

09 | Identification type of numbers

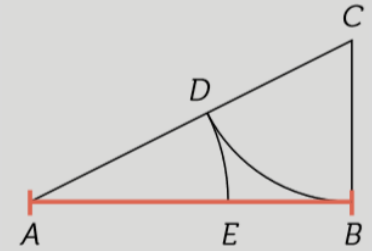
Numerical properties of the golden number

Be a segment \overline{AB} and an interior point E that divides it into two segments \overline{AE} and \overline{EB} . It is said that point E divides the \overline{AB} segment in golden proportion (or in average and extreme ratio) if it is fulfilled that:

$$\overline{AB} \quad \overline{AE} \quad \overline{EB} \quad \frac{\overline{AB}}{\overline{AE}} = \frac{\overline{AE}}{\overline{EB}} = \Phi$$

The ratio of proportionality Φ is known as a gold number or a golden number. To perform the golden division of an \overline{AB} segment, proceed as follows:

1. Draw segment \overline{BC} perpendicular to \overline{AB} such that $\overline{BC} = \frac{1}{2}\overline{AB}$
2. Draw segment \overline{AC}
3. Draw an arc with radius \overline{BC} from C and mark the point of intersection D with the segment to \overline{AC} .
4. An arc is drawn from A of radius \overline{AD} and is considered the point of intersection, E, with segment \overline{AB} .



You can check that $\frac{\overline{AB}}{\overline{AE}} = \frac{\overline{AE}}{\overline{EB}} = \Phi$ with $\Phi = \frac{1+\sqrt{5}}{2}$

Show the following properties of the gold number:

1 It is the positive solution of the quadratic equation $x^2 - x - 1 = 0$.

2 Verify the following equalities: $\Phi^2 = 1 + \Phi$ and $\frac{1}{\Phi} = \Phi - 1$.

3 Its successive powers form a succession of Fibonacci.

4 $\lim \sqrt{1 + \sqrt{1 + \sqrt{1 + \sqrt{1 + \dots}}}} = \Phi$

5 $\lim 1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \dots}}}} = \Phi$

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