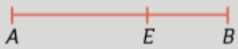
09 | Identification type of numbers
Numerical properties of the golden number

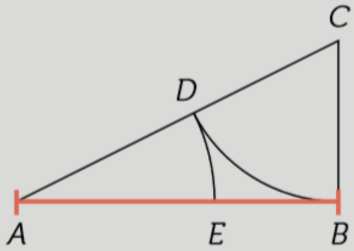
Be a segment $\overline{AB}$ and an interior point E that divides it into two segments $\overline{AE}$ and $\overline{EB}$. It is said that point E divides the $\overline{AB}$ segment in golden proportion (or in average and extreme ratio) if it is fulfilled that:



$$\frac{\overline{AB}}{\overline{AE}} = \frac{\overline{AE}}{\overline{EB}} = \Phi$$

The ratio of proportionality Φ is known as a gold number or a golden number. To perform the golden division of an $\overline{AB}$ segment, proceed as follows:

1. Draw segment $\overline{BC}$ perpendicular to $\overline{AB}$ such that $\overline{BC} = \frac{1}{2}\overline{AB}$
2. Draw segment $\overline{AC}$
3. Draw an arc with radius $\overline{BC}$ from C and mark the point of intersection D with the segment to $\overline{AC}$.
4. An arc is drawn from A of radius $\overline{AD}$ and is considered the point of intersection, E, with segment $\overline{AB}$.



You can check that $\frac{\overline{AB}}{\overline{AE}} = \frac{\overline{AE}}{\overline{EB}} = \Phi$ with $\Phi = \frac{1+\sqrt{5}}{2}$

Show the following properties of the gold number:

1 It is the positive solution of the quadratic equation $x^2 - x - 1 = 0$.

2 Verify the following equalities: $\Phi^2 = 1 + \Phi$ and $\frac{1}{\Phi} = \Phi - 1$.

3 Its successive powers form a succession of Fibonacci.

4 $\lim \sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1..}}}} = \Phi$

5 $\lim 1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \dots}}}} = \Phi$