0 |  | Solution using ClassWiz steps |
|  | –63 + 15*p* + 3*q* = 0 …(1) |  | From equation (1),15𝑥+ 3*y* = 63. (let *p* be 𝑥 |
| From (2), | f(1) = 2(1)3 – (1)2 – 5*p*(1) + 3*q* = 4 |  | and *q* be *y*.) |
|  | 1 – 5*p* + 3*q* = 4 |  | From equation (2), –5𝑥+ 3*y* = 3. |
|  | 3*q* = 3 + 5*p* …(2) |  | Press wQz. |
| Sub (2) into (1) –63 + 15*p* + 3 + 5*p* = 0  20*p* = 60  *p* = 3  *q* =  = 6 | |  | Press 1 for 1: Simul Equation.  Press 2 for 2 unknowns.  Key in equation (1) and (2) accordingly. It should look like this:  Press = to solve the simultaneous equations. It should show 𝑥= 3. Press = again to show *y* = 6.  Hence *p* = 3 and *q* = 6. |



5 . = +

8𝑥– 3 = *A*(𝑥+ 2) + *B*(2𝑥– 1)

8𝑥– 3 = (*A* + 2*B*)𝑥+ (2*A* – *B*)

*A* + 2*B* = 8 …(1)

2*A* – *B* = –3

*B* = 2*A* + 3 …(2)

Sub (2) into (1) *A* + 2(2*A* + 3) = 8

5*A* = 2

*A* =

*B* = 2 + 3 =

= +

1. = + +

7𝑥2 + 7𝑥– 20 = *A*(𝑥+ 1)(𝑥– 3) + *B*(𝑥– 3) + *C*(𝑥+ 1)2

7𝑥2 + 7𝑥– 20 = *A*(𝑥2 – 2𝑥– 3) + *B*(𝑥– 3) + *C*(𝑥2 + 2𝑥+ 1)

7𝑥2 + 7𝑥– 20 = (*A* + *C*)𝑥2 + (–2*A* + *B* + 2*C*)𝑥+ (–3*A* – 3*B* + *C*) *A* + *C* = 7 …(1)

–2*A* + *B* + 2*C* = 7…(2)

–3*A* – 3*B* + *C* = –20 …(3)

Take 3(2) + (3) 3(–2*A* + *B* + 2*C*) + (–3*A* – 3*B* + *C*) = 3(7) + (–20)

–6*A* + 3*B* + 6*C* – 3*A* – 3*B* + *C* = 1

–9*A* + 7*C* = 1 …(4) Take 9(1) + (4) 9*A* + 9*C* – 9*A* + 7*C* = 63 + 1

16*C* = 64

*C* = 4

Unit 3 **Functions**

From (1), *A* + 4 = 7

**TEACHER’S GUIDE**

*A* = 3

From (2), –2(3) + *B* + 2(4) = 7

*B* = 5

= ++

1. is improper.

By long division,

6𝑥2 + 13𝑥– 5 ) 7𝑥2 – 6𝑥+ 8

– (7𝑥2 +𝑥 –)

– 𝑥+

= +

= +

= + +

-𝑥+ = *A*(2𝑥+ 5+ *B*(3𝑥– 1)

When 𝑥= –2.5, *B*[3(–2.5) – 1] = – (–2.5) +

*B* = –

When 𝑥= , *A* [2 + 5] = – +

*A*  =

= + -

1. = +

–2𝑥2 – 5𝑥– 6 = *A*(𝑥2 + 2) + (*B*𝑥+ *C*)(𝑥– 2)

–2𝑥2 – 5𝑥– 6 = (*A* + *B*)𝑥2 + (*C* – 2*B*)𝑥+ (2*A* – 2*C*)

*A* + *B* = –2 …(1)

*C* – 2*B* = –5 …(2)

2*A* – 2*C* = –6 …(3)

From (1), *B* = –2 – *A* …(4) Sub (4) into (2) *C* – 2(–2 – *A*) = –5

*C* + 4 + 2*A* = –5 2*A* + *C* = –9

*C* = –9 – 2*A* …(5) Sub (5) into (3) 2*A* – 2(–9 – 2*A*) = –6

2*A* + 18 + 4*A* = –6

6*A* = –2

*A* = –4

*B* = –2 + 4 = 2

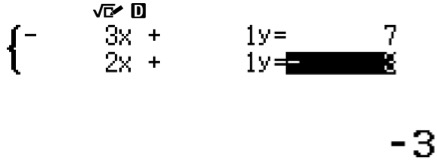
*C* = –9 – 2(–4) = –1

= +

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**TEACHER’S GUIDE**

9. A line *y* = 3𝑥+ 7 intersects a second line *y* = –2𝑥– 3 at point M. (a) *y* = 3𝑥+ 7 …(1)



**ClassWiz steps**

Solution using ClassWiz steps From equation (1), –3𝑥+ *y* = 7. From equation (2), 2𝑥+ *y* = –3. Press wQz.

Press 1 for 1: Simul Equation.

Press 2 for 2 unknowns.

Key in equation (1) and (2) accordingly. It should look like this:

Press = to solve the simultaneous equations. It should show 𝑥= –2. Press = again to show *y* = 1.

Hence, coordinates of M = (–2,1)

*y* = –2𝑥– 3 …(2)

Equating (1) and (2): 3𝑥+ 7 = –2𝑥– 3

5𝑥= –10

𝑥= –2

*y* = 3(–2) + 7 = 1

Coordinates of M = (–2,1)

(b) New line is parallel to the second line: gradient of new line is –2.

*y* = –2𝑥+ *c*

Sub (5, 13) into the equation, 13 = –2(5) + *c*

*c* = 23

*y* = –2𝑥+ 23

10. =

=

6*y* + 18 = 13𝑥– 26

6*y* = 13𝑥– 44

*y* =𝑥–

Unit 3 **Functions**

# Level 4

**TEACHER’S GUIDE**

1. Factorise 𝑥3 – 5𝑥2 – 2𝑥+ 24 first.

Let 𝑥= –2

(–2)3 – 5(–2)2 – 2(–2) + 24 = –8 – 20 + 4 + 24 = 0

𝑥+ 2 is a factor of 𝑥3 – 5𝑥2 – 2𝑥+ 24.

𝑥2 – 7𝑥+ 12

𝑥+ 2 ) 𝑥3 – 5𝑥2 – 2𝑥+ 24

– (𝑥3 + 2𝑥2)

–7𝑥2 – 2𝑥

– (–7𝑥2 – 14𝑥)

12𝑥+ 24

– (12𝑥+ 24)

0

𝑥3 – 5𝑥2 – 2𝑥+ 24 = (𝑥+ 2)(𝑥2 – 7𝑥+ 12)

= (𝑥+ 2)(𝑥– 4)(𝑥– 3)

𝑥3 – 5𝑥2 – 2𝑥+ 24 = (𝑥+ 2)(𝑥2 – 7𝑥+ 12)

= (𝑥+ 2)(𝑥– 4)(𝑥– 3)

= = + +

𝑥2 – 3𝑥– 9 = *A*(𝑥– 4)(𝑥– 3) + *B*(𝑥+ 2)(𝑥– 3) + *C*(𝑥+ 2)(𝑥– 4)

𝑥2 – 3𝑥– 9 = *A*(𝑥2 – 7𝑥+ 12) + *B*(𝑥2 – 𝑥– 6) + *C*(𝑥2 – 2𝑥– 8)

𝑥2 – 3𝑥– 9 = (*A* + *B* + *C*)𝑥2 + (–7*A* – *B* – 2*C*)𝑥+ (12*A* – 6*B* – 8*C*)

*A* + *B* + *C* = 4 …(1)

–7*A* – *B* – 2*C* = –24 …(2) 12*A* – 6*B* – 8*C* = 26 …(3)

(1) + (2) *A* + *B* + *C* – 7*A* – *B* – 2*C* = 4 – 24

–6*A* – *C* = –20

*C* = 20 – 6*A* …(4)

6(1) + (3) 6*A* + 6*B* + 6*C* + 12*A* – 6*B* – 8*C* = 24 + 26

18*A* – 2*C* = 50 …(5) Sub (4) into (5) 18*A* – 2(20 – 6*A*) = 50

18*A* – 40 + 12*A* = 50

30*A* = 90

*A* = 3

From (4), *C* = 20 – 6(3) = 2

From (1), 3 + *B* + 2 = 4

*B* = –1

= + +

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**TEACHER’S GUIDE**

1. Factorise 2𝑥3 + 5𝑥2 – 2𝑥– 15 first. Let 𝑥=

2

2+ 5– 2– 15 = + – 3 – 15 = 0

Hence, 2𝑥– 3 is a factor of 2𝑥3 + 5𝑥2 – 2𝑥– 15.

By long division,

𝑥2 + 4𝑥+ 5

Note that 𝑥2 + 4𝑥+ 5 cannot be factorised! So this is considered a non–factorizable quadratic equation.

2𝑥– 3 ) 2𝑥3 + 5𝑥2 – 2𝑥– 15

–(2𝑥3 – 3𝑥2)

8𝑥2 – 2𝑥

– (8𝑥2 – 12𝑥)

10𝑥– 15

– (10𝑥– 15)

0

2𝑥3 + 5𝑥2 – 2𝑥– 15

= (2𝑥– 3)(𝑥2 + 4𝑥+ 5)

Since is improper, by long division,

4

6𝑥2 + 12𝑥 – 5 ) 8𝑥3 + 17𝑥2 + 41𝑥– 34

– (8𝑥3 + 20𝑥2 – 8𝑥– 60)

–3𝑥2 + 49𝑥+ 26

= 4 +

= 4 +

= 4 + +

–3𝑥2 + 49𝑥+ 26 = A(𝑥2 + 4𝑥+ 5) + (2𝑥– 3)(B𝑥+ C)

–3𝑥2 + 49𝑥+ 26 = A𝑥2 + 4A𝑥+ 5A + 2B𝑥2 + (2C – 3B)𝑥– 3C

–3𝑥2 + 49𝑥+ 26 = (A + 2B) 𝑥2 + (4A + 2C – 3B)𝑥+ (5A – 3C)

*A* + 2*B* = –3 …(1)

4*A* + 2*C* – 3*B* = 49 …(2)

5*A* – 3*C* = 26 …(3)

From (1), *B* = …(4)

2

Sub (4) into (2) 4*A* + 2*C* – 3 = 49

8*A* + 4*C* + 9 + 3*A* = 98

11*A* + 4*C* = 89

4*C* = 89 – 11*A*

*C* = …(5)

4

Sub (5) into (3) 5*A* – 3 = 26

20*A* – 267 + 33*A* = 104

53*A* = 371

*A* = 7

From (1), 7 + 2*B* = –3

2*B* = –10

*B* = –5

From (5), *C* = = 3

= 4 +

Unit 3 **Functions**

4

1. (a) Let 𝑥= 2

**TEACHER’S GUIDE**

3(2)3 – 26(2)2 + 52(2) – 24 = 24 – 104 + 104 – 24 = 0

𝑥– 2 is a factor of 3𝑥3 – 26𝑥2 + 52𝑥– 24. By long division,

3𝑥2 – 20𝑥+ 12

𝑥– 2 ) 3𝑥3 – 26𝑥2 + 52𝑥– 24

– (3𝑥3 – 6𝑥2)

–20𝑥2 + 52𝑥

– (–20𝑥2 + 40𝑥)

12𝑥– 24

– (12𝑥– 24)

You may check your answers using the ClassWiz calculator. Press wQz for Equation/ Function. You will see the 1. Simul Equation

and 2. Polynomial displayed on the calculator. Choose 2 and select 3 for the degree of the polynomial. As shown, the format is given as *a*𝑥3 + *b*𝑥2 + *c*𝑥+ *d*. Key in the values of 3, –26, 52,

–24 for *a*, *b*, *c*, *d* respectively. The calculator will

0

3𝑥3 – 26𝑥2 + 52𝑥– 24 = 0 (𝑥– 2)(3𝑥2 – 20𝑥+ 12) = 0 (𝑥– 2)(3𝑥– 2)(𝑥– 6) = 0

𝑥= 2 or or 6

display = 6 or = 2 or = .

(b) Let 𝑥= *y*.

2

1

3– 26 + 52 – 24 = – + 26𝑥– 24

Hence, 𝑥= 2 or or 6

*y* = 2 or or 6

*y* = 4 or or 12

1. (a) Solve the cubic equation 2𝑥3 – 7𝑥2 – 17𝑥+ 10 = 0. Let 𝑥= –2

2(– 2)3 – 7(–2)2 – 17(–2) + 10 = –16 – 28 + 34 + 10 = 0

𝑥+ 2 is a factor of 2𝑥3 – 7𝑥2 – 17𝑥+ 10. By long division,

2𝑥2 – 11𝑥+ 5

𝑥+ 2 ) 2𝑥3 – 7𝑥2 – 17𝑥+ 10

– (2𝑥3 + 4𝑥2)

–11𝑥2 – 17𝑥

1

You may check your answers using the ClassWiz calculator. Press wQz for Equation/ Function. You will see the 1. Simul Equation

and 2. Polynomial displayed on the calculator. Choose 2 and select 3 for the degree of the polynomial. As shown, the format is given as a𝑥3 + b𝑥2 + c𝑥 + d. Key in the values of 2, –7, –17, 10 for a, b, c, d respectively. The calculator will

display = 5 or = or = –2.

– (–11𝑥2 – 22𝑥)

5𝑥+ 10

– (5𝑥+ 10)

0

2𝑥3 – 7𝑥2 – 17𝑥+ 10 = 0 (𝑥+2)(2𝑥2 – 11𝑥+ 5) = 0 (𝑥+ 2)(2𝑥– 1)(𝑥– 5) = 0

𝑥= –2 or or 5

1. Hence, find the values of *y* for which Let 𝑥= 2*y*

2(2*y*)3 – 7(2*y*)2 – 17(2*y*) + 10 = 16*y*3 – 28𝑥2 – 34𝑥+ 10

Hence 𝑥 = –2 or or 5

*2y* = –2 or or 5

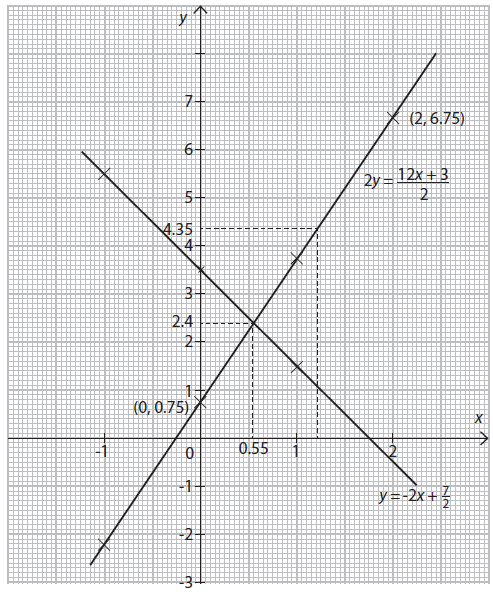
*y* = –1 or or

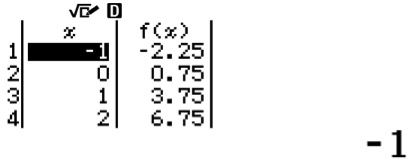
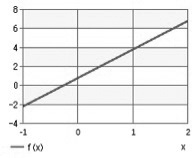
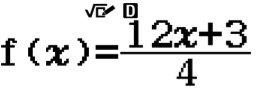
**Casio ClassWiz Mathematics Workbook**

**TEACHER’S GUIDE**

1. (a)

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 𝑥 | –1 | 0 | 1 | 2 |
| *y* | –2.25 | 0.75 | 3.75 | 6.75 |

(b) 　



**ClassWiz steps**

Check your answer using ClassWiz steps

Press w9.

After rearranging the equation, key in a12Q) + 3R4 for f(𝑥).

Press = = to skip g(𝑥) since there is 　　　only 1 equation.

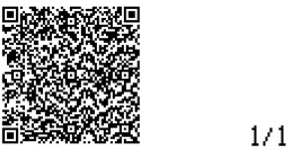
Start the range with –1 and end the range　　 at 2. Set the step as 1.

Press = to see the table of values.

This may also be used to fill in the table of values for question 5(a).

Press qT to generate a QR code.

Scan the QR code with CASIO EDU + mobile application to view graph and compare with the graph you have drawn.



(c) (i) Using 2 coordinates (0, 0.75) and (2, 6.75),

Gradient = = = 3

(ii) When 𝑥= 1.2, *y* = 4.35 from the graph.

(d) Draw *y* = –2𝑥+ .

|  |  |  |  |
| --- | --- | --- | --- |
| 𝑥 | –1 | 0 | 1 |
| *y* | 5.5 | 3.5 | 1.5 |

(e) The intersection point from the graph is (0.55, 2.4).