**Unit**

**12**

**RECURSION, SEQUENCES AND SERIES**

# KEY CONCEPTS

**TEACHER’S GUIDE**

## Arithmetic Series

A series is called an arithmetic series if:

*d* = –

where *d* is the constant, called the common difference.

The general term of an arithmetic series with a first term *a* and a common difference *d* is given as:

= *a* + (*n* – 1)*d*

The sum of the first *n* terms of an arithmetic series with a first term *a* is given as:

when the last term *I* = *Tn* is known.

= (*a* + /)

when the common difference *d* is known.

=

## Geometric Series

A series is called a geometric series if

*r* =

where *r* is a constant, called the common ratio.

The general term of a geometric series with a first term *a* and a constant ratio *r* is given as:

=

The sum of the first *n* terms of a geometric series is given as:

for *r* > 1

=

for *r* < 1

=

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# WORKED EXAMPLES

**TEACHER’S GUIDE**

**Level 1**

### Worked Example 1

Given the arithmetic sequence 5, 9, 13, 17, …,

1. find the next 2 terms.
2. write down the *n*th term.
3. find the sum of the first 10 terms of the series.

Solution (a) 21, 25

(b) *Tn* = 5 + (*n* – 1)(4) = 4*n* + 1

(c) *S*10

= [2(5) + (10 – 1)(4)] = 230

### Worked Example 2

Given the geometric sequence 5, 15, 45, …,

(a) find the next 2 terms.

(b) write down the *n*th term.

(c) find the sum of the first 10 terms of the series.

Solution

(a) 135, 405

(b) *Tn*

(c) *S*10

= 5(3)*n* –1 or

=

### Worked Example 3

Given the following general formula, write out the first 7 terms for *n* = 1, 2, 3, 4, 5, 6, 7.

(a) 5*n* – 3,

(b) *n*2 – 2

Solution

(a) 2, 7, 12, 17, 22, 27, 32

(b) –1, 2, 7, 14, 23, 34, 47

# Level 2

### Worked Example 4

An arithmetic sequence has first term 7 and common difference 6. Find the twelfth term.

Solution

*T*12 = 7 + (12 – 1)(6) = 73

Unit 12 **Recursion, Sequences and Series**

### Worked Example 5

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Find the sum of the first 10 terms of 5 + 11 + 17 + 23 + …

Solution

Since the series is increasing by an addition of 6, it is an arithmetic progression.

*S*10

= [2(5) + (10 – 1)(6)] = 320

### Worked Example 6

Find the sum of the first 10 terms of a geometric progression 2187, –729, 243, …

Solution

### Worked Example 7

Find the two possible values of *x* if for the geometric progression 9, *x*, 81

Solution

*x*2 = 729

*x* = ±27

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# CLASSWIZ WORKSHEETS

**CLASSWIZ WORKSHEETS**

**Level 1 **

1. Given the arithmetic sequence 2, 5, 8, 11, find the next 2 terms. Next, write down the *n*th term.
2. Given the arithmetic sequence 7, 12, 17, 22, find the next 2 terms. Next, write down the *n*th term.
3. Given the arithmetic sequence 11, 19, 27, 35, find the next 2 terms. Next, write down the *n*th term.
4. Given the arithmetic sequence 45, 31, 17, 3, find the next 2 terms. Next, write down the *n*th term.
5. Given the arithmetic sequence 51, 74, 97, 120, find the next 2 terms. Next, find the sum of the first 10 terms of the series.
6. Given the arithmetic sequence 12, 54, 96, 138, find the next 2 terms. Next, find the sum of the first 10 terms of the series.
7. Given the arithmetic sequence 60, 77, 94, 111, find the next 2 terms. Next, find the sum of the first 10 terms of the series.

Unit 12 **Recursion, Sequences and Series**

1. Given the arithmetic sequence 2, 5, 8, 11, 14, find the next 2 terms. Next, find the sum of the first 10 terms of the series.

**CLASSWIZ WORKSHEETS**

1. Given the geometric sequence 2, 4, 8, 16, find the next 2 terms. Next, write down the *n*th term.
2. Given the geometric sequence 6, 18, 54, 162, find the next 2 terms. Next, write down the *n*th term.
3. Given the geometric sequence, 7, 28, 112, 448, find the next 2 terms. Next, write down the *n*th term.
4. Given the geometric sequence 9, –18, 36, –72, find the next 2 terms. Next, write down the *n*th term.
5. Given the geometric sequence 188, 94, 47, 23.5, find the next 2 terms. Next, find the sum of the first 10 terms of the series.
6. Given the geometric sequence 2, 14, 98, 686, find the next 2 terms. Next, find the sum of the first 10 terms of the series.

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1. Given the geometric sequence 3, 12, 48, 192, find the next 2 terms. Next, find the sum of the first 10 terms of the series.

**TEACHER’S GUIDE**

1. Given the geometric sequence 2, 5, 12.5, 31.25, find the next 2 terms. Next, find the sum of the first 10 terms of the series.
2. Given the following general formula, write out the first 7 terms for *n* = 1, 2, 3, 4, 5, 6, 7

Teacher to note that these are odd number series.

(b) 2*n* – 1

(a) 2*n* + 1

(c) 3*n* – 1 (d) 4*n* + 5

(e) 8 – 3*n*

1. Given a general formula, write out the first 7 terms for *n* = 1, 2, 3, 4, 5, 6, 7

(a) 2*n* (b) 2*n* + 1

(c) 3*n*2 (d) *n*3

(e) *n*3 + 2

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# Level 2

**CLASSWIZ WORKSHEETS**

1. An arithmetic sequence has first term 8 and common difference 12. Find the fifteenth term.
2. An arithmetic sequence has first term 18 and common difference 5. Find the sum of the twenty terms.
3. Find the sum of the first 10 terms of the series 2, 8, 14, 20, …
4. Find the sum of the first 10 terms of 14, 23, 32, 41, …
5. The third term of an arithmetic series is 12 and the fifth term is 24. Find the value of the common difference and the first term, hence, evaluate the sum of the first 10 terms.
6. Given that the third term and the twelfth term of an arithmetic progression is 20 and 56 respectively, find the sum of the first 20 terms.

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1. Find the sum of the first 10 terms of a geometric progression 18, 6, 2, …

**CLASSWIZ WORKSHEETS**

1. Find the sum of the first *n* terms, in terms of *n*, of a geometric progression 10, –5, 1, …
2. Find the two possible values of *x* if for the geometric progression 5, *x*, 245
3. Find the two possible values of *x* if for the geometric progression 7, *x*, 35
4. The sum of a geometric progression is *Sn* = (3*n* – 1). Find the first term and the common ratio.
5. The sum of a geometric progression is = 4(1 – ). Find the first term and the common ratio.

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# Level 3

**CLASSWIZ WORKSHEETS**

1. Given that the sum of the first 4 terms of an arithmetic progression is 100 and the sum of the first 7 terms is 238, find the sum of the *n* terms, in terms of *n*.
2. Find the sum of all positive integers less than 40 which are not multiples of 3.
3. Find the sum of all positive integers less than 80 and which are not multiples of 5 or of 7.

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1. Find three numbers in geometric progression such that their sum is 26 and their product 216.

**CLASSWIZ WORKSHEETS**

1. Find three numbers in geometric progression such that their sum is 13 and their product 27.

Unit 12 **Recursion, Sequences and Series**

**CLASSWIZ WORKSHEETS**

1. Find the sum of *n* terms of the geometric series 1, , , … Find the least number of terms needed

for the sum of these terms to exceed 3.

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# SOLUTIONS

**TEACHER’S GUIDE**

**Level 1**

**TEACHER’S GUIDE**

1. 14, 17

*Tn* = 2 + 3(*n* – 1) = 3*n* – 1

2. 27, 32

*Tn* = 7 + 5(*n* – 1) = 5*n* + 2

3. 43, 51

*Tn* = 11 + 8(*n* – 1) = 8*n* + 3

4. –11, –25

*Tn* = 45 + (–14)(*n* – 1) = 59 – 14*n*

5. 143, 166

*S*10 = [2(51) + (10 – 1)(23)] = 1545

6. 180, 222

*S*10 = [2(12) + (10 – 1)(42)] = 2010

7. 128, 145

*S*10 = [2(60) + (10 – 1)(17)] = 1365

8. 17, 20

*S*10 = [2(2) + (10 – 1)(3)] = 155

9. 32, 64

*Tn* = 2(2)*n* – 1 = 2*n*

10. 486, 1458

*Tn* = 6(3)*n* – 1 = 2(3) *n*

11. 1792, 7168

*Tn* = 7(4)*n* – 1 or

12. 144, –288

*Tn* = 9(–2)*n* – 1

13. 11.75, 5.875

*S*10 = = 375

14. 4802, 33614

*S*10 = = 94 158 416

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**TEACHER’S GUIDE**

15. 768, 3072

*S*10

= 3(410 – 1) = 1 048 575

4 – 1

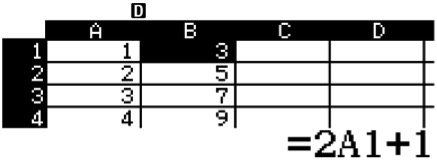
16. 78.125, 195.3125

17.

*S*10

= 2(2.510 – 1) = 12 714.32 (7 s.f.)

2.5 – 1



**ClassWiz steps**

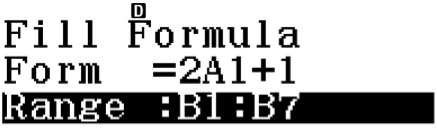
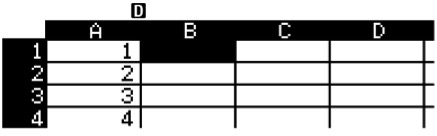
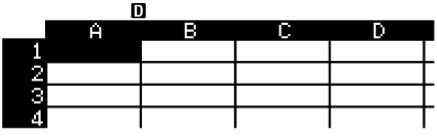
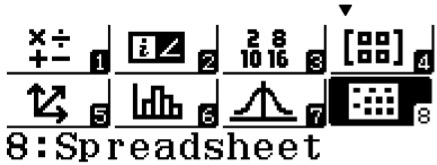
Solution using ClassWiz Press w8

First, fill in cells A1 to A7 with 1, 2, 3, 4, 5, 6, 7.

Next, in cell B1, press T1 to fill formula.

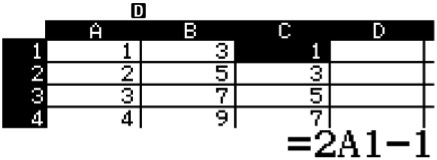
For the Form, press 2Qz1+1=. For the range, press $ to edit “B1:B1” to “B1:B7”. You should see this:

Press = and you should see the list of values for question (a) listed out in column B.



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(a) 3, 5, 7, 9, 11, 13, 15



**ClassWiz steps**

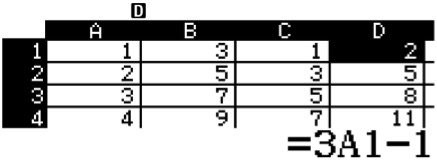
Next, in cell C1, press T1 to fill formula.

For the Form, press 2Qz1p1=. For the range, press $ to edit “C1:C1” to “C1:C7”.

Press = and you should see the list of values for question (b) listed out in column C.

**TEACHER’S GUIDE**

(b) 1, 3, 5, 7, 9, 11, 13



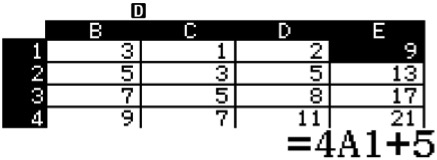
**ClassWiz steps**

Next, in cell D1, press T1 to fill formula.

For the Form, press 3Qz1p1=. For the range, press $ to edit “D1:D1” to “D1:D7”.

Press = and you should see the list of values for question (c) listed out in column D.

(c) 2, 5, 8, 11, 14, 17, 20



**ClassWiz steps**

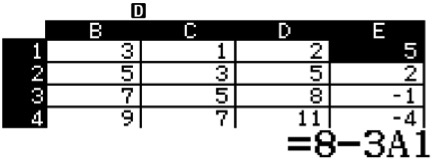
Next, in cell E1, press T1 to fill formula.

For the Form, press 4Qz1+5=. For the range, press $ to edit “E1:E1” to “E1:E7”.

Press = and you should see the list of values for question (d) listed out in column E.

Unit 12 **Recursion, Sequences and Series**

(d) 9, 13, 17, 21, 25, 29, 33



**ClassWiz steps**

As we exhaust the excel spreadsheet space, we can reuse column B,C,D or E. I am going to reuse E.

Next, in cell E1, press T1 to fill formula.

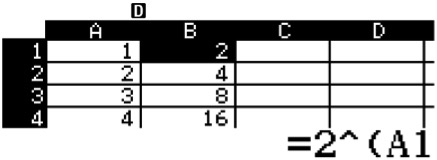
For the Form, press 8p3Qz1=. For the range, press $ to edit “E1:E1” to “E1:E7”.

Press = and you should see the list of values for question (e) listed out in column E.

**TEACHER’S GUIDE**

(e) 5, 2, –1, –4, –7, –10, –13

18.



**ClassWiz steps**

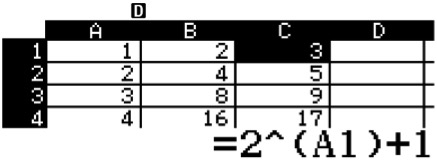
Solution using ClassWiz Press w8

First, fill in cells A1 to A7 with 1, 2, 3, 4, 5, 6, 7. Next, in cell B1, press T1 to fill formula.

For the Form, press 2^Qz1=. For the range, press $ to edit “B1:B1” to“B1:B7”.

Press = and you should see the list of values for question (a) listed out in column B.

(a) 2, 4, 8, 16, 32, 64, 128



**ClassWiz steps**

Next, in cell C1, press T1 to fill formula. For the Form, press

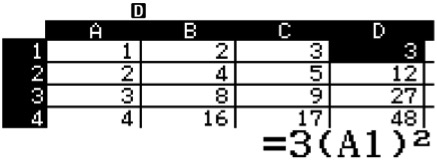
2^Qz1)+1=. For the

range, press $ to edit “C1:C1” to“C1:C7”.

Press = and you should see the list of values for question (b) listed out in column C.

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(b) 3, 5, 9, 17, 33, 65, 129



**ClassWiz steps**

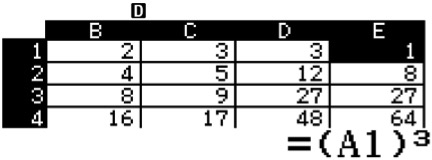
Next, in cell D1, press T1 to fill formula.

For the Form, press 3(Qz1) d=. For the range, press $ to edit “D1:D1” to“D1:D7”.

Press = and you should see the list of values for question (c) listed out in column D.

**TEACHER’S GUIDE**

(c) 3, 12, 27, 48, 75, 108, 147



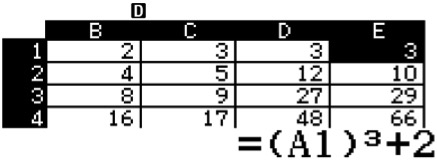
**ClassWiz steps**

Next, in cell E1, press T1 to fill formula.

For the Form, press (Qz1)D=. For the range, press $ to edit “E1:E1” to “E1:E7”.

Press = and you should see the list of values for question (d) listed out in column E.

(d) 1, 8, 27, 64, 125, 216, 343



**ClassWiz steps**

As we exhaust the excel spreadsheet space,

we can reuse column B, C, D or E. I am going to reuse E.

Next, in cell E1, press T1 to fill formula.

For the Form, press (Qz1) D+2=. For the range, press $ to edit “E1:E1” to “E1:E7”.

Press = and you should see the list of values for question (e) listed out in column E.

(e) 3, 10, 29, 66, 127, 218, 345

Unit 12 **Recursion, Sequences and Series**

# Level 2

**TEACHER’S GUIDE**

1. *T*15 = 8 + (15 – 1)(12) = 176

2. *S*20 = [2(18) + (20 – 1)(5)] = 1310

3. *S*10 = [2(2) + (10 – 1)(6)] = 290

4. *S*10 =[2(14) + (10 – 1)(9)] = 545

5. *T*3 = *a* + (3 – 1)*d* = 12 *a* + 2*d* = 12 …(1) *T*5 = *a* + (5 – 1)*d* = 24 *a* + 4*d* = 24 …(2)

Take (1) – (2) –2*d* = –12

*d* = 6

Hence *a* + 2(6) = 12

*a* = 0

*S*10 = [2(0) + (10 – 1)(6)] = 270

6. *T*3 = *a* + (3 – 1)*d* = 20

*a* + 2*d* = 20 …(1)

*T*12 = *a* + (12 – 1)*d* = 56 *a* + 11*d* = 56 …(2)

Take (2) – (1) 9*d* = 36

*d* = 4

Hence, *a* + 2(4) = 20

*a* = 12

*S*20 =[2(12) + (20 – 1)(4)] = 1000

7. *S*10= = 26.999 54…

8. *Sn*= = =

9. =

*x*2 = 1225

*x* = ± 35

10. =

*x*2 = 245

*x* = ± 7

11. *Sn* = (– 1) =

Hence, first term = 3, common ratio = 3.

12. *Sn* = 4[1 – ] =

4 =

4 = 2*a*

*a* = 2

Hence, first term = 2 and common ratio = .

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# Level 3

**TEACHER’S GUIDE**

1. *S*4 = [2*a* + (4 – 1)*d*] = 100

2*a* + 3*d* = 50 …(1)

*S*7 = [2*a* + (7 – 1)*d*] = 238

2*a* + 6*d* = 68 …(2)

Take (1) – (2) – 3*d* = – 18

*d* = 6

2*a* + 3(6) = 50

*a* = 16

*Sn*= [2(16) + (*n* – 1)(6)]

= (6*n* + 26)

= 3*n*2+13*n*

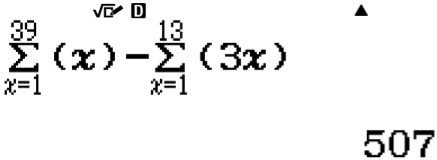
2. *n* – 3*n*

|  |  |
| --- | --- |
|  | **ClassWiz steps** |
|  |
|  | Solution using ClassWiz: |
|  | Press w1 and press q[ for the summation |
|  | function. |
|  | Press [$1$39 and you will see this: |
|  | Press $pq[ for a new summation function. |
|  | Press 3[$1$13= and the |
|  | calculator should display 507. |

= [2(1) + (39 – 1)(1)] – [2(3) + (13 – 1)(3)]

= 780 – 273

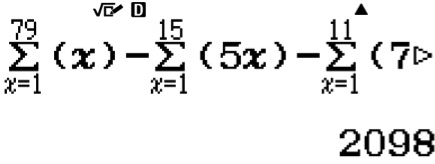
= 507



3.*n* – 5*n* – 7*n*

= [2(1) + (79 – 1)(1)] – [2(5) + (15 – 1)] – [2(7) + (11 – 1)]

= 3160 – 600 – 462



|  |  |
| --- | --- |
|  | ClassWiz steps  Solution using ClassWiz: |
|  |
|  | Press w1 and press q[ for the summation function. |
|  | Press [$1$79. |
|  | Press $pq[ for a new summation function. |
|  | Press 5[$1$15. |
|  | Press $pq[ for a new summation function. |
|  | Press 7[$1$11=. Your calculator should display 2098. |

= 2098

Unit 12 **Recursion, Sequences and Series**

1. Let the three numbers be *a*, *ar*, *ar*2. *a* + *ar* + *ar*2 = 26

**TEACHER’S GUIDE**

*a*(1 + *r* + *r*2) = 26 …(1)

*a*(*ar*)(*ar*2) = 216

*a*3*r*3 = 216

*ar* = 6

*a* = …(2)

Sub (2) into (1) ( )(1 + *r* + ) = 26

6 + 6*r* + 6*r*2 = 26*r*

6 – 20*r* + 6*r*2 = 0

3 – 10*r* + 3*r*2 = 0 (3*r* – 1)(*r* – 3) = 0

*r* = or 3

When *r* = , *a* = 18

Hence the three numbers are 18, 6, 2 When *r* = 3, *a* = 2

Hence the three numbers are 2, 6, 18.

1. Let the three numbers be *a*, *ar*, *ar*2. *a* + *ar* + *ar*2 = 13

*a*(1 + *r* + *r*2) = 13 …(1)

*a*(*ar*)(*ar*2) = 217

*a*3*r*3 = 27

*ar* = 3

*a* = …(2)

Sub (2) into (1) ( )(1 + *r* + ) = 13

3 + 3*r* + 3*r*2 = 13*r*

3 – 10*r* + 3*r*2 = 0 (3*r* – 1)(*r* – 3) = 0

*r* = or 3

When *r* = , *a* = 9

Hence the three numbers are 9, 3, 1 When *r* = 3, *a* = 1

Hence the three numbers are 1, 3, 9.

1. = = = 4[1 –

For 4[1 – ] > 3, 1 – >

<

*n* ln ( ) > ln ( )

a negative value.

taking ln on and gives

There is a sign change as

*n* >

*n* > 4.8188

*n* = 5

At least 5 terms are needed for the sum of these terms to exceed the value of 3.